Informed Traders as Liquidity Providers:

Transparency, Liquidity and Price Formation

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Abstract

I study the consequences of pre-trade transparency on liquidity in an order-driven market with informed and uninformed risk-averse investors and liquidity traders. For a given proportion of informed agents, the more transparent the market, the more liquid it is. With endogenous information acquisition this result can be reversed, since greater transparency reduces the incentive to acquire information and thus lowers the equilibrium number of informed traders. This can reduce liquidity, since in this model informed agents are natural liquidity providers. Informed agents are in a good position to accommodate the liquidity shocks of liquidity traders, because they are not exposed to adverse selection. That transparency can have ambiguous consequences is in line with the results of empirical studies.

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Introduction

The optimal degree of transparency is an important and controversial issue in market design. A number of theoretical studies have shown that transparency can enhance liquidity by mitigating adverse selection (see e.g. Admati and Pfleiderer (1991), Pagano and Röell (1996), and Foster and George (1992)). While these analyses are corroborated by the empirical findings of Theissen (2000) and Harris and Schultz (1997), other empirical studies have found that transparency can reduce liquidity (see Madhavan, Porter and Weaver (1999), Garfinkel and Nimalendran (1998), and Albanesi and Rindi (2000)). Experimental works further suggest that the relationship between transparency and liquidity is complex and ambiguous (see Bloomfield and O’Hara (1999 and 2000), Flood, Huisman, Koedjik and Mahieu (1999), and Perotti and Rindi (2002).)

The present paper offers a theoretical analysis of price formation in an order-driven market, showing why transparency may either increase or reduce liquidity. Like that of Grossman and Stiglitz (1980), my model contemplates informed and uninformed investors who are rational and risk-averse, and liquidity traders; the proportion of informed agents is endogenized, as traders initially decide whether or not to acquire private information, at a
cost. However, there are two fundamental differences from the analysis of Grossman and Stiglitz. First, informed agents not only speculate on their private information but also hedge. Random hedging introduces noise and thus prevents full information revelation. Second, I allow for different regimes of pre-trade transparency. In the anonymous regime, traders can only observe the market-clearing price; under full transparency, they observe both the order flow and personal markers; with partial transparency, they observe all limit and market orders, but not the identities of participants.

I obtain the following results: for a given proportion of informed agents, the more transparent the market, the more liquid it is, a finding in line with earlier theoretical analyses. With endogenous information acquisition, however, this result can be reversed. The reason is that informed and uninformed agents placing limit orders accommodate the liquidity shocks of the liquidity traders. The informed agents are in a particularly good position to supply liquidity, as they do not face adverse selection costs. While uninformed agents are reluctant to accommodate large market orders, suspecting that they are informational motivated, informed agents have no reason to fear. Now, greater transparency reduces the incentives to acquire information and consequently, the proportion of informed agents, hence the amount
of liquidity they supply.

This paper also relates to two prolific strands in the literature: that dealing with information acquisition and aggregation (Fishman and Hagerty 1992, Mendelson and Tunca 2001) and that dealing with the effects of a change in the equilibrium number of informed traders on market quality (see e.g. Holden and Subrahmanyam, 1992). In all these works, a number of competitive uniformed market makers set a price equal to the conditional expected future value of the asset and face either one single insider and a continuum of risk-averse liquidity traders or else a number of insiders and a group of noise traders. In this setting, as the number of insiders increases, so do market makers’ adverse selection costs, and liquidity diminishes. In my framework, instead, as in Kyle (1989) and in Grossman and Stiglitz (1980), liquidity is provided by both uninformed and informed traders, and since the informed traders have the lowest adverse selection costs, an increase in their number may increase rather than decrease liquidity.

Section 1 presents the model with competitive agents, Section 2 extends the model to imperfect competition, and Section 3 concludes.
1 The model with competitive agents

The market consists of two groups of risk-averse agents, N informed and M uninformed, and by Z risk-neutral noise traders. In this Section the model is solved under the assumption that agents are price takers. In Section 2 this hypothesis is removed, and that all risk-averse agents are posited to behave strategically. Let $X_I, X_U$ and $x$, with $x \sim N(0, \sigma_x^2)$, be the size of the informed, uninformed and noise traders’ orders respectively. Agents trade a single risky asset with liquidation value equal to:

$$F = S + \varepsilon, \quad F \sim N(0, \sigma_S^2 + \sigma_\varepsilon^2)$$

As in Glosten (1989) and Madhavan and Panchapagesan (2000) at the beginning of the game informed traders receive an endowment shock equal to $I$, and a signal, $S$, on the future value of the asset, both normally distributed with zero mean and variance equal to $\sigma_I^2$ and $\sigma_S^2$ respectively. The endowment shock is a source of noise that prevents full information revelation, since it induces the informed to act both as hedgers and as speculators. Assuming CARA utility function, the informed trader maximizes end-of-period wealth.
and submit the following demand function:

$$X_I = \frac{E(F|S) - p}{AVAR(F|S)} - I$$  \hspace{1cm} (1)

with $E(F|S) = S$ and $VAR(F|S) = \sigma^2$, where $A$ is the coefficient of risk aversion and $p$ is the market price.

Uninformed traders form conjectures on the equilibrium price and update their expectations on the liquidation value of the asset by extracting a signal from the current price. They receive an endowment shock equal to $I_U$ and submits the following demand function:

$$X_U = \frac{E(F|p) - p}{A \cdot VAR(F|p)} - I_U$$  \hspace{1cm} (2)

A linear rational-expectation equilibrium implies that the equilibrium price observed in the market is a linear combination of $S$, $I$, $I_U$ and $x$. Substituting equations 1 and 2 into the market clearing condition,

$$N X_I + M X_U + Z x = 0,$$  \hspace{1cm} (3)

and solving for $p$, we derive the equilibrium price function and three indi-
cators of market quality: liquidity, \( \left| \frac{dp}{dx} \right|^{-1} \), volatility, \( Var(p) \), and informational efficiency, \( IE = \frac{1}{Var(F|p)} \). Sections 1 compares market quality under three different regimes of pre-trade transparency, Section 2 shows that the results do not change qualitatively when it is assumed that agents behave strategically rather than competitively. Under perfect competition, different regimes of transparency do not affect the strategies of the informed traders, who already have the most precise signal available; but under imperfect competition, where agents take the impact of their own trade into account, transparency modifies the demand elasticity of market price and so influences agents’ perception of their price impact. However, I show that the effects of pre-trade transparency on market quality remain qualitatively the same under imperfect competition.

Since we use the price impact of a noise trader’s order, \( L = \left| \frac{dp}{dx} \right|^{-1} \), as a measure of liquidity (Kyle, 1985), in order to see how transparency affects liquidity one should look at the market clearing condition (3) and consider how the system described by this model reacts to a noise trader’s order. Equation (4) is obtained by substituting the informed and uninformed demand functions (1) and (2) into (3).
\[
N \left[ \frac{S}{\sigma^2} \right] \frac{p}{\sigma^2} - \frac{\sigma^2}{\sigma^2} SELL - I + M \left[ \frac{E(F|p)}{VAR(F|p)} \right] \frac{p}{VAR(F|p)} \frac{p}{VAR(F|p)} - I_U \right] + Z \begin{bmatrix} x \end{bmatrix} = 0
\]

\[X_I : Informed trader’s net demand (response)\]
\[X_U : Uninformed trader’s net demand (response)\]

Assume as an example that a noise trader makes a buy order, \( dx > 0 \), which causes a price increase. The overall price impact depends on two effects: first, the willingness of all other agents to take the other side of the noise trader’s order by selling; and second, on the uninformed traders’ revision of the future value of the asset, \( E(F|p) \), which is revised in the same direction as price changes. Assume as an example that a noise trader makes a buy order, \( dx > 0 \), which causes a price increase. The overall price impact depends on two effects: first, the willingness of all other agents to take the other side of the noise trader’s order by selling; and second, on the uninformed traders’ revision of the future value of the asset,
i.e. on their willingness to place a buy order. Uninformed traders make buy orders because they misinterpret the price rise as due to the arrival of new information, not to noise trading. It is precisely this effect that reduces liquidity under anonymity. Under full transparency, by contrast, after a noise trader’s buy order, uninformed traders do not put their estimate of the future value of the asset up, since they can recognize it as a non-informative order and thus they do not amplify the price impact by placing buy orders.

Equation 4 shows that informed traders are the best liquidity providers and helps explain why liquidity increases with transparency. With full transparency, uninformed traders become "quasi informed" and, incurring lower risk-bearing and no adverse selection costs, they improve liquidity provision.

This intuition explains the present conclusion that pre-trade transparency increases liquidity. Likewise, it explains the opposite result obtained when the entry of informed traders is endogenous. When the number of informed traders is exogenous, transparency increases liquidity by making everyone better informed and thus more willing to supply liquidity to noise traders. But when entry is endogenous, by making everybody better informed, transparency reduces traders’ incentive to acquire information and thus lowers the equilibrium number of informed traders, who are the best providers
of liquidity. In Bloomfield, O’Hara and Saar’ (2002) words: "the superior information of the informed also makes these traders better able to provide liquidity to other traders in the market".

