Cheap Talk Comparisons
by a Clearly Biased Expert*

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Abstract

We consider cheap talk by a biased expert comparing multiple issues, e.g., discussion of different spending proposals by an industry lobbyist, evaluation of different stocks by a sell-side analyst, or analysis of different topics by a biased newspaper. When the expert’s motives are sufficiently transparent, we find that cheap talk is credible and influential even when the expert strongly favors one issue over another, e.g., negative advertising against a rival by a political or marketing campaign. The transparency condition holds for state-independent preferences and for standard Euclidean preferences as the expert’s bias toward a higher (or lower) action on each issue becomes arbitrarily large.

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1 Introduction

Can a biased expert be trusted by a decision maker? If the expert has information on one issue, Crawford and Sobel (1982) show that costless, unverifiable “cheap talk” is credible if the expert’s bias toward a higher action is not too large. For multiple issues, communication possibilities expand because the expert can use comparative statements across issues, e.g., a stock analyst can compare the prospects of different stocks or a lobbyist can rank the benefits of different projects. Such comparisons are credible under standard complementarity conditions if the expert’s biases on each issue and other aspects of the environment are sufficiently symmetric (Chakraborty and Harbaugh, 2006).\(^1\)

This paper examines asymmetric environments where the expert may care arbitrarily more about some issues than others. For instance, a lobbyist is paid to promote certain projects, a stock analyst has an incentive to push certain stocks, or a newspaper has an ideological bias regarding particular topics. In contrast with the literature on one-dimensional cheap talk, we consider situations where the expert’s preferences over actions on each issue are largely independent of the merits of the situation, e.g., a seller wants a buyer to value a product highly regardless of its true quality. Instead, the main issue is how much weight the expert puts on each issue, e.g., whether a salesperson cares more about promoting a particular product due to a higher commission.

If the expert’s motives are transparent, i.e., the decision maker knows the expert’s weights on the issues, we find that influential cheap talk equilibria based on two-message comparative statements exist as long as the expert’s preferences are continuous. In equilibrium, the decision maker is more influenced by the expert on the issues that the expert cares less about, so the expert’s incentive to boost a favored issue is eliminated. For instance, if a stock analyst is known to benefit more from pushing up the value of one stock than another, an investor can adjust his interpretation of the analyst’s comparisons of the stocks so that the analyst has no incentive to lie.

This existence result holds even if the expert puts negative weight on one or more of the issues, e.g., when a political campaign discusses an opposing candidate or a company discusses a competitor’s product. Rather than interpreting the comparative messages as good news about one issue and bad news about the other, the decision maker interprets one message as generally good news about both issues and the other message as generally bad news about both issues. This can explain why positive statements about a rival candidate

\(^1\)For multidimensional cheap talk with more than one expert, see Austen-Smith (1993), Battaglini (2002), and Ambrus and Takahashi (2006).
or product raise the image of both sides, while “going negative” lowers the image of both sides (Ansolabehere and Iyengar, 1997; Jain and Posovac, 2004).

If the expert’s payoffs depend linearly on the decision maker’s actions, we find that more detailed comparisons by the expert are credible. Rather than choosing from just two influential messages, the expert can repeatedly subdivide the information space into smaller regions, implying the existence of $k$ influential messages for any finite $k$. As a result, the decision maker can learn very precise information about the relative values of the issues. For instance, rather than just indicating which of two stocks is recommended, a stock analyst can indicate how much more valuable one stock is than another. Similarly, a lobbyist can indicate not only that one project is better than another, but can credibly indicate how much better.

Linear preferences are of particular interest because they are the limiting case of standard Euclidean preferences as the bias toward a higher (or lower) action in each dimension becomes larger. As these biases increase, Euclidean preferences uniformly converge to linear preferences with known weights corresponding to the ratios of the biases in each dimension. Since there is always a pure cheap talk equilibrium for linear preferences with known weights, it follows from this convergence that there always exists an almost-cheap talk equilibrium (Kartik, 2005) for Euclidean preferences with sufficiently large known biases. Moreover, as the biases become large, the set of such equilibria converges to the set of cheap talk equilibria for the corresponding linear preferences with known weights. These results imply that analysis of Euclidean preferences with sufficiently large biases may best be achieved through analysis of the corresponding linear preferences.

Regarding the informativeness of cheap talk, the equilibrium messages we consider balance out the expert’s favoritism on the different issues, so more information is provided on some issues than others. Even when asymmetries reduce the informativeness of the expert’s messages on a subset of issues, we find that informativeness on the other issues increases, so the equilibria remain informative and influential overall. In particular, for sufficient transparency of the expert’s motives, there always exists an equilibrium in which the expert’s two different equilibrium messages imply two distinct equilibrium actions by the decision maker that each occur with probability of at least one-third. When the action space is at least three dimensional, there exists an equilibrium where each of the two messages have ex-ante probability of one-half. When we consider linear preferences, the ability to further subdivide the information space implies that the informativeness of the equilibria is even higher.

If the expert’s motives are not transparent, communication can break down or become
less informative. For instance, if a salesperson earns a higher commission on one product or another, but the customer does not know which, the consumer has good reason to be suspicious. Nevertheless, the equilibria we analyze still obtain given sufficient transparency. That is, an influential cheap talk equilibrium exists whenever the number of issues is larger than the number of possible expert types, provided only that preferences are continuous. For instance, if there are three products to be compared, communication is always possible if there are two potential commission structures for the salesperson. If we restrict attention to linear preferences, we find that an influential cheap talk equilibrium always exists for any number of issues as long as some type of the expert has positive mass. For instance, if a customer is unsure whether a salesperson is impartial or instead favors the more expensive of two products, the customer can still benefit from the salesperson’s advice. However, communication is less informative than if the seller’s incentives are known.

