Dynamic Modeling of Web Purchase Behavior
and E-Mailing Impact by Petri Net

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Abstract:
In this article, the authors introduce Petri nets to model the dynamics of Web site visits and purchase behaviors in the case of wish list systems. They describe Web site activities and their transition with probability distributions and model the sequential impact of influential factors through links that better explain Web purchase behavior dynamics. The basic model, which analyzes site connections and purchases to explain visit and purchase behavior, performs better than a classical negative binomial regression model. To demonstrate its flexibility, the authors extend the wish list Petri net model to measure the impact of e-mailing intervals on visit frequency and purchase.

Keywords: Internet, wish list, e-mail, Petri net, dynamic model.
1. **INTRODUCTION**

During the past ten years, marketers and researchers have attempted to analyze particular phenomena of the Web and develop appropriate business strategies for using the Internet as a new channel of communication and distribution. The main focus of this research has been improving site design and structure to maximize visit frequency, visit duration, and purchase. For example, Mandel and Johnson (2002) focus on the influence of a Web page’s design on consumer product choice. Ansari and Mela (2003) analyze e-mail content customization and its ability to increase Web site traffic. Lynch and Ariely (2000) and Zauberman (2003) both measure the impact of online information search costs. More particular attention also has been paid to overall Web visit behavior as a means to explain site visits and purchases and to assess the magnitude of their influence.

Modeling Web behavior from a global system perspective is significant for several reasons. First, visits and purchase behavior on the Web are different from those aspects in brick-and-mortar stores, particularly in terms of the Web’s weak order conversion rate. Web sites receive millions of visitors, but only 3% of them purchase, according to an April 2000 study by the Boston Consulting Group and shop.org (Betts, 2001). This finding implies that, as Sismeiro and Bucklin (2004) note, Web visits are not good predictors of purchase intentions, and single-stage models (e.g., probit, logit) that attempt to attribute a direct impact of visits on purchase behavior are not well adapted to analyze the Web. Our main objective in this article is to address this problem with a model that is based on Petri nets and that flexibly takes into account different sequential actions of Web visitors.

Another difference between on- and off-line behavior pertains to the objective of the visit. Moe (2003) distinguishes four types of online shopping visits: directed-purchase visits (the objective is a quick purchase), search and deliberation visits (consumers search for a product with a possibility of purchase), hedonic-browsing visits (Web user is shopping for...
pleasure), and knowledge-building visits (the objective is to learn more about the marketplace). In contrast, a visit to a brick-and-mortar store is probably concentrated on a directed purchase; consumers rarely visit a supermarket, for example, for an exploratory visit. Therefore, each Web visit by a consumer may have a unique goal, and the purchase conversion rate should be linked to the type of visit being made. Whereas previous research has tried to measure the direct impact of influential factors on purchase, we model Web visit and purchase behaviors as a process, in which purchase frequency and volume are the results of Web visits, by capturing the purchase probability of each visit, whose interval we also assess by a probability distribution.

Some particular characteristics of Web visit and purchase behavior require flexible methods for modeling various visitor actions. Because there are virtually no space limits, many actions can happen simultaneously at the same place. For example, an almost unlimited number of visitors can browse a product page, request product information from a Web agent, and purchase the same product simultaneously. For these parallel actions to occur, the Web system must accommodate many parallel visits, which must be processed through different, synchronized sequences, such as a credit information check to convert a visit into a purchase. Thus, Web visit and purchase behavior mirrors other real-world systems that are too complex to be modeled analytically because they require too many computing resources.

Classical causal models, particularly single-stage models, that explore part of this phenomenon are limited to analyzing the dynamics of the whole process of Web visit and purchase behavior. As a remedy, simulation methods may be able to design and analyze such complex systems, as Law and Kelton (2000) suggest. In this article, we propose a Petri net model that can analyze complex systems that consist of parallel and sequential actions that require a high level of synchronization. In addition, the Petri net model is a simulation tool
that can describe the overall, detailed performance of a Web site by incorporating various action variables characterized by probability distributions with flexible model extensions.

We organize the remainder of the article as follows: In Section 2, we describe the theoretical background and present Petri net models. Then in Section 3, we focus on a stochastic Petri net application to model the process of visit and purchase behavior in a Web site that specializes in electronic wish lists (WLs). In section 4, we compare the prediction performance of the Petri net model with that of a negative binomial distribution (NBD) regression model that assesses the direct impact of influential factors. We extend the initial WL Petri net model by incorporating the impact of e-mailing on the visit interval and comparing the performance of various e-mailing strategies in terms of purchase volume in Section 5. Finally, in Section 6, we discuss the advantages of Petri net models for marketing, as well as further research topics.

