Abstract

This paper examines the role of trading and market liquidity on market-based management compensation. The paper recognizes the endogenous nature of a firm’s stock price - it is the outcome of trading motivated by anticipations of future firm value. Our central result is that the degree of market-based compensation is proportional to market liquidity. Even though no outside trader has better information than the inside owner of the firm, their trading results in a stock price that is useful for incentive contracting since it allows the filtering out of noise that is not related to performance.

1 Introduction

A large volume of research considers the growing role of the stock market for executive compensation (see Murphy (1999) for a recent survey). The recent wave of corporate scandals has further increased the focus on the market component in executive pay. Although there are many arguments about the role of the market in providing incentives, surprisingly little attention has been paid to the role of trading and market liquidity in market-based compensation. The lack of analysis may be due to the fact that liquidity and the stock price are themselves endogenous. The stock price is the outcome of trading and liquidity changes as trading conditions or traders’ information change.

This paper recognizes the endogenous nature of the stock price and develops a model of market based compensation and price formation via trading. Our central result is that the degree of market based compensation is proportional to market liquidity.

The intuition is the following. The reason why compensation includes the stock price is that, together with including information about the fundamental value of the firm in the long-run, it filters out the measurement error that is creating the moral-hazard problem between managers and owners in the first place. The filtering is possible since the stock market aggregates traders’ anticipation about the future value of the firm which necessarily includes some anticipation of the measurement error.

Loosely speaking, nobody knows whether the value of the firm is due to managerial skill or luck. The owners of the firm would like to get a handle on luck. A direct handle is impossible but an indirect
handle using the stock price is possible. Although individual traders do not have any better handle on luck vs. skill, they know that every trader has a different handle. This is enough to create trade. Trading results in a price which is the indirect handle the owners wanted.

Filtering out luck is more efficient when there is a large difference between the impact of luck on observable output and on observable stock price. A liquid market is defined to be resilient to shocks in the order flow (caused partly but not entirely by trader anticipation of managerial luck), thus creating the desired difference in impact.

A desirable, and we believe unique, feature of our framework is that it admits and formalizes the following "intuitive" benchmark argument:

"assuming that the market is 'efficient' one might expect stock price to 'correctly' aggregate all of the information signal [...]. This suggests that the optimal compensation contract should depend solely on stock price." (Lambert (1993))

The value of the benchmark is that it spells out precisely what we mean by "efficient" and "correctly aggregate information". Without a benchmark, we would not know what real-world inefficiencies to look out for and what inefficiencies drive the departure from pure market-based compensation.

At first glance, the paper by Holmstrom and Tirole (1993) appears to be closely related to our analysis. A closer inspection however reveals a number of striking differences. In their analysis, the advantage of including a firm’s stock in the firm manager’s incentive contract is that it allows the owners of the firm to access the information of an external monitor. Through costly monitoring, this agent has information about the value of the firm that is otherwise not accessible. The owners cannot write a contract with the monitor but they know that he trades against uninformed noise traders. Due to his insider trading, the price will reveal some of his inside information. The disadvantage is that noise traders’ anticipation of future insider trading against them drives up the initial cost of equity. Their key comparative static is that a higher volume of noise trading leads to more insider trading by the monitor which then leads to a higher cost of capital.1

Our analysis does not rely on some trader having insider information to "motivate" the inclusion of a firm stock price into managerial compensation. In Holmstrom and Tirole (1993) trading is motivates as in Kyle (1985): a single trader with market power has better information than the rest of the market and knows something that the owner of the firm do not know. A price that contains inside information is necessarily useful for incentive contracting.

We motivate trade as in Hellwig (1980): traders in a competitive market have heterogenous information, i.e. no single trader knows better than the market and no trader knows something that the owner of the firm does not know (remember, no one has a handle of luck vs. skill). Ours is a much weaker set of assumptions: the price does not contain any inside, strategic information and yet, it will be useful for incentive contracting.

Paul (1992) uses a multi-tasking framework as in Holmstrom and Milgrom (1991) to show that information that is useful for incentive contracting may not necessarily be useful for valuation and vice versa. Given that both sources of information are useful, it is not surprising that both are used in managerial compensation. Kim and Suh (1993) and Bushman and Injejikian (1993) make a similar point considering the complementarity of accounting and market information. The motivation for all these studies was to find reasons for including information other than the stock price into managerial compensation. Our motivation is different and, we believe, truly intriguing: given that the market does not know better than insider owners of the firm, why include stock price?

1 See also Bolton and Von Thadden (1998) and Maug (1998) for related analysis.
A large literature in accounting starting with Banker and Datar (1989) has considered the role of various source of information for managerial incentive contracting (see Lambert (2001) for an extensive review). However, they focus on different, exogenous sources of information. The stock price in their model is exogenous while it is endogenous in our paper: it is the outcome of trading by agents who anticipate the fundamental value of the firm.

The trading in our model is rational and competitive. Yet the market will not be efficient (in fact, it cannot be efficient as pointed out by Grossman and Stiglitz (1980)). Bolton, Scheinkman and Xiong (2002) depart from rationality altogether and analyze executive competition when traders in the market-place are overconfident. In their model, compensation induces a manager to invest in a worthless but risky project in order increase the speculative value of the firm. Shares of such a "bubble" firm will be valuable as one group of overconfident investors speculates on the resale value to another (differently) overconfident group.

2 The model

The model analyzes the moral-hazard between owners and management inside a publicly traded firm. Active trading of the firm’s shares results in a stock price that is included in the managerial incentive contract. The moral-hazard is modelled as in Holmstrom and Milgrom (1991) and trading takes place in a competitive rational expectations model as in Hellwig (1980), Diamond and Verrechia (1981) or Admati (1985).

2.1 Agents

There are four types of agents. A publicly traded firm is run by a risk-averse manager (the agent) who is acting in his own interest; the manager controls an activity that influences the terminal value of the firm’s shares. The firm is owned by a risk-neutral collective of inside owners (the principal). The inside owners are considered risk-neutral since they have an unlimited access to the financial market, that allows them to diversify the idiosyncratic risk due to their stake in the firm.

The company stock is traded by informed risk-averse traders (rational speculators) and noise traders who trade randomly due to some exogenous shocks.

2.2 Technology, contracting and the sequence of events

The model has three dates: \( t = 0, 1, 2 \). The model ends with the liquidation of the firm for a gross value \( v \). At the beginning, \( t = 0 \), the insider owners (the principal) hire a manager (the agent) to run the firm and sign a management contract with him.

The contract specifies three payments. A fixed payment and two payments that depend on the two observable (and verifiable) variables: the price of shares \( p \) realized at \( t = 1 \) and the net liquidation value of the firm at \( t = 2 \). We write managerial income \( I \) as:

\[
I = a_0 + a_p p + a_v (v - a_0 - a_p p)
\]  

(1)

The coefficient \( a_p \) describes the impact of a dollar change in the stock price at \( t=1 \) on managerial income at \( t=2 \). In the following, we call \( a_p \) as market-based compensation since it does not rely on
fundamentals (they have not realized yet) but on how the market aggregates trader’s anticipation of them. 2 3

After signing the contract, the manager exerts an unobservable effort \( e \) in the productive activity that is privately costly to him, e.g. \( c(e) = \frac{k}{2} e^2 \). More effort improves the expected liquidation value of the firm \( v \):

\[
v = e + \theta
\]

where \( \theta \) is a random variable, \( \theta \sim N(0, \sigma^2_\theta) \), that represents fundamental noise outside the control of the manager. The first-best level of effort, defined as the hypothetical effort that the risk-neutral principal would exert himself, is therefore: \( e^{fb} = \frac{1}{k} \).