These results on the benefits of transparency in multi-dimensional settings contrast with results in the literature on transparency in one-dimensional settings. Morgan and Stocken (2003) consider a model where an investor does not know if a stock analyst is biased or not. They find that, relative to the case where biases are known, an unbiased analyst reveals less information but a biased analyst reveals more information, so the overall impact of transparency is ambiguous. Dimitrakis and Sarafidis (2004) find that if the decision maker does not know the expert’s bias and the bias has full support on the action space then communication is always possible, even when the bias is likely to be so large that communication would rarely be possible if the bias were known. Li (2004) allows the expert’s bias to be positive or negative and finds that disclosure of biases never improves communication.

From a policy perspective, our results imply that an alternative to eliminating bias among experts in multidimensional settings is ensuring sufficient transparency about the expert’s incentives. Rather than requiring that stock analysts be insulated from any incentive to promote a certain stock, it might be sufficient to ensure that their incentives are fully disclosed. And rather than restricting political advertisements, it might be sufficient to require disclosure of funding sources. Similarly, even if a media outlet is biased, this may not impede communication as long as there are multiple issues being reported on and readers are familiar with the bias.

The credibility of cheap talk in our model depends on the ability of the expert to trade off actions across multiple issues. This analysis adds to the more extensive literatures on how communication is affected by other factors such as multiple periods (Sobel, 1985; Morris, 2001; Gentkow and Shapiro, 2006) and multiple competing senders (Gilligan and Krehbiel,
1989; Austen-Smith, 1993; Krishna and Morgan, 2000; Battaglini, 2002; Mullainathan and Shleifer, 2005; Ambrus and Takahashi, 2006). Even when motives are sufficiently transparent to permit cheap talk in our model, the sender’s information is not fully revealed, so the role of other factors in encouraging (or discouraging) communication remains.

The following section provides sufficient conditions for the existence of influential partitional cheap talk equilibria based on the number of dimensions and the transparency of the expert’s motives. Section 3 considers linear preferences and shows that even more information can be revealed. We argue that linear preferences provide a simple and natural model for a number of important economic and social environments. Section 4 then shows the close connection between Euclidean distance preferences and linear preferences. Section 5 concludes the paper.

2 The model

A sender (a.k.a. expert) is privately informed about the ideal actions $\theta \in \Theta$ of a receiver (a.k.a. decision maker) where $\Theta$ is a convex subset of $\mathbb{R}^N$ with a non-empty interior $\text{int}(\Theta)$, and $N \geq 2$. She sends advice in the form of a cheap talk message $m$ from an arbitrary set $M$ to the uninformed receiver whose prior beliefs about $\theta$ are summarized by a joint distribution $F$. The distribution is “well-behaved” in that it admits a continuous density $f$ that has support on $\Theta$ and also has finite moments. The receiver then chooses actions $a \in A \equiv \Theta$ equal to $E[\theta|m]$, the expected value of $\theta$ given his priors and the sender’s message $m$.

The behavioral assumption that the receiver’s actions equal the estimated value of $\theta$ could reflect underlying preferences for ideal actions as close as possible to $\theta$. Alternatively, as is standard in many sender-receiver games, it could capture competition between multiple uninformed receivers who each try to estimate $\theta$. For instance, $\theta$ could represent the value of goods or assets that buyers wish to purchase, with the actions $a$ being the prices paid to the seller by the competing buyers.

The sender’s preferences over receiver actions $a$ are specified by a continuous function $U^S(\cdot; t) : A \to \mathbb{R}$ that depends on her type $t \in T$ that is also her private information (in addition to $\theta$). In most of the literature on cheap talk games it is assumed that $T = \Theta (= A)$.

In contrast, we find it useful to draw a distinction between $t$ and $\theta$, i.e., between uncertainty about the sender’s motives and uncertainty about the receiver’s ideal course of action. This is especially useful when the sender’s bias $b$ is not known to the receiver, i.e., $t = (\theta, b)$.

\footnote{Exceptions are Morgan and Stocken (2003), Dimitrakis and Sarafidis (2004), and Li (2004), who consider one-dimensional models where the sender’s bias $b$ is not known to the receiver, i.e., $t = (\theta, b)$.}
We focus on the case where $T$ is a finite set with $T \geq 1$ elements. When $T = 1$ the sender’s motives are known with certainty. When $T > 1$, we allow $t$ to be a function $\tau(\theta)$ of $\theta$, so that the state $\theta$ captures all the uncertainty about the environment, as is usually assumed in the literature. We also allow arbitrary correlation patterns between $t$ and $\theta$, including letting $t$ be independent of $\theta$.

For our first result, we look for a simple class of influential pure cheap talk equilibria in which the expert partitions $\Theta$ using a single hyperplane. More precisely, denote by $h(s, c)$ the hyperplane passing through a point $c \in \text{int}(\Theta)$ of orientation $s \in S^{N-1}$ where $S^{N-1}$ is the $N - 1$ dimensional unit circle that lives in $\mathbb{R}^N$. We think of a hyperplane of orientation $s \in S^{N-1}$ as being parallel to the tangent hyperplane to $S^{N-1}$ at $s$. The hyperplane $h$ partitions $\Theta$ into two regions $R(h)$ and its complement $R'(h)$, corresponding to two messages $m$ and $m'$. For example, when $\Theta = [0, 1]^2$ the sender may announce whether or not $\theta_2 < \theta_1$ so that $R(h) = \{ \theta \in \Theta | \theta_2 < \theta_1 \}$ and $R'(h) = \Theta \setminus R(h)$ where $h = \{ \theta \in \Theta | \theta_1 - \theta_2 = 0 \}$. Then we can take, for instance, $c = (\frac{1}{2}, \frac{1}{2})$ and $s = (1/\sqrt{2}, -1/\sqrt{2})$.