2. THEORETICAL BACKGROUND

2.1. Modeling Web Visit and Purchase Behavior

Recent research has focused on Web visit and purchase behavior for several reasons. First, many companies have indicated their intention to use the Web as a new communication and distribution channel, but some hesitate to go further because of their lack of understanding of Web user behavior and its potential effects. Consequently, recent research streams progressively have developed the capacity to track Web behavior, particularly for key activities such as visits and purchase. Second, companies have a stake in modeling online visits to measure Web communication effects and online purchases to account for their direct financial impact.

Bucklin and Sismeiro (2003) model browsing behavior on a Web site by integrating two key elements: the visitor’s decision to continue browsing and the duration of each page view. They use a type II Tobit model and link Web users’ propensity to continue browsing
with visit depth and repeat visits. With regard to Web site visit behavior, Telang, Boatwright, and Mukhopadhyay (2004) develop proportional hazards mixture models that capture unobserved heterogeneity to predict search engine visits on a hourly basis. By using mixture models and covariates, they can predict disaggregate data more accurately. Park and Fader (2004), whose stochastic individual model analyzes visit timing behavior across sites, improve sites’ ability to predict future visit timing by capturing the recency and frequency of previous visits to the site and to its competitors’ sites. Their model thus corrects the overestimation of visit numbers that can occur when researchers consider the visit timing to multiple sites independently. Johnson, Moe, Fader, Bellman, and Lohse (2004) model search behavior across competing e-commerce sites with probabilistic models and a logarithmic process. They find that Web users search across few sites and that search intensity is linked to consumer characteristics.

In addition, Sismeiro and Bucklin (2004) propose a predictive model of Web buying behavior that incorporates various tasks accomplished by site visitors (e.g., completion of product configuration, input of complete personal information, order confirmation with provision of credit card data). Using clickstream data and a Bayesian approach, they divide Web users’ purchase process into different sequential tasks and show that their browsing behavior predicts the task completion of all decision levels but that the number of repeat visits does not explain buying propensity. Moe and Fader (2004b) link purchase and visit behavior in a dynamic conversion behavior model that predicts purchase frequency and volume through prior visits to the Web site and purchasing thresholds. By addressing heterogeneity across customers and dynamics over time, they show that their conversion model outperforms more classical models, such as logistic regressions.

One of the limits of these models is their complexity, which makes them challenging to implement and use. In addition, the preceding studies all highlight their limitations with
regard to their incomplete integration of Web user behavior elements, mostly because such a task is extremely complex. To overcome these limitations, we introduce Petri nets, which can model very complex systems by integrating different blocks of submodels, and simplify the complexity by using simple graphical language. Moreover, Petri net models are powerful simulation tools that address the principle objective of Web visit and purchase models: the accurate prediction of online visits and purchase.

2.2. Petri Net Models

Petri nets were created in the 1960s by Carl Adam Petri (1962) to study complex, dynamic systems of communications among automatons. Their application has been expanded to various domains such as computer science, operational research, biology, and organizational management, including human–machine information system modeling (Meldman, 1977), supply chain performance modeling (Viswanadham & Srinivasa Raghavan, 2000), and online order processing modeling (Weitz, 1998). A complete overview of Petri net modeling of workflow systems has been done by Salimifard & Wright (2001). A Petri net is a graphically oriented language for system design, specification, simulation, verification, and optimization. It is particularly well suited for systems in which communication sharing and interactions are important (Jensen, 1992). Petri nets can model dynamic systems that evolve from one state to another when an external or internal event occurs.

A Petri net is a triple $N = \{P, T, F\}$, where $P$ is a set of places, $T$ is a set of transitions, and $F$ is a set of directed arcs. Places describe the states of the system and are graphically represented by circles. Transitions, represented as rectangles, describe the events that occur in the system. Finally, arcs describe how the Petri net changes when a transition occurs. A marking assigns token counts to the various places of the net; each place contains a positive (or 0) number of tokens. The evolution of the tokens through the places and transitions describes the system dynamics.
Each arc links a particular place \( p \) to a transition \( t \) (\( p \) is an input place) or a transition \( t \) to a place \( p \) (\( p \) is an output place). When a transition is enabled, the input place contains at least one token. An enabled transition fires by removing one token from the input place and depositing one token in the output place. The marking again corresponds to the current state of the system.

< Insert Figure 1: Petri Net around here >

In Figure 1, we depict a token in place \( P_1 \) and an enabled transition \( T_1 \). In state 2, after transition \( T_1 \) fires (the event occurs in the system), the token appears in place \( P_2 \). If we assume that the token corresponds to a consumer and the transition represents a purchase, this system models a consumer before the purchase (state 1), a purchase (transition \( T_1 \)), and the consumer after purchase (state 2). In Figure 1, we represent the transitions as black rectangles to indicate that they are immediate (without delay).