1.1 Anonymity

I first solve the model with anonymity where uninformed traders do not observe either the size of other traders’ orders, or their identity. They merely infer the future value of the asset from the price, by extracting the signal \( \Theta = S + \frac{A\sigma^2}{N}x - A\sigma^2 I \). Formally,

**Proposition 1** The following equilibrium price \((p^A)\) and indicators of market quality characterize the anonymity regime.

\[
p^A = \lambda_A \left[ \frac{N}{A\sigma^2} S - NI - M\Psi I_U + Zx \right]
\]

with \( \Psi = \left[ 1 + \frac{M\sigma^2 \sigma_S^2}{NVar(\Theta)Var(F|\Theta)} \right]^{-1} \)
\[ LA = \frac{1}{\lambda_A} = \]
\[ = \left[ \frac{N}{\text{AVar}(F|S)} + MH \right] = \]
\[ = \left[ \frac{N}{A\sigma_z^2} + M \frac{1 - \text{Cov}(F, \Theta)}{\text{AVar}(F|\Theta) + \frac{\text{Cov}(F, \Theta)MA\sigma_z^4}{N}} \right] \]

\[ IE_A = [\text{Var}(F|\Theta)]^{-1} = \]
\[ = \left[ \sigma_s^2 + \sigma_\varepsilon^2 - \frac{\sigma_A^4}{\sigma_s^2 + A^2\sigma_\varepsilon^4 + \frac{A^2\sigma_\varepsilon^4\sigma_I^2 + Z^2\sigma_x^2}{N^2\sigma_z^2}} \right]^{-1} \]

\[ \text{Var}(p^A) = (\lambda_A)^2 \left( \frac{N^2}{A^2\sigma_\varepsilon^4} \sigma_S^2 + N^2\sigma_I^2 + M^2\Psi^2\sigma_{Iv}^2 + Z^2\sigma_x^2 \right) \]

Proof: See Appendix.

Under the anonymous regime, market liquidity \((LA)\), the inverse of the price impact of a noise trader’s order \((\lambda_A)\), is formed by two terms. The first, \(\frac{N}{\text{AVar}(F|S)}\), corresponds to the contribution of the \(N\) informed traders to liquidity; the second, \(MH\), to that of the \(M\) uninformed traders.
Notice that both terms are inverse functions of the conditional variance of $F$, which gauges the risk-bearing costs of risk-averse agents and is greater for uninformed than for informed traders, $\text{Var} (F | S) < \text{Var} (F | \Theta)$. Note, further, that $H$ is the contribution of uninformed traders to liquidity in the presence of both risk-bearing and adverse selection costs, while $\frac{1}{\text{Var} (F | \Theta)}$ would be their contribution without adverse selection costs; it follows that the difference $\left[ \frac{1}{\text{Var} (F | \Theta)} - H \right]$ captures the reduction in uninformed traders’ willingness to offer liquidity due to adverse selection costs and so can be used as a measure of these costs.

$IE_A$, the inverse of the conditional variance of $F$, measures informational efficiency. The more precise the signal traders get from the equilibrium price, the lower the residual variance of the future value of the asset and the more informative the equilibrium price. Finally, note that volatility, $\text{Var}(p)$, depends on the price impact parameter and on the variance of the informed traders’ signal, endowment shocks and noise.

Section 1.2 shows how pre-trade transparency affects these indicators of market quality.
1.2 Full Transparency

Under a regime of full transparency, agents observe traders’ identities and their demands. Uninformed traders extract a signal equal to $\Theta^0_T = S - A\sigma_z^2 I$ from the informed trader’s order $X_I$ and update their expectations on the liquidation value of the asset.

**Proposition 2** *In the full transparency regime*

the equilibrium price is:

$$ p^T = \lambda_T \left[ (\frac{N + M\Omega}{A\sigma_z^2})S - (N + M\Omega)I - MI_U + Zx \right] $$

(9)

with $\Omega = \begin{pmatrix} \text{Cov}(F, \Theta^0_T) & \frac{A\sigma_z^2}{\text{Var}(\Theta^0_T)} \\ \frac{A\sigma_z^2}{\text{Var}(\Theta^0_T)} & \text{Var}(F|\Theta^0_T) \end{pmatrix}$.

Liquidity and informational efficiency are higher than under anonymity:

$$ LT = \frac{1}{\lambda_T} = \left[ \frac{N}{\text{Var}(F|S)} + \frac{M}{\text{Var}(F|\Theta^0_T)} \right] > LA $$

(10)

$$ IE_T = [\text{Var}(F|\Theta^0_T)]^{-1} = \left[ \sigma_s^2 + \sigma_z^2 - \frac{\sigma_s^4}{(\sigma_s^2 + A^2\sigma_z^2\sigma_f^2)^2} \right]^{-1} > IE_A $$

(11)
Depending on the parameter values, under full transparency volatility can be greater than under anonymity:

\[
Var(p^T) = (\lambda_T)^2(\frac{(N + M\Omega)^2}{A^2\sigma^4} \sigma^2_z + (N + M\Omega)^2\sigma^2_I + M^2\sigma^2_{I_I} + Z^2\sigma^2_x) \geq Var(p^A)
\]

(12)

Proof: see Appendix.

With full transparency, uninformed traders behave as if they were informed holding a signal, \(\Theta^0_T\), which is a noisy version of the informed trader’s signal. The more precise the signal they can extract from the equilibrium price, the lower both the risk-bearing and adverse selection costs are, and the greater liquidity is.

Comparing the indicators of liquidity under anonymity and under full transparency, \(LA\) and \(LT\), we can see how transparency affects the two components of trading costs, risk-bearing and adverse selection. The first term in \(LT\) is the contribution of informed traders to liquidity, which is unaffected by the transparency regime. Informed traders do not change their liquidity contribution because of an increase in transparency, since they already possess the best signal and have no adverse selection costs. Pre-trade trans-
parency does affect the contribution of uninformed traders, however. Under full transparency, they too have no adverse selection costs and, as the conditional variance of $F$ is less than under anonymity, they also have lower risk-bearing costs ($AVar(F|\Theta_T') < AVar(F|\Theta)$).

It follows that under full transparency, uninformed traders’ costs decrease by:

$$
\Xi = \left\{ \left[ AVar(F|\Theta_T') \right]^{-1} - AVar(F|\Theta)^{-1} \right\} + \left[ \frac{1}{AVar(F|\Theta)} - H \right] \quad (13)
$$

where the first term shows the reduction of risk-bearing costs and the second measures the adverse selection costs under anonymity. Figures 1, 2 and 3 plot the total cost reduction and the two components respectively. As expected, the adverse selection costs are a positive function of the number of informed traders and a negative function of the noise; the reverse holds for risk-bearing cost reduction.

Inspection of equations 7 and 11 shows that under full transparency the conditional variance of the liquidation value of the asset is less and informational efficiency is greater. Finally, Figure 4 shows that the difference between $Var(p^T)$ and $Var(p^A)$ is positive. Intuitively, in transparency unin-
formed traders behave as "quasi-insiders", which makes the number of "informative shocks" greater than under anonymity. Figure 4 also shows that the difference between \( Var(p^T) \) and \( Var(p^A) \) is an increasing function of \( \sigma_x^2 \) and \( Z \). An increase in the variance of the noise, \( \sigma_x^2 \), or in \( Z \) produces two effects: it directly increases one component \( (Z^2 \sigma_x^2) \) of volatility under both regimes, thus increasing both \( Var(p^T) \) and \( Var(p^A) \); in addition, it reduces the price impact under anonymity \( (\lambda_A) \), the latter effect reducing \( Var(p^A) \) and not \( Var(p^T) \).

### 1.3 Partial Transparency

Under a regime of partial transparency, agents observe prices and quantities but not personal markers. As a consequence, the order that an uninformed trader observes, \( \theta_{pT}^0 \), comes with probability \( \frac{N}{N+Z} \) from an informed trader and with probability \( \frac{Z}{N+Z} \) from a noise trader. Uninformed traders observe a realization of the following random variable:

\[
\{ \Theta_{pT}^0 | \Theta_{pT}^0 \neq X_U \} = qX_I + (1-q)x \quad \text{with} \quad q \sim \begin{cases} 0 & \frac{Z}{N+Z} \\ 1 & \frac{N}{N+Z} \end{cases}
\]  

Moreover, uninformed agents extract the signal \( \Theta_{pT} = S - A\sigma_x^2 I + \frac{Z\lambda}{N}x \)
from the current price by using their conjecture on other uninformed agents’ orders, \( X_U^{PT} = -H^{PT} p + \Omega^{PT} \theta^{0}_{PT} \), and the market clearing condition\(^3\). It follows that uninformed traders use the two signals, \( \Theta_{PT} \) and \( \Theta^{0}_{PT} \), to update their beliefs and evaluate \( E[F|\Theta_{PT}, \Theta^{0}_{PT}] \), which is derived in Lemma 2 in the Appendix.