Let $a(h) = E[\theta|\theta \in R(h)]$ and $a'(h) = E[\theta|\theta \in R'(h)]$ be the actions corresponding to the messages $m$ and $m'$ respectively. The announcement hyperplane $h$ corresponds to a two-element partitional equilibrium if every type $t$ of the sender prefers the action $a(h)$ to $a'(h)$ if $\theta \in R(h)$. In the rest of this paper we refer to such an equilibrium simply by the announcement hyperplane $h$. Such an equilibrium is influential if $a(h) \neq a'(h)$.

Our first result is that such an influential cheap talk equilibrium involving a single hyperplane $h$ always exists when the receiver’s uncertainty about sender motives is small relative to the number of dimensions on which she provides advice.

**Proposition 1** An influential cheap talk equilibrium exists for all continuous $U^S(a; t)$ if the number of dimensions $N$ is greater than the number of possible sender types $T$.

**Proof:** We prove the result by looking for a influential cheap talk equilibrium involving a single hyperplane $h(s, c)$, of orientation $s \in S^{N-1}$ passing through $c \in \text{int}(\Theta)$, that partitions $\Theta$ into two non-empty sets $R(h(s, c))$ and its complement $R'(h(s, c))$, with corresponding receiver actions $a(h(s, c))$ and $a'(h(s, c))$, where we may of think of $R(h(s, c))$ as the region that contains the point $s + c$. To this end, notice first that under the assumed conditions

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3Later in this section we briefly consider non-partitional equilibria, while in the next section we look at equilibria with more than two messages. We focus on a simple class of equilibria involving hyperplanes throughout the paper, although more complex (non-)partitional equilibria may also exist, e.g., where the boundary between two elements of the partition of $\Theta$ may not be a hyperplane. This focus allows us to capture the intuitive idea that comparative statements can be credible and informative, regardless of biases and distributional asymmetries.
on $f$, $a(h(s,c)) \in int(R(h(s,c)))$ and $a'(h(s,c)) \in int(R'(h(s,c)))$, implying in particular that $a(h(s,c)) \neq a'(h(s,c))$ so that any such equilibrium, if it exists, is influential. Furthermore, $a(h(s,c))$ and $a'(h(s,c))$ are continuous functions of $s$ (with the subspace topology for $S^{N-1}$ relative to the usual one on $\mathbb{R}^N$), for any fixed $c \in int(\Theta)$. Notice next that for any two antipodal orientations $-s, s \in S^{N-1}$, we must have $R(h(s,c)) = R'(h(-s,c))$ and so $R'(h(s,c)) = R(h(-s,c))$. It follows that $a(h(s,c)) = a'(h(-s,c))$ implying in particular that the map $G^t(\cdot; c) : S^{N-1} \rightarrow \mathbb{R}$ defined by
\[
G^t(s,c) = U(a'(h(s,c));t) - U(a(h(s,c));t)
\] is a continuous odd function of $s$, for any arbitrary $c \in int(\Theta)$ and all $t \in T$. By the Borsuk-Ulam theorem (see, e.g., Matousek (2003)), when $N-1 \geq T$, there exists $s^* \in S^{N-1}$, such that $G^t(s^*,c) = 0$ for all $t \in T$. Clearly, the hyperplane $h(s^*,c)$ constitutes the announcement hyperplane of an influential equilibrium. $\blacksquare$

Proposition 1 formalizes the intuition that additional dimensions make influential cheap talk easier to sustain as long as the sender’s motives are sufficiently transparent, i.e., $T < N$. This is true regardless of the precise specification of these motives and even with arbitrary asymmetries and correlations across dimensions. When $N > 1$, continuity of sender preferences is sufficient to guarantee that the sender does not have a strict preference ordering over receiver actions, allowing the sender to trade off more favorable actions in some dimensions against less favorable ones in others. In particular, for $N > T = 1$ the expert’s preferences are common knowledge and an influential equilibrium always exists for any continuous $U^S$, in sharp contrast to the case with $N = 1 = T$ where no influential cheap talk equilibrium can exist if $U^S$ is strictly increasing in actions.

If the expectations of $\Theta$ based on the messages are the same then the receiver’s actions are the same, i.e. the equilibrium is not influential. This is not a problem in the equilibria of Proposition 1 because each equilibrium action is in the interior of a different half-space of $\Theta$. The value of cheap talk is also limited if only one of the messages is typically ever sent. This is not a problem because an influential equilibrium exists for any $c \in int(\Theta)$, so $c$ can be chosen to obtain the most informative equilibrium. If $\Theta$ is bounded and $c$ is chosen as the centroid $E[\Theta]$, then each message occurs with ex ante probability of at least

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4 At the opposite extreme are lexicographic preferences in which the expert’s preferences are fully ordered so no tradeoffs across issues are possible. Multidimensional cheap talk can break down in this case (Levy and Razin, 2005).

5 Regarding the multiplicity of equilibria in Proposition 1, when $N > 2$ and $T = 1$, for any such $c$, Dyson’s Theorem implies the existence of $N - 1$ equilibria with announcement hyperplanes that have mutually
\((N/(N+1))^N\) (Grunbaum, 1960), implying a lower bound of \(1/e\) for any \(N\). More generally, the centerpoint theorem implies that there exists a point \(c\) such that each message occurs with probability of at least \(1/(N+1)\). And, if \(N > T + 1\), it can be shown by identical arguments to those in the proof of Proposition 1 that for any \(c\) an influential cheap talk equilibrium exists with the property that each message occurs with ex ante probability of \(1/2\).