At time \( t = 1 \) competitive trading by informed and noise traders determines the market price \( p \) for the firm’s shares (more on this shortly).

At time \( t = 2 \), the gross terminal value of the firm, \( v \), is realized, the manager is paid according to his contract and the model ends with the liquidation of the firm.

The sequence of events is summarized in figure 1.

\[
\begin{align*}
\text{t=0} & \quad \text{Owners give an incentive contract to their manager who then exerts unobservable effort } e \text{ at a private cost.} \\
\text{t=1} & \quad \text{Trading in a competitive market results in stock price } p. \\
\text{t=2} & \quad \text{The firm value } v \text{ is realized, the manager is paid and the firm is liquidated.}
\end{align*}
\]

Figure 1: The timing of events

2.3 The moral-hazard problem

The manager’s preferences are represented by a CARA utility function defined over income and the cost of effort: \( U_m(I, e) = -\exp[-r_m I] - c(e) \), where \( r_m \) is the coefficient of constant absolute risk-aversion. Note that the cost of effort is expressed in (dis)utility terms.

The conflict of interests between inside owners and the manager comes from the fact that his effort is not contractible. Inside owners must choose the incentive contract \( (a_0, a_p, a_v) \) at time \( t = 0 \) that maximizes the expected value of the firm net of managerial income,

\[
\max_{a_0, a_p, a_v} E[v - I]
\]

2 Writing the contract on the net liquidation value \( v - a_0 - a_p p \) instead of the gross liquidation value \( v \) is a modelling device that allows us to abstract from dilution issues (see Holmstrom and Tirole 1993). After a normalization (section 3.1), it also allows us to analyze the trading part and the incentive contracting part of the model separately. We still conform to the standard practice that the contract is linear in \( p \) and \( v \), \( I = a_0(1 - a_v) + a_p (1 - a_v)p + a_v v \), and that the manager is paid at the final period \( t = 2 \).

3 It could also be interpreted as short term versus long term performance (two prices). Effort affect long-term fundamental only. Short term is the markets aggregation of people’s anticipation of long term value.
subject to the manager acting in his own interest,

\[ e = \arg \max_{e'} E[U_m(I, e')] \]  

and subject to the manager wanting to work for the owners,

\[ E[U_m(I, e)] \geq 0 \]

where we have simplified the manager’s outside opportunity to zero.

### 2.4 A competitive rational expectations market

At \( t = 1 \) the firm’s shares are traded on a competitive market with a continuum of risk-averse informed traders indexed by \( i \in [0, 1] \). Their information at \( t=1 \) consists of a noisy signal \( s_i \) about fundamental value of the firm \( v \) one period later at \( t = 2 \),

\[ s_i = v + \varepsilon_i \]

where \( \varepsilon_i \) is a random variable, \( \varepsilon_i \sim N(0, \sigma^2_i) \), that is uncorrelated across traders and independent of \( \theta \).

A key feature of the model is that an informed trader has speculative but not strategic information. A signal \( s_i \) is not strategic in the sense that it does not contain any inside information about the value of the firm \( v \). But it is speculative in the sense that it causes trades. Since each of the many traders receives a different realization of the signal, they will want to trade with each other.

This is very different from the information structure in Holmstrom and Tirole (1993). Their analysis considers a single monopolistic trader who has strategic (i.e. inside) information about the value of the firm. He has some information that is unavailable and hence valuable to owners. Moreover, their informed trader only trades with an uninformed market maker.

We assume that the Law of Large Numbers holds in its strong version, so that on average the information signals are unbiased:

\[ \int_0^1 s_i \, di = v \]

This means that the market as a whole "knows" the true value of the firm. If one could bring all the traders together and induce them to reveal truthfully their signal then the true value of the firm would be known. But that cannot happen since i) there is a countably infinite number of traders and ii) no trader has an incentive to reveal his signal for free. The whole point of having trading and a market clearing price in a competitive market is to work around this coordination problem.

We know from Grossman and Stiglitz (1980) that although the market as whole "knows" the firm value, this knowledge cannot be fully reflected in the price. They showed that information based trading in a competitive rational expectations market cannot be perfect. If it were then everybody would know the exact value of the firm, and there would be no reason to trade on private information. But then how could the price contain any information about the value of the firm in the first place?

At price \( p \), a risk-averse informed trader \( i \) buys (sells) \( x_i(p) \) \((-x_i(p))\) shares of the company stock at \( t = 1 \). One period later at \( t = 2 \) he closes his position at the price that reflects the net liquidation

\footnote{See Admati (1985) for a discussion of this assumption.}
value of the firm $p_2 = v - a_0 - a_p p$. An informed trader’s demand maximizes the CARA utility of his gain in wealth from trading:

$$x_i(p) = \arg \max_{x_i(\cdot)} E[-\exp[-rx_i((v - a_0 - a_p p) - p)]|s_i, p]$$

(7)

where $r$ is the coefficient of constant absolute risk aversion of an informed trader. Since each informed trader has the same degree of risk aversion, $1/r$ denotes the aggregate risk bearing capacity of the market.

Informed traders have rational expectations, i.e. they use all information available to them. This means that they condition their demand not only on their private signal $s_i$ but also on the publicly observable price $p$.

Solving (7), an informed investor’s demand is given by the following standard form:

$$x_i(p) = \frac{E[(v - a_0 - a_p p) - p|s_i, p]}{r \text{Var}[(v - a_0 - a_p p) - p|s_i, p]}$$

(8)

In addition to informed traders, there are noise traders who trade on the market for exogenous reasons. What is important is that their demand is not linked to the information available in the market. Their demand $u$ is assumed to be random according to $u \sim N(0, \sigma_u^2)$ and independent of all other factors in the model.

The stock price is determined by market clearing:

$$\int_0^1 x_i(p) \, di + u = 0$$

(9)

3 Analysis of the incentive contract

The analysis proceeds in a number of steps. First, we carry out a normalization that makes trading independent of incentive contracting. We can therefore analyze trading and incentive contracting separately. Second, we consider those incentive contracts that minimize the income risk to the manager given that he chooses an effort level that is optimal for him. Third, we solve for the price determined by market trading. Without recognizing the endogenous nature of the stock price, we would not be able to answer the following two questions: i) why should the stock price be included in managerial compensation and ii) when, as a benchmark, would managerial compensation be exclusively based on the stock price? The answers to these two questions are steps four and five of the analysis. We are then ready to conclude the analysis of the model by deriving the impact of liquidity on market based compensation.

3.1 A useful normalization

In order to make an informed trader’s demand in (8) independent of the details of the incentive contract, we define $\hat{p}$ as the prize net of contracting:

$$\hat{p} = a_0 + (1 + a_p)p$$

(10)
An informed trader’s demand can then be rewritten in term of \( \hat{p} \)

\[
x_i(\hat{p}) = \frac{E[v - a_0 - a_p(\hat{p} - a_0)(1 + a_p)] s_i, \hat{p}]}{r Var[v - a_0 - a_p(\hat{p} - a_0)(1 + a_p)] s_i, \hat{p}]} = \frac{E[v|s_i, \hat{p}] - \hat{p}}{r Var[v|s_i, \hat{p}]}\]

Note that we can change the conditioning in the expectation and the variance from \( p \) to \( \hat{p} \) since they are informational equivalent. To construct \( \hat{p} \) one only needs public information.