Marking \( M_0 \) is defined by the token distribution during the initial state. The evolution of the system can be described as follows (adapted from Salimifard and Wright, 2001): If \( p_1, p_2, \ldots, p_p \) are different places in the system and \( M(p_p) \) is the number of tokens in place \( p_p \), marking \( M = \{M(p_1), M(p_2), \ldots, M(p_p)\} \) represents one state of the system. An enabling transition \( t \) from marking \( M \) transforms it to a new marking \( M' \) (noted by \( M \rightarrow M' \)) by removing one token from each input place and adding one token to each output place.

Many Web behaviors, such as inter-visit time, are random in nature, whereas others are closely related to their precedent behaviors. For example, online shoppers must provide required information, such as personal characteristics (e.g., name, address), to complete a purchase. Other activities can happen simultaneously (e.g., receiving an e-mail newsletter from a Web site while surfing that Web site), or one activity may have priority over another.
For example, when a Web visitor pays for a product, the site must make checking the consumer’s credit card information a greater priority than other browsing behavior on the site. To model complex Web activities with random natures, it is necessary to develop a model that can incorporate the complex relationships among Web activities by assessing their random nature with appropriate probability distributions.

Stochastic Petri nets (SPN) have been developed to represent such complex stochastic processes. The SPN model enhances the Petri net model by associating a probability distribution with each transition, which are then called timed transitions. Because SPN effectively represent systems that are characterized by concurrency, synchronization, and priorities, they are particularly suited to model online major visit and purchase behavior.

3. **STOCHASTIC PETRI NET APPLICATION**

3.1. **Wish List Concept**

Similar to a traditional wedding registry, an Internet wish list (WL) includes potential gifts a user can specify on a Web site. Many sites, including Amazon.com, currently provide a WL service to enable their customers to receive gifts of their choosing and to acquire new customers who can be registered with a WL. The major advantage of this service on the Web is its usage convenience. With a few clicks, a Web user can build a WL with his or her favorite products from a Web site and its affiliates, and unlike traditional WLs like wedding registries, friends or acquaintances can consult the WL easily to offer a gift through the Web. Because the user makes his or her own WL and contacts potential buyers (gift givers), the company that manages the WL service can attain greater impact but does not need to hire additional salespeople. The WL service concept also is very attractive to marketers because it captures the preferred product categories and brands of a consumer, which substantially enhances the efficiency of targeted promotions.
3.2. Our Data

We use a data set from the French company MilleMercis (www.Millemercis.com). Its main business is to generate sales from linked e-commerce sites through online WLs. Registered customers of MilleMercis can create an electronic WL featuring products available from either its affiliated Web sites or all other existing Web sites. By using what MilleMercis calls its 'facilitator' (a specific program that appears on the menu bar of the Web browser), consumers surfing the Web can include any product from any Web site with a simple click. The list creator then can send an e-mail to friends, family members, or acquaintances who are liable to purchase a gift from his or her WL.

MilleMercis’ WL process is more valuable than standard WL services because it can construct a customer database that includes products from various online shopping sites. This database therefore reflects the best products, and their detailed product information, that consumers want to purchase for themselves. In turn, it provides extremely effective sales leads compared with those generated by traditional marketing databases that provide only limited details of the desired product categories and brands. From a modeling point of view, the WL constitutes a simple site visit and purchase process: site connection, item selection, purchase. Compared with those of other commercial Web sites, the WL process is simple, which minimizes the complexity of our analysis of purchase behavior as a consequence of a Web site connection.

For our research, we extracted a sample of 1000 randomly selected WLs from among the 500,000 managed by MilleMercis. We then chose the 207 most active WLs in terms of the number of site connections, which provides our model with parameter estimates. Because a single Internet user may create more than one WL, our data set finally included 135 customers, whom we refer to as 'agents A'. The observation window covers two and a half
years, from May 2000 to the end of December 2002. Our data set maintains the record of 22,450 connections to MilleMercis Web site by the 135 agents A. 

We provide some descriptive statistics in Table 1. Agent As are rather young (approximately 30 years of age), create a few WLs (1.5 on average), and record purchases of 6 items for 278 site connections.

3.3. Wish List Petri Model

We composed our model of a WL system from several sequential and parallel actions. Site connections are followed by item(s) purchase in sequence, and listed item can be purchased in parallel by either the creator of the list (agent A) or a person who received the list by e-mail, whom we call agent B. Several agents B may exist for a wish list created by one agent A. We apply SPN in our model to assess the dynamics of the sequential and parallel actions, linked by the transition distribution, that have a stochastic nature. We provide an overall description of our Petri net model in Figure 2.