**Proposition 3** *In the partial transparency regime:*

I) the equilibrium price is:

\[
p^{PT} = \lambda_{PT} \left[ \frac{N + M\Omega^{PT} q}{A\sigma^2} \right] S - (N + M\Omega^{PT} q) I - M\Psi^{PT} I_U + (M\Omega^{PT} (1-q) + Z)x \]

(15)

with \( \lambda_{PT} = \left[ \frac{N}{A\sigma^2} + MH^{PT} + \frac{M\Omega^{PT} q}{A\sigma^2} \right]^{-1} \);

II) the price impact is:

\[
LA > E[LPT(q)] > LT
\]

with \( E[LPT(q)] = \frac{\lambda_{PT}^{-1}}{M\Omega^{PT} (1-q) + Z} + 1 \)

(16)
Proof: see Appendix.

Figure 5 shows that with partial transparency, the higher the proportion of informed \((N)\) with respect to noise traders \((Z)\), the greater liquidity is. When the number of informed traders is relatively high, uninformed traders can extract a better signal of the value of the asset from the orders they observe and thus become more willing to offer liquidity.

1.4 Endogenous information acquisition

In this section we assume that informed traders pay a fixed cost equal to \(C\) to capture their signal. They decide to acquire information if the expected utility from trading exceeds that from hedging the endowment shock.

The condition for being informed is:

\[
E[-\exp(-A(\Pi_I - C))] \geq E[-\exp(-A(\Pi_U))]
\]  

(17)

which is derived in Lemma 3.

Lemma 3 When the informed trader can decide either to buy a costly signal and speculate, or not to buy the signal and trade in order to hedge the endowment shock, the equilibrium number of informed traders can be
derived from the following condition.

\[- \exp(AC) \frac{\Upsilon_z}{\sqrt{1 - 2\sigma_U^2(d_z + h_z^2 \sigma_e^2 + L_z F_z^2 + R_z \Lambda_z^2 + J_z U_z^2)}} = (18)\]

\[- \frac{\Upsilon_{zu}}{\sqrt{1 - 2\sigma_{Uz}^2(d_{zu} + h_{zu}^2 \sigma_e^2 + L_{zu} F_{zu}^2 + R_{zu} \Lambda_{zu}^2 + J_{zu} U_{zu}^2)}} \]

**Proof:** see Appendix.

Using Lemma 3, one can solve the model for the equilibrium number of informed traders under the two regimes, (anonymity and full transparency) and evaluate the impact of transparency on market quality (Table 1). The next Proposition summarizes these results.

**Proposition 4** With endogenous information acquisition, the equilibrium number of informed traders and liquidity are higher under anonymity than under full transparency. The results on volatility are mixed.

Tables 1, 1.1 and 1.2 show the equilibrium number of informed traders and the associated difference in liquidity \((LA - LT)\) and volatility \((VA - VT)\) under anonymity, \(N_A\), and under full transparency, \(N_T\), for different values of the parameters \(\sigma_S^2, \sigma_e^2, \sigma_I^2, \sigma_z^2, M, Z, A\) and \(C\). Table 1 shows the results for the model\(^4\) with \(\sigma_U^2 = 0\); Tables 1.1 and 1.2 extend the analysis to different
values of $\sigma_U^2$. Transparency reduces agents’ incentive to acquire information, lowering the equilibrium number of informed traders and decreasing liquidity. A higher $C$ makes information more expensive and the equilibrium number of informed traders smaller. Tables 1 and 1.1 also show that when the number of market participants $(M, Z)$ doubles from 10 to 20, the equilibrium number of informed traders also doubles. In addition, the higher the number of market participants, the stronger are these results, which makes the model with competitive agents robust. Intuitively, the higher the number of uninformed traders, the greater the profits that informed traders can extract from their private information, hence the greater the incentive to acquire such information. The same intuition explains the effect of an increase of $\sigma_x$ (from 0.5 to 0.8) on the equilibrium number of informed traders. Conversely, when $\sigma_S$ decreases from 0.5 to 0.3 (Tables 1 and 1.2), the value of the informed traders’ signal decreases and the equilibrium number of informed traders is generally lower. Lowering the coefficient of risk aversion, $A$, from 2 to 1.9 produces two effects: on the one hand, it makes agents more aggressive when trading on their private information; on the other, it reduces their risk-sharing needs and therefore their incentive to enter the market. The results show that when agents can choose to enter the market and buy information or else to
stay out, the second effect prevails and they enter the market mainly in order to share the higher risk that they perceive; conversely, when the decision to depends only on their willingness to exploit their private information, as they can hedge their endowment anyhow, \( N_A \) increases. Futhermore, a reduction in \( \sigma_U^2 \) (Table 1.2) lowers the equilibrium number of informed traders making the results obtained assuming \( \sigma_U^2 = 0 \) robust. However, it is hard to interpret the consequences of an increase in \( \sigma_U^2 \) and \( \sigma_I^2 \), since many different effects occur simultaneously.

Even if the decision to enter the market is generally associated with the opportunity to buy information, which explains the focus on the endogenous entry of informed traders, it is interesting to check whether the results change when the equilibrium number of uninformed traders is made endogenous\(^5\). Clearly, uninformed traders benefit from the enhanced informational efficiency induced by full transparency and have more incentive to enter a transparent market than an anonymous market. Since this effect increases liquidity, we compare the liquidity effect of an increase in the number of informed traders with that of a reduction in the number of uninformed. Table 2 tells us that for the same parameter values, when transparency increases the reduction of liquidity due to the smaller number of informed
traders \((LAn - LTn)\) outweighs the increase due to the entry of additional uninformed investors \(|(LAm - LTm)|\).

2 The model with strategic traders

Up to here we have posited that risk-averse agents are price takers. Now let us assume that agents behave strategically and weigh the effects of their orders on the equilibrium price. This assumption is especially important in considering the effect of transparency on market depth, which depends on the aggressiveness of liquidity providers. If the latter take the price impact of their trade into account, they will scale back their orders accordingly. Conversely, if they behave competitively they will submit more aggressive orders. We have seen that transparency makes uninformed agents more informed about the value of the asset and thus makes them more willing to provide liquidity and serve as a counterpart to the noise traders. When agents act competitively the effect is stronger, since under imperfect competition agents trade less aggressively.

Proposition 5 Under the assumption that risk-averse traders behave strategically, the equilibrium prices and indicators of liquidity that characterize the
anonymous and fully transparent regimes respectively, are as follows:

anonymity:

\[ p^S = [LSA]^{-1}[NDS - NGI - M\Psi^{SA}I_U + Zx] \]  \hspace{1cm} (19)

\[ LSA = [ND + MH^S]) = \]  \hspace{1cm} (20)

\[ = \frac{N}{A\sigma_\varepsilon^2 + \lambda_I} + M \left( 1 - \frac{\text{Cov}(F, \Theta_S)}{\text{Var}(\Theta_S)} \right) \frac{1 - \frac{\text{Var}(F|\Theta_S) + \lambda_U}{\text{Var}(\Theta_S)} + \frac{\text{Cov}(F, \Theta_S)M}{\text{Var}(\Theta_S)}(A\sigma_\varepsilon^2 + \lambda_I)}{\text{Var}(F|\Theta_S + \Theta_0) + \lambda_U} \]

full transparency:

\[ p^{ST} = [LST]^{-1}[(ND^T + M\Omega^{ST}D^T)S - (NG + M\Omega^{ST})I - M\Psi^{ST}I_U + Zx] \]  \hspace{1cm} (21)

\[ LST = [ND^T + MH^{ST} + M\Omega^{ST}D^T] = \]  \hspace{1cm} (22)

\[ = \left[ \frac{N}{A\sigma_\varepsilon^2 + \lambda_I} + \frac{M}{A\text{Var}(F|\Theta_{ST}, \Theta_0^{ST}) + \lambda_U} \right] \]
Proof: see Appendix.

Proposition 6 Strategic behavior reduces the effect of pre-trade transparency on liquidity.