Next, we also rewrite managerial income using the normalized price:

\[
I = a_0 + a_p\frac{\hat{p} - a_0}{1 + a_p} + a_v(v - a_0 - a_p\frac{\hat{p} - a_0}{1 + a_p})
\]

\[
= (1 - a_v)\frac{1}{1 + a_p}a_0 + (1 - a_v)\frac{a_p}{1 + a_p}\hat{p} + a_vv
\]

The incentive contract is again linear, but now in the gross liquidation value \( v \) and the normalized price \( \hat{p} \). The normalized contract \((\hat{a}_0, \hat{a}_p, \hat{a}_v)\) is related to the original contract \((a_0, a_p, a_v)\) according to:

\[
\hat{a}_0 = (1 - a_v)\frac{1}{1 + a_p}a_0; \quad \hat{a}_p = (1 - a_v)\frac{a_p}{1 + a_p}; \quad \hat{a}_v = a_v
\]

3.2 Optimal minimum variance contracts

Given the incentive contract \( \hat{a}_0, \hat{a}_p, \hat{a}_v \), the manager chooses unobservable effort \( e \) in order to maximize the certainty equivalent of income \( I \) minus the disutility of effort:

\[
e = \arg \max_{e'} E[I] - \frac{r_m}{2} Var[I] - \frac{k}{2} e'^2
\]

The manager’s expected income depends on his expectation of the price of the firm at \( t=1 \) and the true value of the firm at \( t=2 \):

\[
E[I] = \hat{a}_0 + \hat{a}_p E[\hat{p}] + \hat{a}_v E[v]
\]

Since the manager knows what effort level he is taking, he expects the future value of the firm to be: \( E[v] = e \). The expectation of the price is more subtle since the market price will reflect the informed traders’ inference process about the value of the firm. Given that we working within the standard CARA utility-normal distribution framework, we hypothesize at this moment that the equilibrium price will be of the following linear form (we will solve the market model explicitly for the equilibrium price shortly):

\[
\hat{p} = E[v] + \alpha_\theta \theta + \alpha_u u
\]

We suppose that the market price at \( t=1 \) has three elements. First of all, it reflects the average value of the firm \( E[v] \). Second, the price is affected by an information shock about the fundamental noise, \( \alpha_\theta \theta \). And third, the price is perturbed by noise trading, \( \alpha_u u \). Note that on average, the price
reflects the average value of the firm: $E[\hat{p}] = E[v]$. Since the market has rational expectations and the contract is public information, the market’s expectation of firm value correctly reflects the actual effort of the manager: $E[v] = e$.\(^6\)

The variance of income

$$\text{Var}[I] = \hat{a}_p^2 \text{Var}[\hat{p}] + \hat{a}_v^2 \text{Var}[v] + 2\hat{a}_p\hat{a}_v \text{Cov}[\hat{p}, v]$$

is independent of effort. The manager can neither influence the fundamental risk of his environment nor the amount of noise trading.

The first-order condition for (12) characterizing optimal managerial effort is then:

$$ke = \hat{a}_p + \hat{a}_v$$

The condition shows that any appropriate combination of $\hat{a}_p$ and $\hat{a}_v$ induces the risk-averse manager to exert the same effort level. Inside owners’ risk-neutrality however means that the cheapest way to induce such an effort level is to minimize the income risk borne by the manager. An optimal contract must therefore solve:

$$\min_{\hat{a}_p, \hat{a}_v} \text{Var}[I]$$

subject to effort being optimal for manager

$$ke = \hat{a}_p + \hat{a}_v$$

The optimal combination of market based and non-market based compensation that minimizes the manager’s income risk and induces optimal managerial effort satisfies:

$$\hat{a}_v[\text{Var}[v] - \text{Cov}[\hat{p}, v]] = \hat{a}_p[\text{Var}[\hat{p}] - \text{Cov}[\hat{p}, v]]$$

(15)

The condition admits two intuitive conclusions. First, holding the covariance between the price signal and the value signal $\text{Cov}[\hat{p}, v]$ constant, the incentive contract places more weight on the more precise signal, i.e. the signal with the lower variance. Second, as the covariance between price and value, $\text{Cov}[\hat{p}, v]$, increases, the contract places more weight on the more precise signal. For example, suppose that the price is more volatile than firm value. Now if the price and value were to covary more with each other, then the contract would increases the weight on value.

### 3.3 Sensitivity-to-noise ratios

Condition (15) can also be rewritten as

$$\frac{\hat{a}_v}{\hat{a}_p} = \frac{1 - \frac{\text{Cov}[\hat{p}, v]}{\text{Var}[\hat{p}]}}{\frac{\text{Var}[v]}{1 - \frac{\text{Cov}[\hat{p}, v]}{\text{Var}[v]}}}$$

(16)

\(^{6}\)There is more to this statement than perhaps meets the eye. It usually assumed that the market’s expectation is correct only on the equilibrium path. Our statement is stronger. The market must never be fooled. In other words, we assume that a rational expectations market is characterized by the absence of arbitrage opportunities on and off the equilibrium path. If we assumed that the market is fooled off the equilibrium path then the manager has a strong incentive to deviate. In order to counter these incentives, the incentive contract must place less weight on the share price. This leads to problematic comparative statics. The consequence of our assumption is that the market takes care of deviations.
which is the "(adjusted) sensitivity-to-noise" ratio of Banker and Datar (1989) (see also Lambert (2001)). It states that the weight on a signal depends on the ratio of its sensitivity to its precision. The precision of a signal is simply its variance.

The sensitivity of a signal is a more involved notion. It describes how much a change of effort shows up in the signal. The sensitivity is adjusted when the signals are correlated. The sensitivity of a signal $A$ is lower if it is more positively correlated with another signal $B$. The intuition is that if changes in signal $B$ affect signal $A$ a lot, then there is "less room" for changes in effort to show up in signal $A$.

**Example: contracting on one signal**

The adjustment of the sensitivity will be crucial for the rest of the paper. It is therefore useful to illustrate the adjustment with following example. Earlier on we argued that the private information of an informed trader is speculative and not strategic. By this meant that the signal $s_i$ stimulates trade but does not contain any information not known to the owners of the firm. To show that the information is not strategic we can ask: suppose for a moment that owners could include an informed trader’s signal in the managerial contract instead of the price $\hat{p}$, would they want to do it? Or more formally: suppose that $I = b_0 + b_s s_i + b_v v$, what would be the weight $b_s$?

Condition (16) would then state that:

$$
\frac{b_s}{b_v} = \frac{1 - \frac{\text{Cov}[s_i,v]}{\text{Var}[v]}}{\text{Var}[s_i]} = \frac{1 - \frac{\sigma_{s_i}^2}{\sigma_{\theta}^2}}{1 - \frac{\sigma_{s_i}^2}{\sigma_{\theta}^2} + \sigma_{\varepsilon}^2} \frac{\sigma_{\theta}^2}{\sigma_{\theta}^2 + \sigma_{\varepsilon}^2} = 0
$$

This comes as no surprise. Since the signal is just the fundamental value plus noise, it does not add anything. More formally, the adjusted sensitivity of $s_i$ is zero. A regression of $s_i$ on $v$ would produce a regression coefficient $\frac{\text{Cov}[s_i,v]}{\text{Var}[v]}$ of one. A regression coefficient of one formalizes the idea that nothing is learnt by observing $s_i$ in addition to $v$. In other words, firm value at $t=2$ is a sufficient statistic (see Holmstrom (1979)) for managerial effort relative to having access to both, firm value at $t=2$ and informed traders’ information at $t=1$.

Conversely, if we allow the principal-large shareholder to acquire an informative signal in the context of Holmstrom and Tirole (1993), we can show that for some sufficiently high precision, it is optimal for the principal to write the contract on such a signal, and not on the stock price. This example then shows how different are the informational properties of a rational expectation price obtained in a context of dispersed information (ours) and the equilibrium price in a game with monopolistic information held by an insider-monitor (Holmstrom and Tirole (1993)).