Proposition 1 provides a sufficiency condition for the existence of equilibria for any continuous utility function and any well-behaved distribution function. The converse is also true with respect to partitional equilibria with a single hyperplane — it is straightforward to construct examples with continuous \(U^S\), well-behaved \(F\), and \(N = 2 \leq T\) where no such equilibria can exist. In this sense, the sufficient condition \(N > T\) is also necessary.

For particular utility and distribution functions, influential equilibria will still exist even when \(T\) is greater than \(N\). For example, Chakraborty and Harbaugh (2006) show that when \(t = \theta\), (near) symmetry of preferences and distributions across dimensions is sufficient to obtain influential equilibria for additively separable preferences that are (strictly) supermodular in each dimension.\(^6\) Furthermore, as we discuss in the concluding remarks to this section, for finite \(T\) and linear preferences, non-partitional but influential equilibria exist even when \(N \leq T\) if the sender’s type is independent of \(\theta\).

To see Proposition 1 more clearly, assume that \(N = 2\), \(T = 1\) and \(f\) is uniform on \([0,1]^2\). Figure 1(a) shows the sender’s indifference curves for the case where preferences have a Cobb-Douglas form, \(U^S = a_1^{2/3}a_2^{1/3}\). For instance, a lobbyist wants to increase spending on two projects, but cares more about the first project. In this two-dimensional environment, a hyperplane or “announcement line” bisecting \([0,1]^2\) into two regions is an equilibrium when the corresponding actions, i.e., the two pairs of expected values of \(\theta_1\) and \(\theta_2\) given the region that \(\theta\) is announced to lie in, are on the same indifference curve so that the sender has no incentive to lie about which region \(\theta\) lies in. Since the indifference curves are downward-sloping, this indifference condition implies a tradeoff between pushing up the estimate of one issue at the expense of pushing down the estimate of the other issue.

The equilibrium actions are found by picking any interior point, \(c = (1/2, 1/2)\) in the orthogonal orientations. More generally, Bourgin-Yan type theorems can be used to show that the set of possible equilibrium orientations has dimension at least \(N - 1 - T\). See Matousek (2003) for these and other generalizations of Borsuk-Ulam type arguments.

\(^6\)They also show that, under independence, simply adding more dimensions may facilitate communication regardless of asymmetries. In their analyses of comparative statements in bargaining and auction games, Chakraborty and Harbaugh (2003) and Chakraborty, Gupta and Harbaugh (2006) assume that the environment is symmetric.
example, and spinning the orientation $s$ of the announcement line around clockwise. Given the distributional assumption, when the announcement line is vertical the action for the left region is $(E[\theta_1|\theta_1 < 1/2], E[\theta_2]) = (1/4, 1/2)$ and the action for the right region is $(E[\theta_1|\theta_1 \geq 1/2], E[\theta_2]) = (3/4, 1/2)$. Therefore as the announcement line is spun around $c$ from vertical to vertical again, the actions $a, a'$ at the two end points of the operation are respectively $a = (1/4, 1/2)$, $a' = (3/4, 1/2)$ and vice versa. These different actions trace out the near-circular path shown in Figure 1(a). Since the two actions reverse themselves, since the change is continuous, and since the slope of the line that the actions lie on also covers the whole range from negative infinity to infinity, at some intermediate point the actions $a$ and $a'$ must lie on the same indifference curve. Given our choice of parameters, straightforward calculations show the solution occurs with an announcement line that has slope .669.

Proposition 1 shows that this construction works for any continuous $U^S$. Figure 1(b) shows the case where $U^S = a_1 - a_1^{1/2}$ so, instead of just caring more about the first issue than the second, the sender actually favors a lower action on the second issue, e.g., a company benefits from pushing up estimates of its own product and also from pushing down estimates of its competitor’s product. Since the sender’s indifference curves are upward-sloping, in equilibrium the actions for one message must be both higher or lower than for the other

Figure 1: Construction of cheap talk equilibria
message. Therefore, if the sender wants to push down the receiver’s estimate of the second issue, the receiver’s estimate of the first issue must also be lowered, e.g., a company can make a competing product look worse through “negative advertising” but at the cost of inducing consumers to make a negative inference about its own product as well.

When there are more than two dimensions, the cheap talk messages in Proposition 1 essentially compare linear combinations of the issues. For instance, a lobbyist indicates whether the average values of two projects are greater than a third. Or a financial advisor indicates whether the return to one portfolio is likely to be higher or lower than another. Therefore, for \( N > 2 \), the comparative cheap talk equilibria we consider differ qualitatively from those in Chakraborty and Harbaugh (2006) which involve full or partial rankings of the issues.

### 3 Linear preferences

Often the sender has a strong incentive for higher actions in each dimension, e.g., a seller seeks to maximize expected revenue regardless of the true quality of the goods being sold, an analyst seeks to push up each stock price regardless of expected future earnings, or a lobbyist wants to expand government support on alternative energy research programs regardless of the merits of each program. In this case the main uncertainty is over how much the sender cares about a higher action in each dimension, i.e., the sender’s weights or “slant”. A simple way to capture this is the additively separable linear specification

\[
U^S(a; t) = \lambda_1(t)a_1 + \ldots + \lambda_N(t)a_N
\]

where the \( \lambda_i(t) \) are real numbers that measure the relative importance of the action on issue \( i \) for type \( t \) of the sender, so that \( \lambda(t) = (\lambda_1(t), \ldots, \lambda_N(t)) \) summarizes the sender’s possibly uncertain weights on each dimension.