Tables 3.1 through 3.5 compare liquidity \( (L) \) under anonymity \((An.)\) and full transparency \((Tr.)\) on the assumption that agents behave either strategically or competitively. These tables also report the values of the parameters \( \lambda_I^{1,T}, \lambda_U^{1,T}, H^{S,ST}, D^{1,T} \) and \( \Omega^{ST} \). Independently of the regime under analysis and of the parameter values, liquidity is greater when agents behave competitively. In fact, when agents ignore the price impact of their demand, they trade more aggressively, i.e. are more willing to accommodate a noise trader’s order. Looking at Tables 3.1-3.5, one notes that both \( H^{S,ST} \) and \( D^{1,T} \), which measure respectively uninformed and informed traders’ price sensitivity, are always higher when traders behave competitively. However, what drives the results is the assumption on the behavior of informed rather than uninformed traders. Take as an example Table 3.1. Under the anonymous regime, with strategic agents liquidity is 6.309, with competitive traders 7.222. Nevertheless, when only uninformed traders are strategic \( (\lambda_I = 0) \), \( L \) is 7.044, and when only informed traders are strategic \( (\lambda_U = 0) \), \( L \) decreases to 6.566. Notice that this result holds independently of the
relative number of $N$, $M$ and $Z$ (Tables 3.2–3.5). Notice also that when the number of informed traders is four times higher than the number of uninformed traders, liquidity is three times higher (Table 3.2), whereas when $M$ and $Z$ increase to 20, liquidity increases by only 30% and 10% respectively (Table 3.3 and 3.4). As expected, the difference in liquidity under the two regimes diminishes with the number of market participants (Table 3.5). As was explained above, when switching from anonymity to full transparency, liquidity increases because uninformed traders now get a signal similar to that of informed traders. When transparency increases, uninformed traders learn to identify liquidity-motivated orders, albeit imperfectly, and become more willing to supply liquidity. Since traders make more aggressive orders when they behave competitively, the increase in liquidity produced by transparency is greater under competition than under strategic behavior.

3 Conclusion

We have seen how pre-trade transparency affects liquidity and market quality. The issue is analyzed both with competitive and with strategic agents: under the scenario with competitive agents, we allowed the number of informed
traders to be endogenous. This is a crucial feature of the analysis. In fact, when endogenous information acquisition is allowed, the previous results on the effects of pre-trade transparency on liquidity are reversed. I found that transparency lowers the number of informed agents who enter the market and thus reduces liquidity. Most noticeably, I found that the larger the initial number of market participants, the stronger the results. I analyzed the effects of transparency under three different regimes: anonymity, where traders observe only the current price; full transparency, where they also observe personal identities; and partial transparency, where they observe the order flow but not personal identities. Using this set up, we covered all the different degrees of transparency one may find in real markets.

The market is modelled as an open limit-order book where liquidity is provided by all participants. In this framework liquidity, measured as the price impact of a noise trader’s order, depends on two opposite elements: the willingness of traders to accommodate the liquidity shock and where by they update evaluations following the order. When a noise trader places a buy order, the price goes up and the magnitude of the increase will depend on the reaction of the other traders’. On the one hand, after a price increase liquidity suppliers are willing to sell, enhancing liquidity, because
the demand of maximizing traders (both informed and uninformed) is an inverse function of the current price. On the other hand, after a noise trader’s buy order, uninformed traders revise their estimate of the future value of the asset upwards and, increasing their speculative demand, reduce liquidity. It follows that the price impact of the liquidity shock depends on the net effect of these two forces.

Transparency plays a crucial role in this process, since by reducing adverse selection costs it affects uninformed traders’ conditional estimate both of the future price and of its variance. Under full transparency uninformed traders recognize liquidity traders and are therefore more willing to offer liquidity. Hence, transparency increases liquidity. However, when information acquisition is endogenous, transparency reduces the incentive to buy costly information and so reduces the number of informed traders willing to enter the market. This reduces liquidity, since informed agents, with lower adverse selection costs, are the best suppliers of liquidity.

When agents behave strategically, they consider the price impact of their trades and are less willing to supply liquidity; it follows that the effect of transparency on liquidity is smaller.

My theoretical findings are supported by the empirical evidence found in
the literature, which shows mixed results about the effect of pre-trade transparency on liquidity and market quality. In particular, our conclusions here help explain recent empirical results on the Toronto Stock Exchange and the Italian secondary market in government securities (MTS), both screen-based automated exchange. Madhavan, Porter and Weaver (2000) show that after the increase in pre-trade transparency trading costs increase in the Toronto Stock Exchange. Albanesi and Rindi (2000) show that the introduction of anonymity increases liquidity on MTS.

The results obtained in this paper suggest that anonymity may be desirable in automated markets.

As noted above, this model deals with information acquisition and aggregation. Mendelson and Tunca (2001) and Holden and Subrahmanyam (1992) show that in equilibrium, market liquidity and informational efficiency depend on the number of informed traders and on the behavior of the market makers’ counterparts. By contrast, I examine how equilibria in which the type of orders placed by agents is publicly known differ from those in which such information is not known. It would be interesting to extend this analysis by investigating the effects of pre-trade transparency in a dynamic model of price formation where both informed and uninformed traders
act as liquidity suppliers. This is left for future research.

4 Appendix

Proof of Proposition 1.

Each uninformed trader forms a conjecture on other traders’ net demand equal to \( X^A_U = -Hp^A - \Psi I_U \), and, extracting the following signal from the current price,

\[
\Theta = S + \frac{A\sigma^2 Z}{N} x - A\sigma^2 I = \left( \frac{N + A\sigma^2(M-1)H}{N} \right) p^A - \frac{(M-1)\Psi A\sigma^2}{N} I_U - \frac{A\sigma^2}{N} X_U = \gamma_1 p^A + \gamma_2 I_U - \gamma_3 X_U \quad \text{with} \quad \gamma_1 = \left( \frac{N + A\sigma^2(M-1)H}{N} \right), \quad \gamma_2 = \left( \frac{(M-1)\Psi A\sigma^2}{N} \right) \quad \text{and} \quad \gamma_3 = \frac{A\sigma^2}{N},
\]

places the limit order \( X^A_U = E(F|\Theta) - \frac{p^A}{\text{Var}(F|\Theta)} - I_U = \frac{\delta_1 (\gamma_1 p^A + \gamma_2 I_U - \gamma_3 X_U) - p^A}{\text{Var}(F|\Theta)} - I_U, \)

with \( \delta_1 = \frac{\text{Cov}(F, \Theta)}{\text{Var}(\Theta)} = \frac{\sigma^2}{\sigma^2 + A^2\sigma^4 \sigma^2 + \frac{A^4\sigma^2 Z^2}{N^2} \sigma^2} \)

and \( \text{Var}(\Theta) = \sigma^2 + A^2\sigma^4 \sigma^2 + \frac{A^4\sigma^2 Z^2}{N^2} \sigma^2 \).

Solving for \( X^A_U \) and equating the parameters of the realized demand to \( H \) and \( \Psi \), we get:

\[
X^A_U = -\begin{bmatrix} \frac{1 - \text{Cov}(F, \Theta)}{\text{Var}(\Theta)} \frac{\text{Cov}(F, \Theta)}{\text{Var}(\Theta)} \frac{\text{Var}(F|\Theta)}{\text{Var}(\Theta)} \end{bmatrix} p^A - \begin{bmatrix} 1 + \frac{M\sigma^2 \sigma^2}{N\text{Var}(\Theta)\text{Var}(F|\Theta)} \end{bmatrix} I_U
\]

Substituting this equation and equation 1 into 3 and solving for \( p \) we obtain equation 5.

Proof of Proposition 2.

With full transparency each uninformed agent forms a conjecture on the other
uninformed traders’ net demand equal to \( X_U^T = -H^T p^T + \Omega \theta_T^0 - \Psi^T I_U \) and extracts the following signal from the market price, \( p^T: \Theta_T = S - A \sigma_e^2 I + \frac{A \sigma_e^2 Z}{N} x = -\frac{A \sigma_e^2}{N} X_U - \frac{(M-1)A \sigma_e^2 \Psi^T}{N} I_U + \frac{N + A \sigma_e^2 (M-1) H^T}{N} p^T - \frac{A \sigma_e^2 (M-1) \Omega}{N} \theta_T^0 = \gamma_1 T p^T - \gamma_3^T X_U - \gamma_4^T \theta_T^0 = \theta_T \) with \( \gamma_1^T = \frac{N + A \sigma_e^2 (M-1) H^T}{N}, \gamma_2^T = -\frac{(M-1)A \sigma_e^2 \Psi^T}{N} \gamma_3^T = \frac{A \sigma_e^2}{N} \) and \( \gamma_4 = \frac{A \sigma_e^2 (M-1) \Omega}{N} \). Since the trader can observe personal identities, he or she can also extract the signal \( \Theta_T^0 = S - A \sigma_e^2 I = A \sigma_e^2 \theta_T^0 + p^T \) from the informed trader’s demand, \( X_I \). One should notice that \( \theta_T^0 \) is a realization of \( X_I \).