We now adress two central questions: first, why base compensation on the market price? And second, when to base compensation exclusively on the market price? It will become clear at this point that we cannot answer these key questions unless one considers the endogenous nature of the share price. In order to push the argument further, one must recognize the need for an analysis of trading and price formation.

The condition on the sensitivity-to-noise ratio (equation (16)) tells us that the weight on the market price is positive if the regression coefficient of a regression of the price on firm value is less than one: $\alpha_p > 0 \iff \frac{\text{Cov}[\hat{p},v]}{\text{Var}[v]} < 1$. In other words, incentive contracting is based on the market as long as non-market information is not a sufficient statistic for the price. From the hypothesized price (13) we see that this is the case as long as the information shock about fundamental noise does not affect the price one-to-one, i.e. $\alpha_\theta < 1$. The sensitivity of the price to the information shock however

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7 A formal proof of this statement is available upon request.
is endogenous. It depends on the market conditions, e.g. liquidity, at the equilibrium of the trading model.

The same holds for our second question. The contract is based solely on the market price if the regression coefficient of a regression of the firm value on price is one: \( \hat{a}_v = 0 \Leftrightarrow \frac{Cov[p,v]}{\text{Var}[p]} = 1 \). Incentive contracting is based exclusively on the market price as long as it is a sufficient statistic for the value \( v \). This translates into a condition on a ratio of the information-to-noise shock sensitivity of the market price:

\[
\frac{\alpha_\theta(1 - \alpha_\theta)}{\alpha_u} = \frac{\sigma_u^2}{\sigma_{\theta}^2}
\]

(17)

So far, it is impossible to discuss this condition in a meaningful way. Once we bring in a model of trading however, we will be able to provide an intuitive discussion of this benchmark case.

### 3.4 Bringing in the market model

This section characterizes the closed form solution of the market model described above. It is a rational expectations model akin to Hellwig (1980), Diamond and Verrecchia (1981) or to a one-asset version of Admati (1985). The equilibrium price aggregates the different pieces of information embodied in the individual investor’s signal \( s_i \). The price however is not fully revealing because the presence of noise trading. One cannot distinguish between a high price due information-based demand or a high price due to non-information-based (i.e. noise trader) demand.

**Proposition 1** The market price net of incentive contracting \( \hat{p} \) is:

\[
\hat{p} = E[v] + \lambda \theta + \lambda u
\]

where

\[
\lambda = \frac{r + \beta \tau_u}{r \beta + \beta^2 \tau_u + \tau_\theta}
\]

\[
\beta = \frac{\tau_e}{r}
\]

and \( \tau_i = 1/\sigma_i^2 \) is the precision of the random variable \( i \).

**Proof:** In the appendix.

The proposition confirms our guess (13), that the market price is the expected value of the firm \( E[v] \) plus two random shocks.\(^8\) One shock, \( u \), is due to (random) trade that is not base on information. The resilience of the price to these shocks, \( 1/\lambda \), describes the ”depth” of the market as in Kyle (1985). A deep market does not react much to changes in the order flow. According to Black (1971), we can think of depth as a measure of market liquidity.

The market becomes more liquid if there is more non-information based trading (larger \( \sigma_u^2 \)) and/or if there is less fundamental noise (smaller \( \sigma_\theta^2 \)). The trading model therefore builds on Bagehot (1971)’s classic intuition that a market will be more liquid if trading less plagued by adverse selection.

\(^8\)To our knowledge, writing the rational expectations (RE) price as in (18), i.e. analogous to the way Kyle (1985) write his non-rational expectations price, is novel. It allows a new simple interpretation of the RE price function in terms of liquidity and trading intensity (see main text)
The other shock is due to the private information of the informed traders: the price aggregates the traders’ information about the fundamental noise $\theta$. Informed demand is part of the order flow and hence affects price according the depth/liquidity of the market, $1/\lambda$. The coefficient $\beta$ measures an informed investor’s trading intensity, i.e. it is a measure of how strongly his private information $s_i$ affects his demand $x_i$. An investor trades more intensively when is less risk averse (low $r$) and better informed (low $\sigma^2_i$).

3.5 Why base compensation on the market price?

This section answers our earlier question: why base compensation on the market price? We saw earlier on that the incentive contract includes the market price at $t=1$, $\hat{p}$, in addition to firm value at $t=2$, if the information shock about fundamental noise $\theta$ does not affect price one-for-one. Now that we know the equilibrium function (18) we can make a bolder statement: the price will always be included in the incentive contract. The price at $t=1$ always adds information about the firm value at $t=2$. To see this we just need to write out the sensitivity of the price with respect to the information shock:

$$\lambda \beta = r \beta + \beta^2 \tau_u < 1$$

We therefore have the following proposition:

**Lemma 1** The incentive contract always places positive weight on the price at $t=1$: $\hat{a}_p > 0$

At a superficial level, the result seems surprising. If an individual trader’s information is useless for incentive contracting, how the market price contain anything useful for incentive contracting?

Another, “intuitive” objection runs as follows. The competitive market as a whole ”knows” the value of the firm due to the law of large numbers. Therefore, the best the price could reflect at $t=1$ is the future value of the firm. But the future value of the firm is eventually observable and hence available for incentive contracting anyway. As a result, the price cannot add anything.

Our analysis shows that both forms of reasoning are misleading. The market price does contain valuable information for incentive contracting even though it is not fully revealing: it gives information about the fundamental noise $\theta$. This fundamental noise is the reason why there is a moral-hazard problem in the first place. By contracting on the price in addition to output $v$ some noise can be removed and a better risk-incentive trade-off can be achieved.

The ”filtering out” of fundamental noise cannot be achieved by using the information of an individual trader in addition to output since the fundamental noise shows up exactly the same in both signals. The price however is the outcome of trading by risk-averse traders, each of whom receives a different noisy signal of future firm value. Under these circumstances, there is no reason to expect that the fundamental noise shows up in the price at $t=1$ exactly the same way as it shows up later in output at $t=2$.

In fact, Lemma 1 shows that the fundamental noise cannot show up in the price the same way it shows up in output. If it did, there would be no trade. The result is reminiscent of the ”impossibility of informational efficient markets” in Grossman and Stiglitz (1980). To see this we ask: when would the fundamental noise show up in the price the same way it shows up in output, i.e. one-for-one? It does so if the precision of the an informed trader’s signal is infinite. He will then have an infinite

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9 The effect of the trading intensity on market liquidity depends on the market’s risk bearing capacity $1/r$. If it is high, then more intense trading will first decrease and then increase liquidity. If the risk bearing capacity is low, then more intense trading always increases liquidity.
trading intensity resulting in an infinitely deep market where the price’s sensitivity to \(\theta\) is one. But if the price is fully revealing then the trader has no reason to consider his own signal. And if all traders disregard their own signal so that there is no information based trading, how can the price fully be fully revealing in the first place?

The rationale for including the market price of a company in the managerial compensation scheme is related to the idea of relative performance pay. Both schemes allow for the ”filtering out” of fundamental noise. But whereas relative performance pay is rare in practice, market based compensation is ubiquitous. Our model suggests an explanation. Designing a market based compensation scheme is relatively easy compared to drafting a relative performance scheme. There is no need to find other managers who perform exactly the same task in a different but related environment. There is no need to measure the other agent’s output accurately. Many different sources of information can be tapped into. And competitive price setting together with self-interested trading appears much more robust to coordination failures and the problem of collusion.

3.6 A benchmark: when to base compensation exclusively on the market?

This section answers our second question: when would compensation be based solely on the price? What are the characteristics of a hypothetical benchmark case in which the price at \(t=1\) conveys so much information that the fundamental value at \(t=2\) is redundant?