Our model starts with an agent A (place agent A) who creates a WL through the transition Create List. After this first transition step, agent A may start to connect to the MilleMercis Web site (transition Site Connect A). At each connection, agent A may or may not purchase a certain number of items, then return (transition return) to the place where he or she is ready to connect (place Agent A Ready to connect). After agent A creates a WL, he or she sends an e-mail to agents B, who engage in a similar process of Web visit and purchase. They may connect to the Web site (transition Site Connect B) to check the content of agent A’s wish list, make a decision either to purchase (transition purchase) item(s) for agent A or
not to purchase anything (transition no purchase), and then return to their starting state (place
Agent B ready to connect).

< Insert Figure 2: Wish list Petri net around here>

3.4. Analysis of Interconnection Time and Purchase Pattern

We first analyze the transition Site Connect A to assess how often agents A connect to
the MilleMercis Web site. We measure the variable “interconnection time” as the time
between two successive connections of agent A, who is using his or her own login
information for the identification. The distribution of interconnection time, our major concern,
can be captured through probability distributions such as gamma, Weibull, or log-normal. We
exclude exponential distribution because it cannot adequately capture the characteristics of
repeat visit (Moe & Fader, 2004a). Therefore, for the three candidate distributions, we obtain
the parameter estimates that maximize the likelihood function of the site interconnection time
for the 135 agents A. Because these three distributions are not nested, we use Vuong’s (1989)
test to select the distribution that provides the best fit for each agent A, as we show in Table 2.
This test, which is based on the Kullbach Leibler information criterion, measures the distance
between a given distribution and the observed data. No difference exists between the gamma
and Weibull distributions for 56 agents A (41.5% of total agents A) or among the three
distributions for 38 agents A (28.1%). However, the Weibull distribution emerges as the best
distribution for 15.6% of agents A, whereas gamma does for 12.6%. On the basis of Vuong’s
(1989) test results, we first exclude the log-normal distribution because it does not excel in the
model fit. Then, because the mathematical formula for the Weibull distribution is simpler
than that for the gamma distribution, we select the Weibull distribution—whose cumulative
distribution is defined as \( f(X) = c X^{\epsilon-1} \lambda^{\epsilon} e^{-\lambda X^\epsilon} \), \(0 \leq X \leq +\infty\), with \(c\) = shape parameter and \(\lambda\) = scale parameter—to capture the site interconnection time of agents A.

We also perform a analysis for the site interconnection time of agents B. We analyze 9611 connections of agents B and find that the gamma distribution fits best.

We then analyze the purchase behavior after-reach connection on the web site. We find a similar distribution of the numbers of units bought per purchase for agent A and B. We use a classical NBD distribution to capture the purchase rate of items at the site connection state for both the agents.

4. **MODEL PERFORMANCE COMPARISON**

Petri net models can describe the dynamics of complex systems composed of states that are linked both sequentially and in parallel fashion through transition probabilities. In most processes, including purchase activity, the number of purchased items becomes the element that interests marketing managers most. In our WL Petri net model, we incorporate the influence of other actions on the number of purchased items; this influence depends on the transition probabilities between other actions and the target action (purchase). Compared with other methods, such as regression models that explain the causal effect of influential factors on the target variable, the advantage of Petri nets is their flexibility. Petri nets apply the transition probability on the basis of various flexible distributions, which clarifies the impact of influential factors more precisely than do other regression models that are limited to nonlinear forms of the influential factors.
To verify the performance of our WL Petri net model, we compare it with that of a competing NBD regression model. Both models explain the influence of the number of site connections on the total number of purchased items, but they differ in terms of how to incorporate that impact. The WL Petri net model, by including a NBD distribution to capture purchase among other sequential actions (such as site connection), embeds the effect on the basis of the transition distribution of site connection (Weibull or Gamma distribution), whereas NBD regression does so as a change of purchase probability. The NBD regression model can be depicted as follows:

\[
P(Y_i = y) = \frac{\Gamma(y + \gamma)}{\Gamma(\gamma) y!} \left( \frac{\alpha}{\alpha + \exp(\beta' X_i)} \right)^y \left( \frac{\exp(\beta' X_i)}{\alpha + \exp(\beta' X_i)} \right)^y
\]

(1)

where \(\gamma\) is the shape parameter, \(\alpha\) is the scale parameter, and the influential \(X\) affect the purchase probability. With a given observation duration fixed at 1, the NBD regression can be depicted as follows:

\[
P(Y_i = y) = \frac{\Gamma(y + \gamma)}{\Gamma(\gamma) y!} \left( \frac{\alpha}{\alpha + 1} \right)^y \left( \frac{1}{\alpha + 1} \right)^y
\]

(2)

If the given duration changes, “t” can substitute for 1 for the changed duration. The impact of the influential factor works similarly to the change of the observation duration. For example, if a factor has a positive coefficient, it is similar to an increase in the observation duration, and consequently, the probability of purchase increases, whereas that of non-purchase decreases.