**Lemma 1:** \( \Theta_T^0 \) is a sufficient statistic for \( \Theta_T \).

**Proof:**

\[
E \left[ F | \Theta_T, \Theta_T^0 = S - A \sigma_e^2 I \right] = \begin{bmatrix} \text{Cov}(F, \Theta_T) & \text{Cov}(F, \Theta_T^0) \end{bmatrix}^* \begin{bmatrix} \text{Var}(\Theta_T) & -\text{Cov}(\Theta_T, \Theta_T^0) \\ -\text{Cov}(\Theta_T, \Theta_T^0) & \text{Var}(\Theta_T) \end{bmatrix} \begin{bmatrix} \theta_T \\ A \sigma_e^2 \theta_T + p^T \end{bmatrix}^0 = \frac{\sigma_S^2}{\sigma_S^2 + A^2 \sigma_e^2 \sigma_I^2} (A \sigma_e^2 \theta_T + p) = \delta_U^T (A \sigma_e^2 \theta_T + p) = E \left[ F | \Theta_T = S - A \sigma_e^2 I \right], \text{ with } \delta_U = \frac{\text{Cov}(F, \Theta_T^0)}{\text{Var}(\Theta_T^0)} = \frac{\sigma_S^2}{\sigma_S^2 + A^2 \sigma_e^2 \sigma_I^2} \text{ c.v.d.}
\]

From Lemma 1 it follows that uninformed agents, when updating their price beliefs, discard the signal from the current price and submit the net demand schedule

\[
X_U^T = \frac{E(F|\Theta_T^0) - p^T}{\text{Var}(F|\Theta_T^0)} - I_U = \frac{\delta_U^T (A \sigma_e^2 \theta_T + p) - p^T}{\text{Var}(F|\Theta_T^0)} - I_U = -\left( -\frac{1 - \delta_U^T}{\text{Var}(F|\Theta_T^0)} \right) p^T - I_U + \left( \frac{\delta_U^T A \sigma_e^2}{\text{Var}(F|\Theta_T^0)} \right) \theta_T^0.
\]

By equating the parameters obtained to those previously conjectured, we have:
\[ H^T = \left( 1 - \frac{\text{Cov}(F, \Theta_0^T)}{\text{Var}(\Theta_0^T)} \right), \quad \Omega = \left( \frac{\text{Cov}(F, \Theta_0^T)A\sigma_2^2}{\text{Var}(\Theta_0^T)} \right) \quad \text{and} \quad \Psi^T = 1 \]

Using the market clearing condition, it is straightforward to derive the equilibrium price, \( p^T \), and the results on market quality from equation (10):

\[
LT - LA = \frac{\sigma_2^2A[2A^2\sigma_2^2M N + \sigma_2^2MN + A^2\sigma_2^2Z^2\sigma_2^2 + + A^2\sigma_2^2N^2 + \sigma_2^2MN + \sigma_2^2A^2\sigma_2^2Z^2\sigma_2^2 + + A^2\sigma_2^2N^2 + \sigma_2^2MN]A\sigma_2^2(\sigma_2^2A^2\sigma_2^2\sigma_2^1 + \sigma_2^2 + A^2\sigma_2^4\sigma_2^2)}{\sigma_2^2A^2\sigma_2^2Z^2\sigma_2^2 + \sigma_2^2A^2\sigma_2^2\sigma_2^1N^2 + \sigma_2^2N^2 + A^2\sigma_2^4\sigma_2^2Z^2 + + A^2\sigma_2^4\sigma_2^2N^2 + \sigma_2^2MN} > 0
\]

Proof of Proposition 3.

Under partial transparency uninformed traders update their beliefs on the future value of the asset by using the information from other traders’ net demands and the signal \( \Theta_{PT} \) they can extract from the market price:

\[
\begin{align*}
\Theta_{PT} &= S - A\sigma_2^2I + \\
\frac{ZA\sigma_2^2}{N}x &= \gamma_1^PT p^PT + \gamma_2^PT I_U - \gamma_3^PT X_U^PT - \gamma_4^0\Theta_{PT}^0 & \text{with} & \quad \gamma_1^PT = \frac{N + A\sigma_2^2(M-1)\Omega^PT}{N}, \\
\gamma_2^PT &= \frac{A\sigma_2^2(M-1)\Psi^PT}{N}, & \gamma_3^PT = \frac{A\sigma_2^2}{N}, & \gamma_4^PT = \frac{A\sigma_2^2(M-1)\Psi^PT}{N}.
\end{align*}
\]

Lemma 2 Assuming that \( \text{Var}(x) = \text{Var}(X_I) \) and that \( \frac{ZA\sigma_2^2}{N} = 1 \), then

\[
E[F|\Theta_{PT}, \Theta_{PT}^0] = \delta_T^T [\mu_1p^PT + \mu_2I_U - \mu_3X_U + \mu_4\Theta_{PT}^0].
\]

Proof:

\[
E [F|\Theta_{PT}, \Theta_{PT}^0] = E [F|\Theta_{PT}, \Theta_{PT}^0 = S - A\sigma_2^2I] \Pr \{ q = 1 | \Theta_{PT}, \Theta_{PT}^0 = S - A\sigma_2^2I \} + + E [F|\Theta_{PT}, \Theta_{PT}^0 = x] \Pr \{ q = 0 | \Theta_{PT}, \Theta_{PT}^0 = x \}
\]

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Using Lemma 1, it is straightforward to show that: $E[F|\Theta_{PT}, \Theta_{PT}^0 = S - A\sigma_e^2 I] = \delta^T_U(p^PT + A\sigma_e^2 \theta_{PT}^0)$ and that $E[F|\Theta_{PT}, \Theta_{PT}^0 = x] = \delta^T_U(\gamma_1^PT p + \gamma_2^PIU - \gamma_3^PTX_U - (\gamma_4 + \frac{Z\sigma^2}{N})\theta_{PT}^0)$,

therefore: $E[F|\Theta_{PT}, \Theta_{PT}^0] = \delta^T_U(p^PT + A\sigma_e^2 \theta_{PT}^0) \Pr \{q = 1|\Theta_{PT}, \Theta_{PT}^0\} +$ $+ \delta^T_U(\gamma_1^PT p + \gamma_2^PIU - \gamma_3^PTX_U - (\gamma_4 + \frac{Z\sigma^2}{N})\theta_{PT}^0) \Pr \{q = 0|\Theta_{PT}, \Theta_{PT}^0\}$.

Assuming that $\text{Var}(x) = \text{Var}(X_I)$, and that $\text{Cov}(\Theta_{PT}, x) = \text{Cov}(\Theta_{PT}, X_I)$, we have: $\Pr \{q = 1|\Theta_{PT}, \Theta_{PT}^0\} = \Pr \{q = 0\} = \frac{N}{N+Z}$ and

\[\Pr \{q = 0\} = \frac{Z}{N+Z}; \text{ hence, we obtain:}\]

$E[F|\Theta_{PT}, \Theta_{PT}^0] = \delta^T_U[\mu_1 p^PT + \mu_2 IU - \mu_3 X_U + \mu_4 \theta_{PT}^0]$ with $\mu_1 = \frac{N+\gamma_1^PTZ}{N+Z} = (N + \frac{Z\sigma^2(N-1)}{N})/(N+Z)$, $\mu_2 = \frac{\gamma_2^PT\theta_{PT}^0}{N} = \frac{Z\sigma^2(N-2)}{N} + \frac{Z\sigma^2(N-1)}{N+Z}$ and $\mu_3 = \frac{\gamma_3^PTX_U}{N+Z} = \frac{N\sigma^2(N+1)}{N}$ and $\mu_4 = \frac{\gamma_4^PTX_U}{N+Z}$ and

We can now substitute $E[F|\Theta_{PT}, \Theta_{PT}^0]$ into the uninformed trader’s demand, $X_U^PT$, and solve for the parameters from previous conjecture: $H^PT = (1 - \delta^T_U)[\text{Var}(F|\Theta_{PT}, \Theta_{PT}^0) + \frac{Z\sigma^2}{N+Z}(1 + \delta^T_U(M - 1))]^{-1}$ and $\Omega^PT = ((N - Z)\sigma^2\delta^T_U)[\text{Var}(F|\Theta_{PT}, \Theta_{PT}^0)N + \frac{Z\sigma^2(N-1)}{N+Z}]^{-1}$ and

$\Psi^PT = [1 + \frac{\sigma^2Z(N+1)}{\text{Var}(F|\Theta_{PT}, \Theta_{PT}^0)N+Z}]^{-1}$ with $\text{Var}(F|\Theta_{PT}, \Theta_{PT}^0) = \sigma_S^2 + \sigma_e^2 - \frac{\sigma^2}{\sigma_S^2 + 3\sigma_e^2M^2}$.

Using the market clearing condition $N\left[\frac{S - P^PT}{A\sigma^2_e} - I\right] - MH^PT p^PT - M\Omega^PT\Theta_{PT}^0 - M\Psi^PT IU + ZX = 0$, where $\{\Theta_{PT}^0|\Theta_{PT}^0 \neq X_U\} = qX_I + (1-q)x$,
we obtain equations 15 and 16.