Our initial attempt to answer the question only produced the unintelligible condition (17). Rewriting the condition in terms of \(\lambda\) and \(\beta\) yields:

\[
\lambda = \frac{\beta \tau_u \tau}{\beta^2 \tau_u + \tau \theta} \equiv \lambda^N
\]  

(19)

If we compare \(\lambda^N\) to the \(\lambda\) given in proposition 1, we see that they are identical when \(r = 0\) (and \(\beta\) not being affected by \(r = 0\)). The comparison seems to suggest that compensation is only based on the market if it is infinite risk bearing capacity (but individual traders remain risk averse). Indeed, Vives (1995) analyzes a situation where there is competitive trading by risk-averse traders but the price is not set by market clearing. Instead, it is set by a risk-neutral market making sector: \(\hat{p} = E[v|\hat{p}]\). Note that risk-neutral pricing makes the market semi-strong efficient: the price is a martingale, i.e. price at \(t=1\) allows a fair gamble on the value of the firm at \(t=2\). Vives then obtains exactly the same result, namely that with risk-neutral pricing the inverse of depth is \(\lambda^N\).

Our benchmark then is:

**Lemma 2** The incentive contract places no weight on firm value at \(t=1\), \(\hat{a}_v = 0\), if the price was semi-strong efficient, \(\hat{p} = E[v|\hat{p}]\).

The intuition is the following. A risk-neutral market making sector in our (hypothetical) benchmark incorporates the information in the order flow with maximum aggressiveness. Being risk-neutral, his trading is not affected by the noise trader component in the order flow. If he was risk-averse, he would hold back and the price would be ”polluted” by noise trading.

Proposition ?? formalizes the following common intuition:

”assuming that the market is ’efficient’ one might expect stock price to ’correctly’ aggregate all of the information signal […]. This suggests that the optimal compensation contract should depend solely on stock price.” (Lambert (1993))

Our discussion however makes it clear that this can only be the (hypothetical) benchmark case, e.g. the case if there was a market-making sector with infinite risk-bearing capacity. Outside the benchmark,
the incentive contract must contain both market and non-market based compensation. The relative weight of these two components and the impact of market liquidity on the relative weight is analyzed next.

### 3.7 Market based compensation and liquidity

Having shown that except in the benchmark case of a market with infinite risk-bearing capacity, optimal compensation will always include both, market and non-market based elements, we now relate the relative weight of these two elements to the liquidity of the market.

The following proposition contains our central result:

**Proposition 2** The ratio of the optimal weights on the market price at $=1$, $\hat{p}$, and on the value of the firm at $=2$, $v$, satisfies:

$$\frac{\hat{a}_p}{\hat{a}_v} = \frac{1}{\lambda r \sigma_n^2}$$

(20)

The proposition shows that the ratio of market to non-market based compensation is proportional to three market characteristics: liquidity ($1/\lambda$), risk bearing capacity ($1/r$) and non-information (noise) trading volatility ($\sigma_n^2$).

The intuition for the positive effect of liquidity on the amount of market based compensation is the following. We saw that point of market based compensation was to filter out the fundamental noise that is causing the moral-hazard problem in the first place. The filtering out is more efficient if there is a large difference between the impact of fundamental noise on market and non-market information, i.e. if the price at $=1$ and firm value at $=2$ covary little with each other. A liquid market is resilient to shocks in the order flow (caused partly by fundamental noise), thus creating the desired difference in impact.

Note that noise trading has opposing direct and indirect effects on the degree of market based compensation. Less noise trading directly reduces the volatility of the price. This makes the price a more precise measure of managerial effort. Less noise trading however also decreases liquidity since it worsens the adverse selection problem when trading. This makes the price a less sensitive measure of managerial effort.\(^{10}\)

An appealing feature of our result in proposition 2 is that the ratio of market to non-market based compensation is independent of the characteristics of the manager. The ratio is independent of managerial risk aversion or the manager’s marginal cost of effort. The reason is that these characteristics affect market and non-market based compensation in the same proportion. This property will be especially valuable when designing empirical tests since it allows us to avoid the omitted variables problem due to non-observability of managerial preferences for risk and effort.

### 3.8 Market based compensation and effort

In the last step of the analysis we consider the impact of market based compensation on the level of managerial effort. For this, we need to find the optimal contract $(\hat{a}_0, \hat{a}_p, \hat{a}_v)$ that maximizes the

\(^{10}\)The overall effect on the ratio however is positive, i.e. the precision effect prevails. The role of the risk bearing capacity is more ambiguous. A market with a higher risk bearing capacity provides a less volatile price but the impact via liquidity is ambivalent. A higher risk bearing capacity increases liquidity only if $\tau_e > \tau_\theta/2$. 

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expected net value of the firm at t=0 \((3)\) subject to the first-order condition of the manager’s effort choice problem \((14)\), the minimum income risk condition \((20)\) and the manager’s participation constraint \((5)\)\(^{11}\).

The following proposition solves the optimization problem in terms of effort:\(^{12}\)

**Proposition 3** The manager exerts a (second-best) effort level:

\[
e^\text{sb} = \frac{1}{k(1 + kr_m\sigma^2_\theta K)}
\]

where

\[
K = \frac{1}{1 + \frac{\tau_\theta}{(\tau + \beta \tau_u)^2}}
\]

**Proof:** In the appendix. ■

In order to interpret the proposition, consider for a moment the (third-best) effort the manager would take if the incentive contract was only based on the value of the firm at t=2, \(I = b_0 + b_vv\):

\[
e^{tb} = \frac{1}{k(1 + kr_m\sigma^2_\theta)}
\]

Comparing \((21)\) and \((23)\), remembering the first-best level of effort would be \(e^{fb} = 1/k\), we have: \(e^{fb} > e^{sb} > e^{tb}\). Adding market-based compensation to compensation based solely on final output increases managerial effort. The increase is more pronounced if there is less fundamental noise (high \(\tau_\theta\)) or the market has a higher risk bearing capacity (low \(r\)). The impact of less noise trading (high \(\tau_u\)) is again ambiguous. Note that effort increases if informed traders trade less intensively on their private information (low \(\beta\)). This is not immediately clear since the trading intensity has a direct effect via stock price volatility and an indirect effect via market liquidity. The direct effect however prevails (see the Appendix for the details). Finally, if we write the parameter \(K\) as

\[
K = \frac{\lambda r + \lambda \beta \tau_u}{\lambda r + \tau_u}
\]

we see that a more liquid market (low \(\lambda\)) leads to more effort.

The comparative statics of the expected net value of the firm \(E[v - I]\) at the optimum with respect to market liquidity are ambiguous. For example, a higher liquidity decreases the volatility of the price (which increases net firm value since it decreases the income risk borne by the manager) but it also increases the weight given to the price in the incentive contract (which decreases net firm value since it exposes the manager to more income risk).

### 3.9 An extension: endogenous collection of information

The question we address here is whether higher liquidity trade volatility induces the competitive trader to collect a more precise information signal, that is, formally, whether higher \(\sigma^2_u\) induces a lower \(\sigma^2_\varepsilon\) when the information collection is costly. This effect is a key element in the argument of Holmstrom

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\(^{11}\)Which is binding as usual without limited liability constraints.

\(^{12}\)Solving the optimization for effort is equivalent since the three constraints \((14)\), \((20)\) and \((5)\) define the optimal contract in terms of effort: \((\hat{a}_0(e), \hat{a}_\mu(e), \hat{a}_v(e))\)
and Tirole (1993), since they argue that quoting a larger share of the firm on the secondary market increase the incentive to collect information by a single insider and then provides a more informative price.