To compare the performance of both models, we select a data set of 94 agents and their WLs for 16 months after their registration. The data set includes visit and purchase behavior, such as the number of site connections, purchased items, and created lists, as well as the profile of the agent, including gender and age. We divide this data set into two distinct periods: before and after month 12. We use the pre-month 12 data set to calibrate the model parameter estimates and the post-month 12 data set to compare the predictive power of the
two models. For the WL Petri model, we use SPNP software (Trivedi, Duke University) For the NBD regression, we use Excel Solver for the scale and shape parameter estimates and Stata 8.0 for the influential factors’ coefficient estimates and standard error computation.

We calculate the Petri net six parameters following the method described in the previous sections. We build a Petri net for each of the 94 agents A by calculating the parameters of the Weibull distribution for agent A’s interconnection time, the gamma distribution for all the agent B’s interconnection time, and the NBD distribution for agent (A or B)’s purchase after each connection. The results from the NBD regression show that the number of site connections and the age of the agent have significant impacts on the number of purchased items (Table 3). Active site connection Web behavior enhances the purchase volume, and younger customers show a tendency to purchase more than do older ones. However, other Web behavior factors such as the number of created WLs and the visitor’s gender are not statistically significant enough to use to test the model’s predictive power.¹

<Insert Table 3: NBD Regression Results of Impacts on Purchased Items around here>

On the basis of these results, we then compare the WL Petri model and the NBD regression model on their performance in terms of the number of purchased items. We estimate the parameters of both models using the pre-month 12 data set. We obtain individual-level parameter estimates for the WL Petri model and estimate aggregate-level parameters for the NBD regression model.² We use the same parameter estimates for the

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¹ In NBD regression, we take the log function for 'Site connection' and 'Age' variables and add '1' to 'Site connection' before taking the log to avoid the invalid value error for cases having no site connection.
² In the NBD regression model, we retain only statistically significant variables. For the Web behavior variable, Ln (site connection+1), .629 and for the profile variable, Ln (age), -
model prediction in the post-month 12 period (months 13–16). For the WL Petri model, we can predict the purchase volume by fixing the simulation duration at 4 months. For the NBD regression model, we must reduce the duration by one-third, because the length of the prediction period (4 months) is one-third that of the calibration period (12 months), so we introduce the number of site connections by each agent during the prediction period to compute the predicted purchase volume. For the comparison, we use two metrics that frequently are employed to test the forecast power: MAD (mean absolute deviation) and MSE (mean squared error). The MAD takes the absolute difference between the observed and the model-predicted values of the number of purchased items, then divides it by the number of observations (agents in this case). The MSE complements MAD by comparing how this difference is spread across observations (agents); it then sums the square of the difference and divides it by the number of observations.

As we present in Table 4, the WL Petri model outperforms the NBD regression in predictive power and calibration errors. The flexibility of the transition distributions appears to offer the WL wish list Petri model the edge. Because each transition probability is obtained through the best fitting distribution, WL Petri nets can provide the maximum flexibility for the model calibration, which then can be realized in terms of the model’s predictive power. The validity of Petri nets’ simulation-based method is precisely described by Haas (2002). In contrast, the NBD regression model faces more constraints in refining the model calibration to provide better predictive power than the WL Petri model because the .509 are obtained from the model calibration. The values of $\gamma$ and $\alpha$ are .423 and .912 respectively.
shape of the probability distribution is fixed, regardless of the magnitude of the influential
factors that refine the model.

5. **AN EXTENSION OF WISH LIST PETRI MODEL**

5.1. **E-Mail Impact on Purchase**

Another advantage of the Petri model is its flexible ability to add actions into an
already structured process. In our case, we add the action “E-mailing” to our WL Petri model
so that we can measure the impact of the e-mail interval on whether an e-mail is opened.
When an agent opens a sent e-mail, he or she automatically is connected to the associated lists.
Therefore, opened e-mails can increase purchase volume by accelerating the process of Web
visits and purchases, as we show in the example of the Figure 3, in which an opened e-mail
implies a visit (with no purchase) and diminishes the delay before the next connection and
purchase.

< Insert Figure 3: Purchase Accelerating Effect of E-Mail around here>

The marketing literature contains several articles about modeling e-mail efficiency.
Some recent research has focused on e-mail response and the factors that influence it
(Chittenden & Rettie, 2003; Ken & Brandal, 2003; Sheehan & McMillan, 1999). In addition,
Ansari and Mela (2003) model the customization of e-mail content using clickstream data and
show that optimizing the content of permission-based e-mails can increase the number of
click-throughs by 62%.