**Proof of Lemma 3 and of Proposition 4**

Let \( \Pi_{I,U} = X_{I,U}(F - p^\ast) + I_{U}F \) with \( z = A, T \), be the end of period profits of each informed and uninformed trader respectively under the anonymous (A) and the fully transparent (T) regime. Let \( p^\ast, F - p^\ast \) and \( X_{I,U}^z \) be:

\[
p^\ast = \alpha_1^z S + \alpha_2^z x + \alpha_3^z I + \alpha_4^z I_U, \quad F - p^\ast = (1 - \alpha_1^z)S - \alpha_2^z x - \alpha_3^z I - \alpha_4^z I_U + \varepsilon
\]

and

\[
X_{I,U}^z = \beta_1^z S + \beta_2^z x + \beta_3^z I + \beta_4^z I_U. \quad \text{It follows that the expected utility of each trader’s profits is equal to:}
\]

\[
E[- \exp (-A \Pi_{I,U}^z)] = \quad (23)
\]

\[
= -E[\exp (b_{z,zu}S^2+c_{z,zu}I^2+d_{z,zu}I_U^2+e_{z,zu}x^2+f_{z,zu}(\varepsilon S)+

g_{z,zu}(\varepsilon I) + h_{z,zu}(\varepsilon I_U) + i_{z,zu}(\varepsilon x) + l_{z,zu}(SI) + m_{z,zu}(SI_U) +
+n_{z,zu}(Sx) + p_{z,zu}(II_U) + q_{z,zu}(Ix) + r_{z,zu}(I_U x))]\]
with \( b_{z, tu} = -\beta_{1,1u}(1 - \alpha_i^z)A \), \( c_{z, tu} = \beta_{3,3u}\alpha_3^z A \),

\[
d_{z, tu} = \beta_{4,4u}\alpha_4^z A, \quad e_{z, tu} = \beta_{5,2u}\alpha_2^z A, \quad f_{z, tu} = -\beta_{1,1u}^z A, \quad g_z = -(\beta_3^z + 1)A,
\]

\[
g_{zu} = -\beta_{3,1u}^z A, \quad h_z = -\beta_{1,2u}^z A, \quad h_{zu} = -(\beta_{4,1u}^z + 1)A, \quad i_{z, zu} = -\beta_{2,2u}^z A,
\]

\[
l_z = (\beta_1^z\alpha_3^z + \beta_2^z(1 - \alpha_i^z) + 1)A, \quad l_{zu} = -(\beta_{1,2u}^z\alpha_3^z + \beta_{3,1u}^z(1 - \alpha_i^z))A,
\]

\[
m_z = -(\beta_1^z\alpha_4^z + \beta_4^z(1 - \alpha_i^z))A, \quad m_{zu} = -(\beta_{1,4u}^z\alpha_4^z + \beta_{3,1u}^z(1 - \alpha_i^z) + 1)A,
\]

\[
n_{z, zu} = -(\beta_{1,1u}^z\alpha_2^z + (1 - \alpha_i^z)\beta_2^z)A, \quad p_{z, zu} = -(\beta_{3,3u}^z\alpha_3^z - \beta_{4,4u}^z\alpha_3^z)A,
\]

\[
q_{z, zu} = -(\beta_{2,2u}^z\alpha_3^z - \beta_{3,3u}^z\alpha_3^z)A, \quad r_{z, zu} = -(\beta_{2,2u}^z\alpha_4^z - \beta_{4,4u}^z\alpha_4^z)A
\]

Using the Law of Iterative Expectations we obtain \( E_x = E[\exp(-A\Pi_{I, U}) | S, I, I_U, x] \),

\[
E_S = [E_x | I, I_U, x], \quad E_x = [E_S | I, I_U], \quad E_I = [E_x | I_U] \quad \text{and}
\]

\[
E_{I_U} = [E_I] = \frac{\Upsilon_{z, zu}}{\sqrt{1 - 2\sigma_I^2(d_{z, zu} + h_{z, zu}^2 \frac{\sigma_I^2}{2} + L_{z, zu} F_{z, zu}^2 + R_{z, zu} \Lambda_{z, zu}^2 + J_{z, zu} U_{z, zu}^2)}}
\]
with:

\[
\begin{align*}
\Upsilon_{z,zu} &= \frac{R_{z,zu}}{\sqrt{1 - 2\sigma^2_I V_{z,zu}}}; \quad J_{z,zu} = \frac{\sigma^2_I}{2(1 - 2\sigma^2_I V_{z,zu})}; \\
V_{z,zu} &= R_{z,zu} \chi^2 + L_{z,zu} C_{z,zu} + \frac{\sigma^2}{2} g_{z,zu} + C_{z,zu}; \quad Q_{z,zu} = \frac{H_{z,zu}}{\sqrt{1 - 2\sigma^2_S M_{z,zu}}}; \\
R_{z,zu} &= \frac{\sigma^2_x}{2(1 - \Upsilon_{z,zu} \sigma^2_M z,zu)}; \quad \chi_{z,zu} = L_{z,zu} 2C_{z,zu} G_{z,zu} + \frac{\sigma^2}{2} 2g_{z,zu} i_{z,zu} + e_{z,zu}; \\
M_{z,zu} &= L_{z,zu} G_{z,zu}^2 + \frac{\sigma^2}{2} i_{z,zu}^2 + e_{z,zu}; \quad H_{z,zu} = \frac{1}{\sqrt{1 - 2\sigma^2_S (\frac{\sigma^2}{2} f_{z,zu}^2 + b_{z,zu})}}; \\
L_{z,zu} &= \frac{\sigma^2_S}{2[1 - 2\sigma^2_S (\frac{\sigma^2}{2} f_{z,zu}^2 + b_{z,zu})]}; \quad C_{z,zu} = \frac{\sigma^2}{2} \frac{f_{z,zu} g_{z,zu} + l_{z,zu}}{2}; \\
F_{z,zu} &= \frac{\sigma^2_S}{2} r_{z,zu} f_{z,zu} h_{z,zu} + m_{z,zu}; \quad G_{z,zu} = \frac{2}{2} f_{z,zu} i_{z,zu} + n_{z,zu}.
\end{align*}
\]

In order to solve for the equilibrium number of informed traders, we need to evaluate agents’ profits under different transparency regimes. Looking at the equilibrium price, given by expressions 5 and 9, under anonymity and under full transparency respectively, we infer that:

\[
p^A = \alpha^A_1 S + \alpha^A_2 x + \alpha^A_3 I + \alpha^A_4 I_U \quad \text{with } \alpha^A_1 = \lambda_A \frac{N}{\lambda x}, \quad \alpha^A_2 = \lambda_A Z, \quad \alpha^A_3 = -\lambda_A N, \quad \alpha^A_4 = -\lambda_A M \Psi \quad \text{and} \quad p^T = \alpha^T_1 S + \alpha^T_2 x + \alpha^T_3 I + \alpha^T_4 I_U \quad \text{with } \alpha^T_1 = \lambda_T \frac{N + M \Omega}{\lambda x}, \quad \alpha^T_2 = \lambda_T Z, \quad \alpha^T_3 = -\lambda_T (N + M \Omega), \quad \alpha^T_4 = -\lambda_T M.
\]

Substituting the expressions for the equilibrium price into each trader’s demand \(X_{I,U}\) we get:
\[ X_i = \frac{S - p^z}{A \sigma_z^2} - I = (1 - \alpha_i A \sigma_z^2) S - \frac{\alpha_i}{A \sigma_z^2} x - (\frac{\alpha_i}{A \sigma_z^2} + 1) I - \frac{\alpha_i}{A \sigma_z^2} I_U = \beta_i^z S + \beta_i^z x + \beta_i^z I + \beta_i^z I_U \]

with
\[ \beta_i^z = (1 - \alpha_i A \sigma_z^2), \quad \beta_i^z = -\frac{\alpha_i}{A \sigma_z^2}, \quad \beta_i^z = -\frac{\alpha_i}{A \sigma_z^2} + 1, \quad \beta_i^z = -\frac{\alpha_i}{A \sigma_z^2} \]

and
\[ X_U = -H^{1,T} p^z + \Omega^{-\infty,1} \theta_T^0 + \Psi^{1,0} I_U = \beta_{1u}^z S + \beta_{2u}^z x + \beta_{3u}^z I + \beta_{4u}^z I_U \]

with
\[ \beta_{1u}^z = -H^{1,T} \alpha_1^z + \Omega^{-\infty,1} \beta_1^z, \quad \beta_{2u}^z = -H^{1,T} \alpha_2^z + \Omega^{-\infty,1} \beta_2^z, \quad \beta_{3u}^z = -H^{1,T} \alpha_3^z + \Omega^{-\infty,1} \beta_3^z, \quad \beta_{4u} = -H^{1,T} \alpha_4^z - \Omega^{0,1} + \Omega^{-\infty,1} \beta_4^z \]

where \( z = A, T \).

Now, substituting the values of \( \alpha^z \) and \( \beta^z \) into (24) and (26) the expected utility of each trader’s profit under the anonymity and transparency regimes can be evaluated; and using (25) one can obtain the results in Tables 1 and 2 and Proposition 4.

**Proof of Propositions 5 and 6.**

When traders behave strategically, transparency influences not only uninformed, but also informed traders’ strategies. It follows that, unlike the competitive framework, we now need to evaluate informed traders’ strategies under the
three different regimes, namely, anonymity, full transparency and partial transparency.