We start computing the profit (in terms of utility, since our traders are risk-averse) for an informed speculator who received the signal \( s_i \) with precision \( \tau_e \) in equilibrium:

\[
E[\pi_i|s_i, \hat{p}] - \frac{r}{2} \text{Var}[\pi_i|s_i, \hat{p}]
\]

with \( \hat{p} = E(v) + \lambda \theta + \lambda u \) and (Proposition 1):

\[
\lambda = \frac{r + \beta \tau_u}{r \beta + \beta^2 \tau_u + \tau \theta} \\
\beta = \frac{\tau_e}{r}
\]

The interim profit is \( \pi_i \) (conditional to the actual realization of the signal \( s_i \) and of the normalized price \( \hat{p} \) is equal to:

\[
\pi_i = x_i (v - \hat{p})
\]

where \( \hat{p} \) is a random variable ex-ante (i.e. before the realization of the signals \( s_i \) and the trade \( u \)). Computing the conditional expectations and variance of \( \pi_i \) and substituting for \( x_i \):

\[
E[x_i (v - \hat{p})|s_i, p] = x_i (E[v|s_i, p] - \hat{p})
\]

\[
= \frac{E[v|s_i, p] - \hat{p}}{r \text{Var}[v|s_i, \hat{p}]} (E[v|s_i, p] - \hat{p}) = \frac{(E[v|s_i, \hat{p}] - \hat{p})^2}{r \text{Var}[v|s_i, \hat{p}]}
\]

\[
\text{Var}[\pi_i|s_i, p] = (x_i)^2 \text{Var}[v - \hat{p}|s_i, p]
\]

\[
= \frac{(E[v|s_i, \hat{p}] - \hat{p})^2}{r^2 \text{Var}[v|s_i, \hat{p}]}
\]

so the (interim-) objective is then:

\[
E[\pi_i|s_i, \hat{p}] - \frac{r}{2} \text{Var}[\pi_i|s_i, \hat{p}] = \frac{1}{2r} \frac{(E[v|s_i, \hat{p}] - \hat{p})^2}{\text{Var}[v|s_i, \hat{p}]}
\]

and using the projection theorem:

\[
E(v|s_i, \hat{p}) = \frac{s_i \tau_e + \hat{p} \tau \hat{p}}{\tau_e + \tau \hat{p}} = \hat{p} + \frac{\tau_e}{\tau_e + \tau \hat{p}} (s_i - \hat{p})
\]

\[
\text{Var}(v|s_i, \hat{p}) = \frac{\sigma^2 \beta^2 \sigma^2 + \sigma^2 - \sigma^2 \beta^2 \sigma^2}{\sigma^2 \beta^2 \sigma^2 + \sigma^2 - \sigma^2 \beta^2 \sigma^2}
\]

\[
= \frac{\sigma^2 \beta^2 \sigma^2}{\sigma^2 \beta^2 \sigma^2 + \sigma^2 - \sigma^2 \beta^2 \sigma^2}
\]

\[
= r^4 \left( \sigma^2 - \sigma^2 \right) \frac{\sigma^2 \beta^2 \sigma^2}{\sigma^2 \beta^2 \sigma^2 + \sigma^2 - \sigma^2 \beta^2 \sigma^2}
\]

\[
= \frac{\sigma^2 \beta^2 \sigma^2}{\sigma^2 \beta^2 \sigma^2 + \sigma^2 \beta^2 \sigma^2}
\]

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Finally:

\[
E[\pi_i|s_i, \hat{p}] - \frac{r}{2} Var[\pi_i|s_i, \hat{p}] = \frac{1}{2r} \left( \frac{E[v|s_i, \hat{p}] - \hat{p}}{Var[v|s_i, \hat{p}]} \right) = \frac{1}{2r} \left( \frac{r}{\tau_i + \tau_s} (s_i - \hat{p}) \right)^2
\]

and the ex-ante objective function (unconditional to the observation of the private signal and the price) is then obtained integrating the above expression for the (joint) distribution law \( F \) of \( s_i, \theta, u \), (hence \( \hat{p} \)):

\[
\frac{1}{2r} \left( \frac{\tau_i}{\tau_i + \tau_s} \right)^2 \int (s_i - \hat{p})^2 dF = \frac{1}{2r} \left( \frac{\tau_i}{\tau_i + \tau_s} \right)^2 \int (s_i - \hat{p})^2 dF
\]

\[
= \frac{1}{2r} \left( \frac{\tau_i}{\tau_i + \tau_s} \right) \int \left( \theta (1 - \lambda \beta) + \varepsilon_i - \lambda u \right)^2 dF
\]

\[
= \frac{1}{2r} \left( \frac{\tau_i}{\tau_i + \tau_s} \right) \left( \lambda \beta \right)^2 \int \left( \theta (1 - \lambda \beta) + \varepsilon_i - \lambda u \right)^2 dF
\]

since \( \theta \) and \( u \) and \( \varepsilon_i \) are not correlated and they are normally distributed with zero mean.\(^{13}\)

Substituting for the equilibrium values of \( \lambda \) and \( \beta \) we obtain the ex-ante profit for an informed speculator as a function of the precisions of his informative signals and other noises of the market:

\[
\frac{1}{2r} \left( \frac{\tau_i}{\tau_i + \tau_s} \right)^2 \int (s_i - \hat{p})^2 dF = \frac{1}{2r} \left( \frac{\tau_i}{\tau_i + \tau_s} \right)^2 \int (s_i - \hat{p})^2 dF
\]

\[
= \frac{1}{2r} \left( \frac{\tau_i}{\tau_i + \tau_s} \right) \left( \lambda \beta \right)^2 \int \left( \theta (1 - \lambda \beta) + \varepsilon_i - \lambda u \right)^2 dF
\]

\[
= \frac{1}{2r} \left( \frac{\tau_i}{\tau_i + \tau_s} \right) \left( \lambda \beta \right)^2 \int \left( \theta (1 - \lambda \beta) + \varepsilon_i - \lambda u \right)^2 dF
\]

with \( \Upsilon \) being a complex function of the parameters variances.\(^{14}\)

If the cost of information collection \( c(\sigma_e^2) \) is convex the problem of choosing the optimal signal precision is:

\[
\max_{\sigma_e^2} \Upsilon \left( \sigma_e^2, \sigma_o^2, \sigma_u^2, r \right) = \frac{1}{2\sigma_e^2}
\]

Imposing \( \sigma_o^2 = r = 1 \) we can simplify the function \( \Upsilon \) :

\[
\Upsilon \left( \tau_e, \tau_u \right) = \frac{\left( \frac{\tau_e}{\tau_u} + 4 + \tau_u \right)^2}{\tau_u \left( 1 + \tau_u \right)} \left( \frac{\tau_u \left( \tau_u + 1 \right)}{\tau_u \left( 1 + \tau_u \right)} \right)^2 \left( \frac{\tau_u \left( \tau_u + 1 \right)}{\tau_u \left( 1 + \tau_u \right)} \right)^2 \left( \frac{\tau_u \left( \tau_u + 1 \right)}{\tau_u \left( 1 + \tau_u \right)} \right)^2 \left( \frac{\tau_u \left( \tau_u + 1 \right)}{\tau_u \left( 1 + \tau_u \right)} \right)^2
\]

obtaining the following plot:

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\(^{13}\)Hence, given that \( \theta \) (and \( u \) and \( \varepsilon_i \)) are normally distributed, \( E(\theta^2) = Var(\theta) \).