Figure 2(a) shows the case where \( N = 2, T = 1 \), with \( f \) uniform on \([0, 1]^2\) for \( \lambda_1 = 4 \) and \( \lambda_2 = 1 \). The equilibrium announcement line with slope .359 is found by same process as described in the previous section.\(^7\) By interpreting one message as slightly positive news about the favored stock and more negative news about the unfavored stock, and interpreting the other message as slightly negative news about the favored stock and more positive news

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\(^7\)For an i.i.d. uniform distribution and \( c = (1/2, 1/2) \), the slope equals \(-\lambda_1/\lambda_2 + (3 + (\lambda_1/\lambda_2)^2)\) for \( \lambda_1/\lambda_2 > 1 \) and equals \(-\lambda_1/\lambda_2 - (3 + (\lambda_1/\lambda_2)^2)^{1/2} \) for \(-\lambda_1/\lambda_2 > 1 \). The case of \( \lambda_1, \lambda_2 > 0 \) for this example is discussed in section 4.1.3 of Chakraborty and Harbaugh (2006).
Figure 2: Multiple-message cheap talk equilibria with linear preferences

about the unfavored stock, the investor eliminates any incentive for the analyst to misreport her information.\footnote{The comparison can be implicit rather than explicit. For instance, the alternative messages can be ranking both stocks equally or ranking the favored stock higher, in which case the former message is effectively better news about the unfavored stock than the latter message is about the favored stock.}

Can the sender reveal more information than in the two-message equilibrium of Proposition 1? Consider if, after stating message \( m \), the sender is given an opportunity to again make a two-message comparative statement for the region \( R \) (a convex set) corresponding to the area above the announcement line. From Proposition 1, we know that an equilibrium exists for this subgame, but it might seem that anticipation of the subgame will distort the sender’s incentives in the first stage.

To check this, observe from Figure 2(a) that if we set \( c = (E[\theta_1], E[\theta_2]) \) then, by the law of iterated expectations, \( c \) is a convex combination of \( a = (E[\theta_1|m], E[\theta_2|m]) \) and \( a' = (E[\theta_1|m'], E[\theta_2|m']) \) so all three points must lie on the same linear indifference curve. Similarly, if in the second stage we set \( c_2 = a = (E[\theta_1|m], E[\theta_2|m]) \), then this point of region \( R \) is a convex combination of the expectations for the two new subregions, and again all three points are on the same linear indifference curve. Consequently, not only is the sender indifferent between sending either message in the subgame after sending \( m \) in the
first stage, but the sender is also indifferent between sending either message in the subgame and sending \( m' \) in the first stage. So the sender has no incentive to lie in the first stage.

Clearly this same logic applies for any number of stages - the sender can keep making additional comparative statements that subdivide the space further, and the potential for further stages does not affect incentives at earlier stages. Figure 2(b) shows the case where the top region is subdivided several times so that progressively narrower slices are revealed. As a result, the receiver is given an increasingly accurate estimate of the relative values of the two issues.

This logic also extends to more than two dimensions. In this case the expectations for each subregion need not lie on a line as in the two-dimensional case, but must lie on the same hyperplane so the sender still has no incentive to lie at any stage. Note also that the logic applies to simultaneous statements so the sender need not make continually finer statements but can simply reveal the whole set of hyperplanes at the first stage. Our next result therefore follows.

**Proposition 2** If sender preferences are linear as in (2), then a cheap talk equilibrium with \( k \) influential messages exists for all \( k > 1 \) if the number of dimensions \( N \) is greater than the number of possible sender types \( T \).

**Proof.** Formalizing the discussion above, note that the first hyperplane \( h_1 = h(s_1, c_1) \) obtained via Proposition 1, the region \( R(h_1) \) is a convex set, since it is the intersection of a a convex set \( \Theta \) and one of the half-spaces associated with \( h_1 \). Further, it has a non-empty interior since \( a(h_1) \in \text{int}(R(h_1)) \). Picking \( c_2 = a(h_1) \) and applying Proposition 1 again, we are guaranteed the existence of a second hyperplane \( h_2 = h(s_2, c_2) \) and associated convex regions \( R(h_2) \) and \( R'(h_2) = R(h_1) \setminus R(h_2) \), and corresponding actions \( a(h_2) \in \text{int}(R(h_2)), a'(h_2) \in \text{int}(R'(h_2)) \), such that \( U^S(a(h_2); t) = U^S(a'(h_2); t) \) for all \( t \in T \). Since the actions are all conditional expectations, by the law of iterated expectations, there must exist \( q \in (0, 1) \), \( q = \Pr[\theta \in R(h_2) | \theta \in R(h_1)] \), such that

\[
a(h_1) = qa(h_2) + (1 - q)a'(h_2)
\]

Since \( U^S \) is given by (2), we conclude that

\[
U^S(a(h_2); t) = U^S(a'(h_2); t) = U^S(a(h_1); t) = U^S(a'(h_1); t) \text{ for all } t \in T.
\]

so that a three-message influential equilibrium exists with induced actions \( a(h_2), a'(h_2) \) and \( a'(h_1) \). The \( k \)-message case uses the argument above as an inductive step. ■
In contrast with Proposition 1, this result depends strongly on the preference specification. If preferences are not linear then the different actions in the different subregions will not in general lie on the same indifference curve, so the sender will typically have an incentive to lie at early stages. Linear preferences are of particular interest because, as the following section shows, they are the limiting case of standard Euclidean preferences as the sender’s bias towards a higher action on each issue becomes larger. Therefore linear preferences represent a simple way to capture situations where the sender’s incentive to exaggerate on each issue is very large.