With anonymity each informed trader submits a limit order equal to \(X_I^S = \frac{S - p^S - A\sigma^2 I}{A\sigma^2 + \lambda_I} = D(S - p^S) - GI\), being \(D = \frac{1}{A\sigma^2 + \lambda_I}\) and \(G = \frac{A\sigma^2}{A\sigma^2 + \lambda_I}\). The latter parameters can be easily derived from the first order condition, \(S - p^S - \frac{\partial p^S}{\partial X} X_I^S = \frac{1}{2}[2(X_I^S + I)]\sigma^2 = 0\), being \(\lambda_I = \frac{\partial p^S}{\partial X}\) the price elasticity each informed trader calculates solving for \(p\) the conjectured market clearing conditions: \((N - 1)(D(S - p^S) - GI) - MH^S p^S + M\Psi^{SA} I_U + Zx + X_I^S = 0\). Each uninformed trader extracts the following signal from the market price, \(\Theta_S = S - A\sigma^2 I + \frac{(A\sigma^2 + \lambda_I)Z}{N} x = -(A\sigma^2 + \lambda_I) X_U^S + \frac{N + (A\sigma^2 + \lambda_I)(M-1)H^S}{N} p^S + \frac{(A\sigma^2 + \lambda_I)(M-1)\Psi^{SA}}{N} I_U = \gamma_1^S p^S - \gamma_2^S X_U^S + \gamma_3^S I_U\), which is used to update expectations on \(F\) and to formulate a limit order equal to \(X_U^S = \frac{E[F|\Theta_S] - p^S}{AVar[F|\Theta_S] + \lambda_U} - \frac{AVar[F|\Theta_S]}{AVar[F|\Theta_S] + \lambda_U} I_U = -H^S p^S - \Psi^{SA} I_U\) with \(H^S = N \left[ \frac{\text{Cov}(F, \Theta_S)}{\text{Var}(\Theta_S)} \right] [N + (M - 1)(A\sigma^2 + \lambda_I)] + N(AVar[F|\Theta_S] + \lambda_U + \frac{\text{Cov}(F, \Theta_S)}{\text{Var}(\Theta_S)} (A\sigma^2 + \lambda_I)) \right]^{-1}, \lambda_I = [(N - 1)D + MH^S]^{-1}\) and \(\lambda_U = [ND + (M - 1)H^S]^{-1}\).

The equilibrium value for \(H^S\) is obtained by equating the parameter from the solution of each uninformed trader’s first order condition to the previous conjecture for that parameter, while \(\lambda_U\) can be derived as before solving for \(p\) the conjectured market clearing conditions: \((N - 1)(D(S - p^S) - GI) - (M - 1)H^S p^S - (M - 1)\Psi^{SA} I_U + Zx + X_U^S = 0\). Substituting \(D\) into \(H^S, \Psi^{SA}, \lambda_U\) and \(\lambda_I\), the
solution to the model with strategic traders and anonymity can be obtained by solving the system with 4 equations and 4 unknowns.

With full transparency, each informed trader submits $X_I^{S} = \frac{S - p^{ST} - A\sigma^2_I}{A\sigma^2_ST + \lambda_T^I} = D^T(S - p^{ST}) - G^TI$ with $D^T = \frac{1}{A\sigma^2_ST + \lambda_T^I}$ and $G^T = \frac{A\sigma^2_ST + \lambda_T^I}{A\sigma^2_ST + \lambda_T^I}$. This demand is derived from the first order conditions: $S - p^{ST} - \frac{\partial p^{ST}}{\partial X_I} X_I^{S} - \frac{A}{2} [2(X_I^{S} + I)]\sigma^2 = 0$ with $\lambda_T^I = \frac{\partial p^{ST}}{\partial X_I} = [(N - 1)D^T + MH^{ST} + M\Omega^{ST}D^T]^{-1}$. Each uninformed trader places a net demand equal to $X_U^{ST} = \frac{E(F|\Theta_0^{ST}) - p^{ST}}{A\text{Var}(F|\Theta_0^{ST}) + \lambda_U^T} = -H^{ST}p^{ST} - \Psi^{ST}I_U + \Omega^{ST}\theta_0^{ST}$ where $H^{ST} = (1 - \frac{\text{Cov}(F, \Theta_0^{ST})}{\text{Var}(\Theta_0^{ST})})/(A\text{Var}(F|\Theta_0^{ST}) + \lambda_U^T)$, $\Omega^{ST} = \frac{\text{Cov}(F, \Theta_0^{ST})}{\text{Var}(\Theta_0^{ST})}([A\sigma^2_ST + \lambda_T^I]/[A\text{Var}(F|\Theta_0^{ST}) + \lambda_U^T])$, $\Psi^{ST} = A\text{Var}(F|\Theta_0^{ST})/[A\text{Var}(F|\Theta_0^{ST}) + \lambda_U^T]$ and $\lambda_U^T = [ND^T + (M - 1)H^{ST} + (M - 1)\Omega^{ST}D^T]^{-1}$. $\lambda_T^I$ and $\lambda_U^T$ can be obtained solving for $p$ the informed and uninformed traders’ conjectured market clearing conditions, which are equal to $(N - 1)(D^T(S - p^{ST}) - G^TI) - MH^{ST}p^{ST} + M\Omega^{ST}\theta_0^{ST} + M\Psi^{ST}I_U + Zx + X_I^{S} = 0$ and $N(D^T(S - p^{ST}) - G^TI - (M - 1)H^{ST}p^{ST} + (M - 1)\Omega^{ST}\theta_0^{ST} + (M - 1)\Psi^{ST}I_U + Zx + X_U^{ST} = 0$ respectively. Notice that uninformed traders use the signal $\Theta_0^{ST}$ to update their expectations on $F$. Using Lemma 1, it is straightforward to show that uninformed traders discard the signal from the market price and update their beliefs on $F$ by observing $X_I^{S}$ and extracting $\Theta_0^{ST} = S - A\sigma^2_ST = p^{ST} + (A\sigma^2_ST + \lambda_T^I)\theta_0^{ST}$. Solving the system with 5 equations and 5 unknown,
\(H^{ST}, \Omega^{ST}, \Psi^{ST}, \lambda_U^T\) and \(\lambda_T^T\), allows for numerical simulations and comparisons with the anonymous regimes.

With partial transparency the model can be solved analogously with informed and uninformed traders submitting the following net demands respectively: 

\[X^{SPT}_I = \frac{S-p^{SPT} - A\sigma^2 I}{A\sigma^2 + \lambda^T_I} = D^{PT}(S-p^{SPT}) - G^{PT}I \quad \text{with} \quad D^{PT} = \frac{1}{A\sigma^2 + \lambda^T_I} \quad \text{and} \quad G^{PT} = \frac{A\sigma^2}{A\sigma^2 + \lambda^T_I}\]

\[X^{SPT}_U = \frac{\mathbb{E}(I\Theta^{SPT}) - p^{SPT}}{\mathbb{A}\, \mathbb{V} \mathbb{a} \mathbb{r}(F|\Theta^{SPT}, \Theta^0_I) - \mathbb{P} \mathbb{T}} = -H^{SPT}p^{SPT} + \Omega^{SPT}\theta^0_{SPT} - \Psi^{SPT}I_U\]

where \(H^{SPT} = [1 - \mathbb{C} \mathbb{o} \mathbb{v}(F|\Theta^{SPT})] \, \mathbb{V} \mathbb{a} \mathbb{r}(\Theta^0_{SPT})] \, \mathbb{A} \, \mathbb{V} \mathbb{a} \mathbb{r}(\Theta^{SPT}) N(N + Z)\] and \(\Omega^{SPT} = \mathbb{C} \mathbb{o} \mathbb{v}(F|\Theta^{SPT})(\mathbb{A}\sigma^2 + \lambda^T_I)\, \mathbb{N}(N + Z)\) and \(\lambda^T_I = \frac{\mathbb{C} \mathbb{o} \mathbb{v}(F|\Theta^{SPT})(\mathbb{A}\sigma^2 + \lambda^T_I)\, \mathbb{M} \mathbb{Z}}{\mathbb{V} \mathbb{a} \mathbb{r}(\Theta^{SPT}) N(N + Z)}\) \[\psi^{SPT} = [1 + \frac{(\mathbb{A}\sigma^2 + \lambda^T_I)\mathbb{Z}(1 + \delta^2(M-1))}{\mathbb{A}\, \mathbb{V} \mathbb{a} \mathbb{r}(F|\Theta^{SPT}, \Theta^0_{SPT}) N(N + Z)}]^{-1}, \mathbb{V} \mathbb{a} \mathbb{r}(\Theta^0_{SPT}) = \sigma^2_S + A^2\sigma^4_I, \mathbb{V} \mathbb{a} \mathbb{r}(F|\Theta^{SPT}, \Theta^0_{SPT}) = \sigma^2_S + \sigma^2_\varepsilon - \frac{\sigma^4_I}{\sigma^2_S + \lambda^T_I}\] and \(\mathbb{C} \mathbb{o} \mathbb{v}(F|\Theta^{SPT}) = \sigma^2_\varepsilon\). As before, from the informed and uninformed traders’ conjectured market clearing conditions, it is straightforward to show that \(\lambda^S\) and \(\lambda^U\) are equal to: 

\[\lambda^{SPT}_I = [(N - 1)D^{PT} + MH^{SPT} + M\Omega^{SPT}D^{PT}]^{-1} \frac{N}{N + Z} + [(N - 1)D^{PT} + MH^{SPT}]^{-1} \frac{Z}{N + Z} \quad \text{and} \quad \lambda^{SPT}_U = [N D^{PT} + (M - 1)H^{SPT} + (M - 1)\Omega^{SPT}D^{PT}]^{-1} \frac{N}{N + Z} + [N D^{PT} + (M - 1)H^{SPT}]^{-1} \frac{Z}{N + Z}.