Generally, an e-mail consumer reaction works as follows: A user receives many e-
mails and decides whether to open each one. All e-mails sent to the same person are nested
by that person, such that some characteristics of the person influence the opening rate, and
each e-mail also has its own characteristics. Because this phenomenon has two layers of
influence (email characteristics and individual profile), we must separate them to analyze their impact. Therefore, we consider a hierarchical linear model (HLM) with two levels (level 1: email characteristics; level 2: individual profile) and a logit link for binary outcomes (Raudenbush & Bryk, 2002). The probability that user j will open e-mail i can be described as prob(opening email_{ij}) = f(individual profile_i, email profile_{ij}). In our case, we structure the analysis in two levels. In Level 1, we assess the opening probability and the magnitude of the influential factors at the e-mail level. As we observe the number of sent e-mails (Y) and the number of opened e-mails (m), we can obtain the e-mail open rate from a binomial distribution: m openings of Y sent. Because the e-mail open rate therefore is a probability whose value varies from 0 to 1, we take its logit form to capture the impact of the influential factor in a regression. In Level 2, we measure the impact of the user’s profile on the magnitude of the influential factors determined in Level 1.

5.2. Hierarchical Linear Model Structure

In this model, we start to assess the e-mail open rate with a binomial distribution, then transform it through a link function for regression analysis. Let \( Y_{ij} \) be the number of opened e-mails in \( m_{ij} \) trials (the number of sent e-mails), and let \( \phi_{ij} \) be the probability of opening an e-mail. Then \( Y_{ij} | \phi_{ij} \sim B(m_{ij}, \phi_{ij}) \) denotes that \( Y_{ij} \) has a binomial distribution with \( m_{ij} \) deliveries, and \( \phi_{ij} \) is the probability of opening per trial.

Several link functions—such as probit, log-normal, and logit—are possible when the Level 1 sampling model is binomial (Hedeker & Gibbons, 1994) and we tried the most common and convenient one, the logit link: 

\[
\eta_{ij} = \log\left(\frac{\phi_{ij}}{1 - \phi_{ij}}\right)
\]

where \( \eta_{ij} \) is the log of the odds of opening the e-mail. We relate the transformed predicted values \( \eta_{ij} \) to the predictors of the model through the linear structural model, 

\[
\eta_{ij} = \beta_{0j} + \beta_{1j}X_{1ij} + \beta_{2j}X_{2ij} + \ldots + \beta_{nj}X_{nij}
\]

and

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convert the predicted log-odds to a predicted probability by computing 
\[ \varphi_{ij} = \frac{1}{1 + \exp(-\eta_{ij})}. \]
Whatever the value of \( \eta_{ij} \), Equation 8 will produce a value of \( \varphi_{ij} \) (probability of opening an e-mail) between 0 and 1.

The Level 2 model has the following form of 
\[ \beta_{qj} = \gamma_{q0} + \sum_{s=1}^{S_q} \gamma_{qs} W_{js} + u_{jq}, \]
where the random effects \( u_{jq}, q = 0, \ldots, Q \), constitute a vector \( u_j \) with a multivariate normal distribution with component means of 0 and a variance-covariance matrix \( T \).

### 5.3. Results of the Level 2 Analysis

From the 135 agents listed, 12 of them are removed due to inadequate data. The agents received an e-mail every 23 days on average and opened them 54% ('OPEN') of the time. In this analysis, the interval between email is transformed into a logarithmic scale ('LN INT') as the expected interval effect is not proportional to the original scale. For all 123 agents together, the profile variables 'AGE', 'LIST', and 'CELLBIN' are significant at \( \alpha = 10\% \) in Level 2. However, the interval effect ('LN INT') on e-mail opening behavior is not significant, which may be because the individual-level heterogeneity of e-mail opening behavior is not captured correctly when the agents are aggregated. To solve this problem, we decide to split the agents into two groups showing the different pattern of email opening behavior. We check the regression coefficients of two interval variables (the log of interval, 'LN INT' and the log of the square of interval 'LN INT2'). Only agents whose opening rate is an inverted U-shape (the coefficient of 'LN INT' is positive and that of 'LN INT2' is negative) are selected and grouped as group 1. The rest of agents are grouped into 2. Group 1 (N = 33) reflects an inverted U-shaped effect; the probability of opening increases as the interval increases and then decreases after passing a peak. However, for this group, none of the profile variables, such as 'AGE', 'LIST', 'CELLBIN', and 'GENDER', appears statistically
significant. On the other hand, Group 2 (N = 90) displays a U-shaped effect of emailing opening behavior.