Regarding limited uncertainty over the sender’s motives, consider the case where $N = 3 > T = 2$, i.e., the conversation concerns three issues of interest to the receiver about which the sender has expertise and the sender is one of two possible types (e.g., a liberal or a conservative). For example, the three issues could involve the receiver’s ideal allocation of spending on cancer research (issue 1), education (issue 2), and welfare (issue 3). A liberal sender ($t = L$) may want the receiver to take a higher action on all three issues, whereas the conservative type ($t = C$) may have identical preferences regarding cancer research but opposed preferences on the other two issues. Using linear preferences, we may write

$$U^S(a'; L) = a_1 + a_2 + a_3$$
$$U^S(a; C) = a_1 - a_2 - a_3$$

as the preferences of the two types. Let $a$ and $a'$ be the equilibrium actions chosen by the receiver for the first-stage two-message equilibria of Proposition 1. Since both types must be indifferent in any such an equilibrium, we must have

$$a_1 + a_2 + a_3 = a'_1 + a'_2 + a'_3$$
$$a_1 - a_2 - a_3 = a'_1 - a'_2 - a'_3.$$

It follows that the receiver is not influenced on the issue of cancer research (i.e., $a_1 = a'_1$) but can obtain useful information about the relative importance of the other two issues (i.e., $a_2 \neq a'_2$, $a_3 \neq a'_3$ but $a_2 - a'_2 = a'_3 - a_3$), regardless of his priors about $t$ or $\theta$.\footnote{For independent $\theta$, keeping $a_1 = a'_1$ can be achieved by just ignoring issue 1, i.e., having the announcement plane parallel to dimension 1. If there is any correlation then statements regarding any one issue affect the receiver’s estimates of other issues so keeping the sender indifferent is more complicated, but can still be done by Proposition 1.} Proposition 1 shows that cheap talk holds quite generally in this case. Moreover, since preferences are linear, Proposition 2 implies that issues 2 and 3 can be compared in detail. In general, when the weights $\lambda_i(t)$ for the two types $t = L, C$ across issues $i$ are not identical in magnitude
(although possibly opposed in sign), the decision maker will be influenced on all three issues by the comparative statements.

Considering this example with two sender types further, clearly cheap talk can still be credible even if there are only two issues, but it involves accepting that one of the sender types will always give uninformative advice. For instance if preferences are as given above but there is no issue 3, then it is an equilibrium for the receiver to make decisions assuming that a liberal sender will be indifferent if \( a_1 + a_2 = a'_1 + a'_2 \) while a conservative sender will always prefer a higher action on issue 1. To see this, assume \( f \) is uniform on \([0,1]^2\) and that \( t \) and \( \theta \) are independent. Consider the announcement line with slope one intercepting \( c = (1/2, 1/2) \). Suppose message \( a \) is interpreted as meaning that \( \theta_2 \geq \theta_1 \) and that the sender is liberal. Then the actions are just \((a_1, a_2) = (1/3, 2/3)\). And suppose message \( a' \) is interpreted as meaning that either: a) \( \theta_2 < \theta_1 \) and the sender is liberal or b) no information about \( \theta \) and the sender is conservative. Then the actions are the average of \((2/3, 1/3)\) and \((1/2, 1/2)\), or \((a'_1, a'_2) = (5/9, 4/9)\). Given these actions neither type of sender has an incentive to lie, so a cheap talk equilibrium exists.\(^{10}\) Note that the messages in this non-partitional equilibrium are less informative than in the partitional equilibria that exist with transparent sender preferences.

4 Euclidean preferences

Following Crawford and Sobel (1982), sender preferences based on the distance between the sender’s ideal action and the receiver’s ideal action (the sender’s “bias”) are widely used in the literature on one-dimensional cheap talk and have also been adopted in the nascent literature on multidimensional cheap talk. In the one-dimensional case, the sender makes a trade-off between actions that the sender thinks are too small and actions that the sender thinks are too large. Therefore, influential communication is only possible if the sender’s bias toward a higher action is limited. In the multi-dimensional case, the tradeoff is across issues rather than within issues, so it is possible to analyze situations where the sender’s bias for a higher (or lower) action on each issue is arbitrarily strong.

In this section, we show that Euclidean distance preferences converge uniformly to linear preferences with known weights as the sender’s bias toward a higher (or lower) action on each issue increases. As a result, pure cheap talk is an equilibrium at the limit even for arbitrary

\(^{10}\)This construction depends on the linearity of the preferences to ensure that the expectations remain on the same indifference curve. Stronger distributional assumptions, namely affiliation of \( f \), are needed to ensure an equilibrium more generally.
asymmetries. We then use this result to show that the set of “almost cheap talk” equilibria
(Kartik, 2005) for Euclidean distance preferences converges to the set of equilibria for linear
preferences. This result provides a justification for modeling many environments where the
sender is very biased using simple linear preferences rather than distance preferences.

Consider the following specification of Euclidean distance preferences:

\[
U^S(a; t) = -d(a, \tau(\theta)) = -\left( \sum_{i=1}^{N} (a_i - (\theta_i + b_i))^2 \right)^{1/2}
\]

(4)

where \(d(\cdot, \cdot)\) is the Euclidean distance function and \(b = (b_1, \ldots, b_N) \in \mathbb{R}^N\) is the vector
of known biases representing the distance between the receiver’s ideal action \(\theta\), and the
sender’s ideal action \(\tau(\theta) = \theta + b\). Since \(\theta\) is the sender’s private information, the sender’s
type is \(t = \tau(\theta)\). The set of such types is not finite so existence results from Proposition 1
do not apply.\(^{11}\)

For bounded \(\Theta\), influential communication in any single dimension is impossible when
the biases are large enough. To see whether comparative statements across dimensions are
credible with arbitrarily large biases, let \(b = (\rho_1 B, \ldots, \rho_N B)\) for \(B \geq 0\) and \(\rho = (\rho_1, \ldots, \rho_N) \neq 0.\(^{12}\) As \(B\) increases without bound, the sender’s ideal point becomes more and more distant
from \(\theta\), so the circular indifference curves corresponding to Euclidean preferences converge
to those of known linear preferences of the form in (2) with weights \(\rho_i\) instead of \(\lambda_i.\(^{13}\) This
is seen in Figure 3 where \(\rho_1 = 4, \rho_2 = 1,\) and \(\theta = (0.95, 65)\). As \(B\) increases,
the ideal action \(\tau(\theta)\) moves away from \(\theta\) in the direction with slope \(\rho_2/\rho_1 = 1/4\), and the indifference
curves corresponding to \(\tau(\theta)\) become straighter, converging to the linear indifference curves
of Figure 2 for \(\lambda_1 = 4\) and \(\lambda_2 = 1.\) Clearly this holds for any \(\theta\) since the ideal points move
outwards in the same direction \(\rho_2/\rho_1\) as \(B\) increases.