Notice that with agents behaving strategically, the signal that uninformed traders can extract from the order they observe is equal to: 

\[\{\Theta^{SPT}_{SPT} | \Theta^{SPT}_I \neq X^{SPT}_U\} = qX^{SPT}_I + (1 - q)x.

40
References


Footnotes

1. Section 1.2, shows that under full transparency this term is equal to
   \[ [Var(F|\Theta^0_T)]^{-1}, \Theta^0_T \] being the signal uninformed traders extract from the informed traders’ demand.

2. Numerical simulations also show that the difference between \( Var(p^T) \) and \( Var(p^A) \) is an inverse function of \( A \) and \( \sigma^2_e \).

3. Under the regime of partial transparency, uninformed agents observe a price and a quantity and they do not know whether this is an informed trader’s limit order or a noise trader’s market order, since the latter is displayed together with the best market price. This feature is modelled here by assuming that uninformed agents update their estimates of the future value of the asset by using both other traders’ orders and the market price.

4. The hypothesis that uninformed traders do not receive an endowment shock at the beginning of the trading game makes the model analytically simple and does not alter the results qualitatively.

5. I thank Alexander Guembel for recommending this extension.
Table 1: Equilibrium number of informed traders, \((N_A, N_T)\), difference in liquidity \((LA - LT)\) and volatility \((VA - VT)\).

Results from Lemma 3: \(\sigma_I = 0\)

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<th>(\sigma_I)</th>
<th>(\sigma_\varepsilon)</th>
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Table 1.1: Equilibrium number of informed traders, \((N_{Au}, N_{Tu})\), difference in liquidity \((LA – LT)\) and volatility \((VA_u – VT_u)\).

Results from Lemma 3: \(\sigma_S = \sigma_x = \sigma_I = \sigma_x = \sigma_U = .5\)

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Table 1.2: Equilibrium number of informed traders, \((N_{Au}, N_{Tu})\), difference in liquidity \((LA - LT)\) and volatility \((VA_u - VT_u)\).

Results from Lemma 3: \(M = Z = 20, A = 2, C = .02\)

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Table 2: Equilibrium number of informed and uninformed traders: \((N_A, N_T)\), \((M_A, M_T)\) and differences in liquidity: \((LAn - LTn)\), \((LAm - LTm)\)

Results from Lemma 3 with \(A = 2\), \(C = .02\), \(\sigma_s = 0.5\), \(\sigma_I = 0.5\).

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<th>(M)</th>
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TABLE 3.1: Equilibrium parameter values and liquidity for:

\[ A = 1, \sigma_z^2 = .5, \sigma_y^2 = 1, \sigma_f^2 = .5, \sigma_U^2 = .5, N = 5, M = 5, Z = 5 \]

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Informed and uninformed traders’ demand under the two regimes:

An: \[ X_s^I = D(S - p^S) - GI, \quad X_U^S = -H^S p^S - \Psi^SA_I \]

Tr: \[ X_I^{ST} = D^T(S - p^{ST}) - G^T I, \quad X_U^{ST} = -H^{ST} p^{ST} + \Omega^{ST} \theta^{ST}_I - \Psi^{ST} I_U \]
TABLE 3.2: Equilibrium parameter values and liquidity for:

\[ A = 1, \sigma_S^2 = .5, \sigma_Z^2 = 1, \sigma_Z^2 = .5, \sigma_I^2 = 5, \sigma_U^2 = .5, N = 20, M = 5, Z = 5 \]

<table>
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<th>( \lambda_U^{1,T} )</th>
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<th>( D^{1,T} )</th>
<th>( \Omega^{ST} )</th>
<th>( LSA, LST )</th>
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Informed and uninformed traders’ demand under the two regimes:

**An:** 
\[ X_I^S = D(S - p^S) - GI, \quad X_I^S = -H^S p^S \Psi^{SA} I_U \]

**Tr:** 
\[ X_I^{ST} = D^T(S - p^{ST}) - G^T I, \quad X_I^{ST} = -H^{ST} p^{ST} + \Omega^{ST} \theta_{ST}^0 - \Psi^{ST} I_U \]
TABLE 3.3: Equilibrium parameter values and liquidity for:

\[ A = 1, \sigma_x^2 = .5, \sigma_z^2 = .5, \sigma_s^2 = .5, \sigma_f^2 = 5, \sigma_U^2 = .5, N = 5, M = 20, Z = 5 \]

<table>
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<tr>
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<th>(\lambda_U^{1,T})</th>
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<th>(D^{1,T})</th>
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<table>
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<th>(\lambda_U^{T})</th>
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<th>(D^{T})</th>
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<th>(LSA, LST)</th>
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<td>.048</td>
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<td>.954</td>
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Informed and uninformed traders’ demand under the two regimes:

An: \[X_I^S = D(S - p^S) - GI, \quad X_U^S = -H^S p^S - \Psi^{SA} I_U\]

Tr: \[X_I^{ST} = D^T(S - p^{ST}) - G^T I, \quad X_U^{ST} = -H^{ST} p^{ST} + \Omega^{ST} \theta^{0}_{ST} - \Psi^{ST} I_U\]
TABLE 3.4: Equilibrium parameter values and liquidity for:

\[ A = 1, \sigma_x^2 = 1, \sigma_z^2 = 1, \sigma_s^2 = 1.1, \sigma_I^2 = 1, N = 5, M = 5, Z = 20 \]

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<td>9.010</td>
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<td>( \lambda_U^T = 0 )</td>
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Informed and uninformed traders’ demand under the two regimes:

An: \( X^S_I = D(S - p^S) - GI, \quad X^S_U = -H^S p^S - \Psi^{SA} I_U \)

Tr: \( X^{ST}_I = D^T(S - p^{ST}) - G^TI, \quad X^{ST}_U = -H^{ST} p^{ST} + \Omega^{ST} \theta^0_{ST} - \Psi^{ST} I_U \)
TABLE 3.5: Equilibrium parameter values and liquidity for:

\[ A = 1, \sigma_x^2 = 1, \sigma_z^2 = 1, \sigma_s^2 = 1.1, \sigma_I^2 = 1, N = 20, M = 20, Z = 20 \]

<table>
<thead>
<tr>
<th>An.</th>
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<th>( \lambda_U^{1,T} )</th>
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</table>

Informed and uninformed traders’ demand under the two regimes:

An: \( X_I^S = D(S - p^S) - GI, \quad X_U^S = -H^S p^S - \Psi^S A_I \)

Tr: \( X_I^{ST} = D^T(S - p^{ST}) - G^T I, \quad X_U^{ST} = -H^{ST} p^{ST} + \Omega^{ST} \theta_{ST}^0 - \Psi^{ST} I_U \)
Figure 1: Total cost reduction due to pre-trade transparency. On the vertical axes: \[ \Xi = \left[ \text{AVar} (F|\Theta) \right]^{-1} - \left[ \text{AVar} (F|\Theta) \right]^{-1} + \left[ \frac{1}{\text{AVar} (F|\Theta)} - H \right] \] with \( A = 2, \sigma_e^2 = .5, \sigma_I^2 = .5, \sigma_x^2 = .5, \sigma_S^2 = .5, M = 20 \):
Figure 2: Adverse selection costs under anonymity. On the vertical axes: 
\[
\frac{1}{\text{Var}(F(\Theta))} \cdot H
\]
with \(A = 2, \sigma_e^2 = .5, \sigma_f^2 = .5, \sigma_x^2 = .5, \sigma_S^2 = .5, M = 20\)
Figure 3: Risk-bearing cost reduction due to pre-trade transparency. On the vertical axes: $[AV ar (F|\Theta_T)^{-1}] - [AV ar (F|\Theta)]^{-1}$ with $A = 2, \sigma^2_F = .5, \sigma^2_I = .5, \sigma^2_x = .5, \sigma^2_S = .5, M = 20$. 

\[\text{AV ar}(F|\Theta_T)^{-1} - \text{AV ar}(F|\Theta)^{-1}\]
Figure 4: On the horizontal axes: $SX = \sigma_x, Z$ and on the vertical axis: $\text{Var}(p^T) - \text{Var}(p^A)$ with $A = .6, \sigma_S = .8, \sigma_e = .5, \sigma_I = .8$ $M = N = 20$. 
Figure 5: $LT - E[LPT(q)]$ with $A = 1, \sigma_S = .8, \sigma_x = 1, \sigma_x = .1, \sigma_I = 1, \sigma_U = 1$ $M = 20$. 