In Group 1, the inverted U-shaped effect interval is revealed to be significant at $\alpha = 5\%$ with the maximum opening rate reached when the interval is 18.9 days for a positive effect of opening the e-mail. Therefore, the firm need to send agents in Group 1 an e-mail every 18.9 days to maximize their opening rate.

In contrast to the results for Group 1, in Group 2 the opening rate decreases until the interval hits the point of 28 days and rebounds. This pattern may seem to be weird unless the plausible range of emailing interval is taken into consideration. As the plausible range spans from 7 to 58 days and it covers the 80% of email intervals in our data set, agents in Group 2 would have actively opened email if it had been sent in either a shorter or a longer interval than the minimum response interval of 28 days. In addition, the opening rate is found higher among agents who created more WLs and provided their cell telephone number to the firm than others.

5.4. Integrating E-Mailing Impact in the Wish List Petri Model

To further our analysis, we decide to compare the impact of the e-mailing interval on purchase volume. In our data, 80% of e-mails were sent during a 7–60 day interval, and on average, the firm sent e-mails with the interval of 23 days. Therefore, we compare the
purchase volume (number of purchased items) for the optimal e-mail interval, which maximizes the number of opened e-mails, versus the firm’s 23-day interval. We select the optimal e-mailing interval from the effective range of 7–60 days. In the first two columns of Table 7, we present the probability that each group will open the e-mail; in the next two columns, we denote the number of average opened e-mails during 60 days. We also include the trade-off between the volume of sent e-mails (frequent or infrequent) and the opening rate as a function of the interval between two e-mails. For Group 1, the opening rate reaches its maximum at an 18.9-day interval, but the number of opened e-mails is maximized when the e-mails are sent every 7 days which is located at the 10% lower bound of our data set. Within the effective interval range, the 7-day interval strategy maximizes the number of opened e-mails because the increased sending emailing frequency compensates largely the decrease of opening rate. Compared with the 23-day interval, the 7-day interval can almost double the number of opened e-mails by Group 1 (from 2.12 to 4.25) and more than double the number for Group 2 (from .55 to 1.43) during the 60 days we investigate.

< Insert Table 7: Opening Rate and Opened Emails around here>

To integrate this e-mailing effect into our WL Petri model, we create e-mailing places and transitions parallel to the creation of the list and the site connection. However, this parallel connection considers e-mailing optionally; that is, if e-mailing does not occur, the agent still will connect to the site and purchase items according to the previous parameters of transition probability. We present the WL Petri model with e-mailing in Figure 4.

< Insert Figure 4: Wish List Petri Model with Emailing around here>
We represent e-mailing by the transition mailpub (sending e-mail to agent A). When an e-mail is sent (place mailed), agent A can open it (transition open) or not (transition no open). When the e-mail is not opened, the system returns to the initial state agent A ready to connect. The open rate parameters included in the Petri net (firing probabilities for transitions open and no open) are those previously calculated by the HLM model. When an e-mail is opened (transition open, then place opened), agent A either clicks (transition click) or does not click (transition nclick) on the e-mail. The click rate to define these transitions is the average group click rate (Group 1 and 2). When agent A clicks on the e-mail, he or she connects to the Web site (transition connect) and then may or may not purchase an item on the WL. If agent A does not click on the e-mail, he or she returns to the initial state (place agent A ready to connect).

We extend our WL Petri net model to include e-mailing for each agent, using the parameters we obtained previously to describe the agents’ connections and purchase behavior. Then we simulate the impact of e-mailing frequency on two variables: the number of total connections by agents A and B and the number of purchased items. For each agent, we simulate a 16-month period (the longest common period with data availability per agent) for two different strategies: sending e-mail every 7 days and sending e-mail every 23 days. We present the results in Table 8.

As we expect from the comparison of the number of opened e-mails between the two strategies (Table 7), e-mailing performance directly influences the volume of purchase. First, the optimal e-mailing strategy of a 7-day interval results in an increased number of site connections. The number of site connections increases almost by 3% and 5% respectively for
Group 1 and 2 when the firm changes its e-mailing strategy. Both differences are statistically significant at \( \alpha = 5\% \) (both p-values are .00). Second, the increase in site connections leads to an increase in the purchase volume. For both groups, the number of purchased items increases by 5% when the firm implements the optimal e-mailing policy and sends e-mails every 7 days instead of every 23 days. The increase of purchased item is relatively significant in Group 1 (the p-value of the paired sample t-test is .15) and statistically significant in Group 2 (the p-value is .00).