To understand this convergence further, consider whether a sender with the given Euclide-
an preferences is willing to truthfully report according to the two-message equilibrium for

\(^{11}\)These preferences satisfy the complementarity conditions for influential cheap talk in multiple dimensions
for the (near) symmetric case (Chakraborty and Harbaugh, 2006), but here we are interested in arbitrary
asymmetries.

\(^{12}\)The sender’s biases can be negative in some or all dimensions and can be zero in \(N - 1\) dimensions.
Negative bias in a one-dimensional model is allowed in Li (2004) and Gordon (2006). Negative biases also
appear in the literature on cheap talk with multiple senders.

\(^{13}\)If in (4) there are different weights on the dimensions then the space is stretched so that the indifference
curves are elliptical. The total weight with linear preferences is then the product of the weight and the bias
with Euclidean preferences, leaving the analysis qualitatively unchanged. This approach is taken in Levy
and Razin (2005).
linear preferences from Figure 2(a). This will not be the case if for any $\theta$ above the announcement line the sender’s ideal point $\tau(\theta)$ is closer to the action for the half-space below the announcement line or vice-versa. In fact, as is clear from the indifference curves in Figure 3(a) the ideal point $\tau(\theta)$ for the given $\theta$ is slightly closer to $a'$ than to $a$, so a sender of type $\theta$ will misreport. The utility gain from lying is simply the difference in the distance $d(a', \tau(\theta)) - d(a, \tau(\theta))$, which is the minimum distance between $a'$ and the indifference curve through $a$. From Figure 3(b) this gain to lying falls as $B$ rises, and clearly converges to 0 in the limit. Since the same holds for any $\theta$, it follows that the equilibrium from Figure 2(a) for linear preferences obtains for Euclidean preferences at the limit as the biases go to infinity. By similar arguments, the equilibria with more messages identified in Figure 2(b) also obtain.

This limiting result may seem to contradict the result of Levy and Razin (2005) that influential cheap talk can become more difficult with Euclidean preferences if asymmetries are held constant and the biases in each dimension are made sufficiently large.\(^\text{14}\) To reconcile

\[^{14}\text{Cheap talk with Euclidean preferences requires that the announcement line plus } b \text{ bisect the line joining the two actions and be perpendicular to it. Arguments like those used for Proposition 1 cannot guarantee the latter condition in general, though in this particular example the environment is sufficiently symmetric that an announcement line satisfying both conditions can be found by appropriate choice of } c \neq (1/2, 1/2).\]
these results, note that even though a sender of type $\theta$ in Figure 3 becomes indifferent between actions $a$ and $a'$ at the limit, action $a$ is still preferred for any finite $B$. Therefore pure cheap talk based on the given announcement line is not an equilibrium. However, because any gain to lying goes to zero as $B$ increases, communication based on the messages identified in Proposition 2 become easier to sustain if there is any cost at all to lying.

To see this more formally, we investigate when “almost-cheap talk” (Kartik, 2005) equilibria exist. We modify the game so that the sender’s payoff from any action $a$ and message $m$ given $\theta$ is $U^S(a, m; \theta) = -d(a, \tau(\theta))$ less a small cost $\varepsilon > 0$ of lying if the message $m$ is not consistent with $\theta$.\footnote{We are implicitly assuming that messages have some intrinsic meaning so that punishment for lying is feasible.} We study if influential equilibria exist in such an almost-cheap talk game for large $B$. In particular, we say that an announcement strategy $h$ is an almost-cheap talk equilibrium for large biases of the game with distance preferences (4) if and only if for all $\varepsilon > 0$ there exists $B$ such that for all $B > B$ and any $\theta$, the incentive to lie for a sender is at most $\varepsilon$, i.e., for a single hyperplane equilibrium with two influential messages we must have $|U^S(a; \tau(\theta)) - U^S(a'; \tau(\theta))| < \varepsilon$.\footnote{For the equilibria with $k > 2$ influential messages of Proposition 2, an announcement strategy is summarized by $k - 1$ hyperplanes $h_1, \ldots, h_{k-1}$. In such a case we need $|U^S(a^j; \tau(\theta)) - U^S(a^{j'}; \tau(\theta))| < \varepsilon$ for every pair of action profiles $a^j, a^{j'}$ and $j, j' = 1, \ldots, k$.}

Proposition 3 Let $\Theta$ be compact. An announcement strategy is an influential almost cheap talk equilibrium for large biases with Euclidean preferences in (4) if and only if it is an influential cheap talk equilibrium for the limiting linear preferences in (2) with $\rho = \lambda$.

Proof: Fix $h$ and let $L$ be the line joining $a = a(s, c)$ and $a' = a'(s, c)$. Pick any $\theta \in \Theta$ and let $p(\theta)$ be the perpendicular on to $L$ from $\tau(\theta) = \theta + B\rho$. Then

$$p(\theta) = q(\theta)a + (1 - q(\theta))a'$$

where $q(\theta) \in \mathbb{R}$ is given by

$$q(\theta) = \frac{\langle \theta - a' \rangle \cdot (a - a') + B\rho \cdot (a - a')}{(a - a') \cdot (a - a')}.$$ 

Notice that this is well-defined since $a \neq a'$.