\(^{14}\)\( \Upsilon \equiv \frac{1}{2r} \left( \frac{\tau_i}{\tau_i + \tau_s} \right)^2 \int \left( \theta (1 - \lambda \beta) + \varepsilon_i - \lambda u \right)^2 dF \).
From the figure above, it is clear that the locus of the maxima $\tau_\varepsilon (\tau_u)$ is approximately constant in $\tau_u$: in particular, with a lower the precision $\tau_u$ of the liquidity trade, the speculator does not choose a higher precision $\tau_\varepsilon$, which is in contrast with the prediction of Holmstrom and Tirole (1993). In our model, then, higher liquidity trade does not provide incentives to collect more precise information. This is easy to understand: in our model, contrarily to Holmstrom and Tirole (1993) no rational investor possess a superior information than other rational investors. In other words, all the investors are (ex-ante) the same, and if it is profitable for one of them to acquire a more precise costly signal when the noise trade is more volatile, this is true also for all the others: but then their profit, due to the competitive behavior, does not increase; hence it cannot be profitable for a single investor to spend more in information acquisition just because the volume of noise trade is higher.

4 Conclusion

This paper analyzed the role of trading and market liquidity on market-based management compensation. Our starting point was to recognize that the stock price is itself endogenous - it is the outcome of trading motivated by anticipations of future firm value.

We motivate trade as in the standard competitive rational expectations framework: traders in a competitive market have heterogenous information, i.e. no single trader knows better than the market and no trader knows something that the owner of the firm does not know (remember, no one has a handle of luck vs. skill). Ours is a much weaker set of assumptions than Holmstrom and Tirole (1993): the price does not contain any insider information and yet, it will be useful for incentive contracting. Even though no outside trader has better information than the inside owner of the firm, their trading results in a stock price that is useful for incentive contracting since it allows the filtering out of noise that is not related to performance.

Our central result is that the degree of market-based compensation is proportional to market liquidity. Filtering out luck is more efficient when there is a large difference between the impact of luck on observable output and on observable stock price. A liquid market is defined to be resilient to shocks in the order flow (caused partly but not entirely by trader anticipation of managerial luck), thus creating the desired difference in impact.
The paper also admits a useful (hypothetical) benchmark: if the market was (semi-strong) efficient, all compensation would be based on the market price. The value of the benchmark is that it spells out precisely what we mean by "efficient". Without it, we would not know what real-world inefficiencies to look out for and what inefficiencies drive the departure from pure market-based compensation. Our model traces pay that is not linked to the stock price directly to market inefficiency or its limited risk-bearing capacity.
5 Bibliography


6 Appendix

**Proof of Proposition 1**: We follow here the proof of Admati (1985), adjusting it in a one-asset setting. Conjecture a linear price function for \( \hat{p} \): \( \hat{p} = \alpha_0 + \alpha_1 \bar{v} - \alpha_2 \bar{Z} \). The conditional distribution of \( \bar{v} \) given \((s_i, \hat{p})\) is normal (Projection Theorem for Normal variables), with a conditional expectation linear in the realizations equal to:

\[
E[\bar{v}|s_i, \hat{p}] = B_{0i} + B_{1i}s_i + B_{2i}\hat{p}
\]

while the conditional variance \( V_i \) is a constant, independent of the realizations of \( s_i, \hat{p} \):

\[
Var[\bar{v}|s_i, \hat{p}] = V_i
\]

Given the conjectured price function \( \hat{p} = \alpha_0 + \alpha_1 \bar{v} - \alpha_2 \bar{u} \), then the optimal demand for \( i \) with CARA utility is:

\[
x^d_i = \frac{B_{0i} + B_{1i}s_i + (B_{2i} - 1)\hat{p}}{r_iV_i} \tag{25}
\]
and the market-clearing condition reads:

\[
\int_0^1 \frac{B_0i + B_1i s_i + (B_2i - 1) \hat{p} i}{r_i V_i} di = u
\]

\[
\hat{p} \int_0^1 \frac{1 - B_2i}{r_i V_i} di = \int_0^1 \frac{B_0i + B_1i s_i}{r_i V_i} di - u
\]

\[
\hat{p} = \left( \int_0^1 \frac{1 - B_2i}{r_i V_i} di \right)^{-1} \left( \int_0^1 \frac{B_0i}{r_i V_i} di + \int_0^1 \frac{B_1i s_i}{r_i V_i} di - u \right)
\]

The pricing function \( \hat{p} = \alpha_0 + \alpha_1 \alpha_2 \tilde{Z} \) is a REE only if, identifying the coefficients:

\[
\alpha_2 = \left( \int_0^1 \frac{1 - B_2i}{r_i V_i} di \right)^{-1}
\]

\[
\alpha_0 = \alpha_2 \int_0^1 \frac{B_0i}{r_i V_i} di
\]

\[
\alpha_1 = \alpha_2 \int_0^1 \frac{B_1i s_i}{r_i V_i} di
\]

since \( \int_0^1 \frac{B_0i s_i}{r_i V_i} di = \int_0^1 \frac{B_1i (v + \epsilon)}{r_i V_i} di = v \int_0^1 \frac{B_1i s_i}{r_i V_i} di \) since \( \int_0^1 \epsilon_i di = 0 \). To obtain the closed-form solution of parameters \( (\alpha_0, \alpha_1, \alpha_2) \) just apply in our one-dimensional case Lemma 3.2 of Admati (1985) that gives the following:

\[
\alpha_0 = \frac{1}{r} \frac{\tau_0 E(v)}{\tau_0 + \frac{1}{r} Q^2 \tau_u + Q} + \frac{1}{r} \frac{Q \theta}{\tau_0 + \frac{1}{r} Q^2 \tau_u + Q}
\]

\[
\alpha_1 = \alpha_2 Q
\]

\[
\alpha_2 = \frac{1 + \frac{1}{r} Q \tau_u}{\frac{1}{r} \tau_0 + \frac{1}{r} Q^2 \tau_u + Q}
\]

where \( Q = \int_0^1 \frac{\tau_u}{r_i V_i} di \) is the weighted average precision of the informational signals, and \( \frac{1}{r} = \int_0^1 \frac{1}{di} \) is the average risk-aversion. This corresponds exactly to Theorem 3.1 in Admati (1985) in a setting with a single asset, which is also obtained by Hellwig (1980) (cfr. Proposition 5.2).

In order to obtain the expression for \( \hat{p} \) simply assume that \( \tilde{Z} = 0 \) and then notice that \( \frac{1}{r} \frac{\tau_0}{\tau_0 + \frac{1}{r} Q^2 \tau_u + Q} + \alpha_1 = 1 \). Rewriting:

\[
\hat{p} = \frac{1}{r} \frac{\tau_0}{\tau_0 + \frac{1}{r} Q^2 \tau_u + Q} E(v) + \alpha_1 (E(v) + \theta) + \alpha_2 u
\]

\[
= E(v) + \alpha_2 Q \theta + \alpha_2 u
\]

and denoting \( \beta \equiv Q \) and \( \alpha_2 = \lambda \) we obtain (18). The result in \( p \) is simply obtained by the relation \( p = \hat{p} - Ap - W \).