6. DISCUSSION AND LIMITATIONS

In contrast with previous research, our SPN model incorporates various actions in a flexible structure to explain their impact on purchase. We show that Petri nets can decompose a global, complex system into different blocks of sub-models that each describe a sub-phenomenon (agent connections, purchase behavior, e-mailing) using probability theory and stochastic models. We then link all the sub-models dynamically to achieve the global phenomenon to be modeled. Thereby, we can run several simulations with different input strategies (e.g., e-mailing intervals) and determine their impact on the key measures of output performance. Our findings enrich Moe and Fader’s (2004b) model, which analyzes the trade-offs between two components of a purchase conversion rate. Whereas their major result shows the positive effect of accumulated non-purchase visits and the negative effect of the purchase threshold effect on visit and purchase conversion rates, our model also includes the positive accumulating effects of previous non-purchase visits. Because we repeatedly apply the same purchase probability to each visit, the purchase probability increases across non-purchase repeat visits. But the major advantage of our model is its flexibility; it can incorporate other variables, such as e-mailing, that influence the purchase rate and volume. Unlike Sismeiro and Bucklin’s (2004) model, in which they analyze the visit and purchase process by decomposing them into sequential processes, our WL system allows both agent A,
who created the list, and agent B, who was contacted by agent A, to connect in parallel to the Web site to consult agent A’s WLs. This kind of parallel action cannot be analyzed in a model based on sequential analysis only and represents an advantage of using Petri nets, which can represent parallelism, synchronization, sequential actions, and even non-Markov chain systems without stationarity.

We show that our model flexibility not only delivers high performance in terms of model fit and predictive power but also provides a comprehensive simulation tool for incorporating marketing actions such as e-mailing. Our performance comparison with the NBD regression model proves that our model can analyze complex systems that include sequential and parallel actions. The flexibility and extendibility of Petri net models also can help marketers assess the impact of their marketing activities on extremely complex purchase behavior both off- and on-line. In addition, we show that SPN models are particularly well suited to Web phenomena. Many visit and purchase processes on the Web are random in nature, and Web user behavior often consists of various sequential, parallel, or synchronous tasks. Moreover, these actions are performed by thousands of users, and this large amount of data makes the modeling task difficult. By simplifying the complexity, Petri net models provide simple graphical representations that enable both researchers and practitioners to understand the modeling of the systems.

However, our model cannot address some aspects of visit and purchase behavior on the Web. One limitation is the form of the probability distribution we use to explain the number of purchased items. In our model, we apply NBD without any consideration of the number of listed items; during each visit, we provide the number of listed items. The application of NBD may provide worse model fit and predictive power than that of a binomial distribution in which information about the exact number of listed items at the moment of the site visit is provided. In addition, in line with our finding about the impact of e-mailing, we
did not assess a saturation effect for repeated e-mails, which led us to choose the 7-day interval arbitrarily instead of determining the optimal interval from the HLM analysis. Because we developed our model and its performance using data specific to a single firm, our model should be tested with empirical data that describe different types of consumer behavior. Furthermore, advanced applications of the Petri net model may improve marketing performance related to the following topics:

- Web site design, for which the model could analyze multi-page browsing and purchase behavior;

- Communication pattern analysis in virtual communities;

- Consumer behavior during online actions;

In addition, Web promotional games that attempt to create e-mail prospect databases, reposition a service, or improve sales using various players and questions would constitute interesting and complex systems that also might be simplified and modeled by Petri nets. The results of different simulation strategies would provide added value to marketers by proposing powerful, simple-to-use decision aid systems. New mobile telephone networks, including both interactive communications between companies and customers and all in-between user exchanges, remain a vast domain of marketing research open to the application of Petri nets. More generally, Petri nets should be considered for any problematic phenomenon that is linked to complex networks and are particularly suited to analyze consumer behavior in the technological marketing age.
REFERENCES


Table 1: Wish List Statistics

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Table 2: Vuong Test Results

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Table 3: NBD Regression Results of Impacts on Purchased Items

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Table 4: Model Comparison of Predictive Power

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Table 5: Impacts on Opening Rate (Group 1)

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Table 7: Opening Rate and Opened Emails

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Table 8: Simulation Results

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<td>23 days</td>
<td>154.77</td>
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<td>+ 5%</td>
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<td>23 days</td>
<td>125.19</td>
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<td>7 days versus 23 days</td>
<td>+ 5%</td>
<td>+ 5%</td>
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Figure 1: Petri Net

![Petri Net Diagram](image-url)
Figure 2: Wish List Petri Net

Agent A
Create List
Agent A
Ready to connect
No purchase
Connected
Purchase

• Interconnection time A
  – Weibull distribution
• Interconnection time B
  – Gamma distribution

Figure 3: an example of purchase Accelerating Effect of Email

List-Up

E-mail

List-Up

No Purchase  Purchase  Purchase

No Purchase  Purchase  Purchase

t

t
Figure 4: Wish List Petri Model with E-Mailing

Graph 1: Interval Impact on Opening Rate (Group 1)