For the if part suppose $h$ is a cheap talk equilibrium when $U^S$ is given by (2), so that $\rho \cdot (a - a') = 0$. Then $q(\theta)$ and $p(\theta)$ do not depend on $B$. Furthermore, using Pythagoras’
theorem we obtain

$$\begin{align*}
|U^S(a; \tau(\theta)) - U^S(a'; \tau(\theta))| &= |d(a', \theta + B\rho) - d(a, \theta + B\rho)| \\
&= \left| \frac{d^2(a', p(\theta)) - d^2(a, p(\theta))}{d^2(a', \theta + B\rho) + d^2(a, \theta + B\rho)} \right| \\
&\leq \max_{\theta \in \Theta} \left| \frac{\rho (a' - a)}{\sqrt{\rho \cdot \rho}} \right|.
\end{align*}$$

Let $\theta_B$ be the solution to the last maximization problem. As $B$ rises, $\theta_B$ stays bounded in the compact set $\Theta$, so that $p(\theta_B)$ stays bounded as well implying that the numerator stays bounded. However the denominator becomes arbitrarily large. It follows that for any $\varepsilon > 0$, for $B$ large enough, $|U^S_D(a, \theta) - U^S_D(a', \theta)| < \varepsilon$ for all $\theta$.

An analogous argument obtains for the $k$-message equilibria of Proposition 2, $k \geq 2$ and finite, if we consider pairs of equilibrium actions that must all lie on the same line $L$ and run the logic above.

For the only if part, suppose that $h$ is not a cheap talk equilibrium when $U^S$ is given by (2). W.l.o.g., suppose that $\rho \cdot a < \rho \cdot a'$. Consider type $\theta = a$ and observe, via Pythagoras’ theorem, that

$$\lim_{B \to \infty} [U^S_S(a; \tau(\theta)) - U^S_S(a'; \tau(\theta))] = \lim_{B \to \infty} [d(a, a + B\rho) - d(a', a + B\rho)]$$

$$= \lim_{B \to \infty} \left[ \frac{d^2(a, p(a)) - d^2(a', p(a))}{d(a, a + B\rho) + d(a', a + B\rho)} \right]$$

$$= \frac{\rho \cdot (a' - a)}{\sqrt{\rho \cdot \rho}} > 0.$$ 

It follows that when $\varepsilon < \frac{\rho \cdot (a' - a)}{\sqrt{\rho \cdot \rho}}$, and $B$ is large enough, type $\theta = a$ would gain by more than $\varepsilon$ from lying (i.e., by inducing the receiver to choose the action $a'$ instead of $a$), implying in turn that $h$ is not an almost-cheap talk equilibrium for large $B$, when $U^S$ is given by (4). An identical argument obtains for the $k$-message equilibria of Proposition 2, $k \geq 2$ and finite, if we consider some pair of actions for which $\rho \cdot a \neq \rho \cdot a'$.

Since $\Theta$ is bounded, the sender’s ideal point will often be higher than any of the feasible receiver actions, which might be inappropriate in some cases. For $\Theta = [0, 1]^N$, one simple way to represent large but bounded ideal points is, for positive biases, to set $\tau_i(\theta) = \min\{\theta_i + b_i, 1\}$. For $N = 1$ the set of cheap talk equilibria coincides with that when $\tau(\theta)$ is unbounded and cheap talk is impossible for $b_i \geq 1$. However, for $N \geq 2$ and $b_i \geq 1$, indifference curves do not depend on $\theta$ so $T = 1$ and the existence result from Proposition 1 applies directly.
The cheap talk literature sometimes uses a quadratic rather than Euclidean specification in which the square root term in (4) is dropped. Applied to our context, the same conclusions about almost cheap talk still hold as long as the cost of lying is of the same order of magnitude as $B$ so that the “unit” of payoffs does not artificially increase with $B$. Another alternative to Euclidean preferences is the Manhattan metric $U^S(a; t) = -d_M(a, \tau(\theta)) = -\sum_i |a_i - \tau_i(\theta)|$, with $\tau(\theta) = \theta + b$. For $N = 1$, the set of cheap talk equilibria again coincides with the case of Euclidean distance so cheap talk is impossible for $b_1 \geq 1$. And again for $N \geq 2$ and $\Theta = [0, 1]^N$, indifference curves corresponding to $d_M$ do not depend on $\theta$ when $b_i \geq 1$ so $T = 1$ and the existence result from Proposition 1 applies. Similar arguments then extend the result to the equilibria identified in Proposition 2.

These results show that large biases are not a fundamental barrier to communication with distance-based preferences even in very asymmetric environments. This does not imply that cheap talk or almost cheap talk is always feasible since there still remains the problem that uncertainty over the different biases can impede communication. Uncertainty over the different $\rho$ is equivalent at the limit to uncertainty over the different weights $\lambda$, so the same results on sufficient transparency of the sender’s motives from Propositions 1 and 2 apply.

5 Conclusion

When there is sufficient transparency over the expert’s motives, we show that influential cheap talk equilibria exist in multidimensional environments even under arbitrary asymmetries. The preferences we consider represent a natural specification for a large class of important economic environments, and include the limiting case of standard Euclidean preferences. When the expert’s motives are known, our results provide an intuitive solution where the decision maker treats comparative statements with enough skepticism for communication to be credible. When the expert’s motives are not known for sure but the uncertainty is limited, our results indicate when communication is still possible.

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17 With quadratic preferences, as $B$ increases the sender’s utilities from actions $a$ and $a'$ rapidly become very negative and the difference in utilities from the actions increases, but at a slower rate than $B$. Therefore it is straightforward to show that the same convergence result for Proposition 3 holds with quadratic preferences if the cost to lying is $\varepsilon B$ for any $\varepsilon > 0$. 

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References