Proof of Proposition 2: Substitute into (16) the expressions for

\[
Var(\hat{p}) = \alpha_1^2 \sigma_\theta^2 + \alpha_2^2 \sigma_u^2
\]

\[
Cov(\hat{p}, v) = \alpha_1 \sigma_\theta^2
\]
obtaining

\[
\frac{\hat{a}_v}{a_p} = \frac{\alpha^2 \sigma^2_u - \alpha_1 \sigma^2_\theta (1 - \alpha_1)}{\sigma^2_\theta (1 - \alpha_1)} = \frac{\alpha^2 \sigma^2_u - \alpha_2 Q \sigma^2_\theta (1 - \alpha_2 Q)}{\sigma^2_\theta (1 - \alpha_2 Q)}
\]

\[
= \alpha_2 \left( \frac{\alpha^2 \sigma^2_u}{\sigma^2_\theta (1 - \alpha_2 Q)} - Q \right)
\]

\[
= \alpha_2 \left( \frac{1}{\tau_u} - \frac{\tau_c}{\tau} \right)
\]

\[
= \alpha_2 \left( \frac{r}{\tau_u} + Q - Q \right) = \alpha_2 \frac{r}{\tau_u} = \alpha_2 r \sigma^2_u
\]

**Proof of Proposition 3:** From (IC) and (MV) one can obtain the explicit expression for \((\hat{a}_2, \hat{a}_3)\) as functions of \(e\):

\[
\hat{a}_2 + \hat{a}_3 = ke
\]

\[
\frac{\hat{a}_3}{\hat{a}_2} = \frac{ke}{1 + \alpha_2 r \sigma^2_u}
\]

\[
= \frac{ke}{1 + \alpha_2 r \sigma^2_u}
\]

so that, using these expression into the objective function:

\[
e - \frac{r_m e}{2} \text{Var}(I) - \frac{k}{2} e^2
\]

\[
= e - \frac{r_m}{2} \left( \left( \frac{ke}{1 + \alpha_2 r \sigma^2_u} \right)^2 \text{Var}(\hat{p}) + \left( \frac{\alpha_2 r \sigma^2_u}{1 + \alpha_2 r \sigma^2_u} \right)^2 \frac{2 \alpha_1 \sigma^2_\theta}{(1 + \alpha_2 r \sigma^2_u)} \hat{a}_2 + 2 \alpha_1 \sigma^2_\theta \left( \frac{ke}{1 + \alpha_2 r \sigma^2_u} \right) \left( \frac{\alpha_2 r \sigma^2_u}{1 + \alpha_2 r \sigma^2_u} \right) e \right) - \frac{k}{2} e^2
\]

so that the f.o.c. in \(e\) reads:

\[
\frac{\partial(e - r_m \text{Var}(I) - k e)}{\partial e} = 1 - \frac{r_m}{2} \left( 2 e \left( \frac{ke}{1 + \alpha_2 r \sigma^2_u} \right)^2 \text{Var}(\hat{p}) + 2 e \left( \frac{\alpha_2 r \sigma^2_u}{1 + \alpha_2 r \sigma^2_u} \right)^2 \frac{2 \alpha_1 \sigma^2_\theta}{(1 + \alpha_2 r \sigma^2_u)} \hat{a}_2 + 4 \alpha_1 \sigma^2_\theta \left( \frac{1 + \alpha_2 r \sigma^2_u}{1 + \alpha_2 r \sigma^2_u} \right) e \right) - k e = 0
\]

\[
e \left( kr_m \left( \frac{1}{1 + \alpha_2 r \sigma^2_u} \right)^2 \text{Var}(\hat{p}) + kr_m \left( \frac{\alpha_2 r \sigma^2_u}{1 + \alpha_2 r \sigma^2_u} \right)^2 \frac{2 \alpha_1 \sigma^2_\theta}{(1 + \alpha_2 r \sigma^2_u)} \hat{a}_2 + 2 k r m \alpha_1 \sigma^2_\theta \left( \frac{1 + \alpha_2 r \sigma^2_u}{1 + \alpha_2 r \sigma^2_u} \right) e \right) = 1
\]

\[
e = \frac{1}{k + k r_m} \left( \frac{1}{1 + \alpha_2 r \sigma^2_u} \right)^2 \text{Var}(\hat{p}) + \left( \frac{\alpha_2 r \sigma^2_u}{1 + \alpha_2 r \sigma^2_u} \right)^2 \frac{2 \alpha_1 \sigma^2_\theta}{(1 + \alpha_2 r \sigma^2_u)} \hat{a}_2 + \frac{1}{k + k r_m} \left( \frac{2 k r m \alpha_1 \sigma^2_\theta}{(1 + \alpha_2 r \sigma^2_u)} \right)
\]

and with \(\text{Var}(\hat{p}) = (\alpha_2 Q)^2 \sigma^2_\theta + \alpha^2_2 \sigma^2_\theta\):

\[
e = \frac{1}{k + k r_m} \left( \frac{1}{1 + \alpha_2 r \sigma^2_u} \right)^2 ((\alpha_2 Q)^2 \sigma^2_\theta + \alpha^2_2 \sigma^2_\theta) \left( \frac{2 \alpha_1 \sigma^2_\theta}{(1 + \alpha_2 r \sigma^2_u)} \right)^2 + \frac{1}{k + k r_m} \left( \frac{2 k r m \alpha_1 \sigma^2_\theta}{(1 + \alpha_2 r \sigma^2_u)} \right)
\]

\[
= \frac{1}{k + k r_m} \left( \frac{1}{1 + \alpha_2 r \sigma^2_u} \right)^2 ((\alpha_2 Q)^2 \sigma^2_\theta + \alpha^2_2 \sigma^2_\theta) \left( \frac{2 \alpha_1 \sigma^2_\theta}{(1 + \alpha_2 r \sigma^2_u)} \right)^2 + \frac{1}{k + k r_m} \left( \frac{2 k r m \alpha_1 \sigma^2_\theta}{(1 + \alpha_2 r \sigma^2_u)} \right)
\]

\[
= \frac{1}{k + k r_m} \left( \frac{1}{1 + \alpha_2 r \sigma^2_u} \right)^2 ((\alpha_2 Q)^2 \sigma^2_\theta + \alpha^2_2 \sigma^2_\theta) \left( \frac{2 \alpha_1 \sigma^2_\theta}{(1 + \alpha_2 r \sigma^2_u)} \right)^2 + \frac{1}{k + k r_m} \left( \frac{2 k r m \alpha_1 \sigma^2_\theta}{(1 + \alpha_2 r \sigma^2_u)} \right)
\]
Comparative statics: difference from Holmstrom and Tirole. Our (inverse of) liquidity measure is: \( \lambda = \frac{r + \beta \tau_u}{r \beta \tau^2 u + \tau \theta} \). It is immediate to verify that liquidity decreases (i.e. \( \lambda \) increases) when the volatility of the noise trading decreases (i.e. when its precision \( \tau_u \) increases):

\[
\frac{\partial \lambda}{\partial \tau_u} = \frac{\tau \theta}{(r \beta + \beta^2 \tau_u + \tau \theta)^2} > 0
\]

This is according to HT and Garvey and Swan (1999) empirically verify this prediction.

A different empirical implication from HT is the following: our model predicts that the ratio between the weight on the market price and the weight on \( v \) in the optimal contract increases with the precision (i.e. decreases with the volatility) of liquidity trade.

\[
\frac{\partial \hat{\rho}}{\partial \tau_u} = \frac{1}{\lambda r \sigma_u^2} = \frac{\tau_u \tau^2 + \tau^2 \tau u + \tau \theta \tau^2}{r^2 + \tau \tau_u}
\]

\[
\frac{\partial \left( \frac{\hat{a}_u}{\hat{a}_v} \right)}{\partial \tau_u} = \frac{\tau \tau^4 + 2 \tau^2 \tau^2 v + \tau^2 \tau^4 + \tau \theta \tau^4}{r^2 (r^2 + \tau \tau_u)^2} > 0
\]

In HT the relation is the opposite, since higher liquidity trade volatility induces the risk-neutral insider to acquire more precise information, that creates a less volatile price, hence a higher weight on the market price in the optimal contract.

Theoretically, the difference is important, since it is due to the different kind of information we focus on. In our model, rational speculators trade among themselves because they have received different information signals who do not give them an informational advantage as in HT against the uninformed market makers (the so called strategic information) but simply induce heterogeneous beliefs over the future value of the stock. More noisy liquidity trade, then, does not lead to more monitoring in our model, contrarily to HT, but it simply makes the price ex-ante more volatile, hence less precise as a contracting variable.