How to Use Natural Experiments to Estimate Misallocation *

David Sraer
UC Berkeley, NBER & CEPR

David Thesmar
MIT, NBER & CEPR

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Abstract

We propose a method to estimate the effect of firm policies (e.g., bankruptcy laws or subsidized credit) on allocative efficiency using (quasi-) experimental evidence. Our approach takes general equilibrium effects into account and requires neither a structural estimation nor a precise assumption on how the experiment affects firms. Our aggregation formula relies on treatment effects of the policy on the distribution of output-to-capital ratios, which are easily estimated in (quasi-) experimental data. We show that this method is valid as long as the true data-generating process belongs to a large class of commonly-used models in macro-finance. Finally, we apply this method to the French banking deregulation episode of the mid-1980s and find that this reform led to an increase in aggregate TFP of 2.7%.

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1 Introduction

The misallocation of resources is a central question in economics. Starting with Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), the literature measures equilibrium misallocation by estimating the cross-sectional dispersion of marginal products across firms. This approach suffers from several well-known limitations. For instance, measurement errors in inputs, or adjustment costs, can generate productivity dispersion without resource misallocation (Asker et al. (2014), Bils et al. (2020), Gollin and Udry (2020)). Also, the *policy relevance* of such misallocation measures is questionable. Misallocation is typically measured relative to a frictionless benchmark, which can be difficult to achieve in practice. Finally, this approach is mostly silent on the particular frictions that generate misallocation and the potential policies that may improve allocative efficiency.

In contrast, a large literature in applied microeconomics exploits (quasi-) experimental settings to estimate the causal effect on firm-level outcomes of economic policies such as financial deregulation, bankruptcy reform, banking regulation, or corporate taxation.[1] These policies are designed to alleviate firm-level frictions, so they should reduce misallocation in the economy. While this empirical literature uses these experiments to measure their effect on firm growth and investment, it does not *quantify* how they affect aggregate allocative efficiency.

Our paper bridges these two approaches. We offer a method to measure allocative efficiency in a (quasi-) experimental settings. This method works as follows. An econometrician observes firm-level data in an economy where a (quasi-) natural experiment has taken place. This experiment changes the set of frictions faced by *treated* firms while leaving control firms unaffected. Under the appropriate identifying assumption, the econometrician can estimate the causal effect of the experiment on the distribution of log marginal products of capital (lMRPKs), using classic difference-in-difference estimators. These estimates can then be injected in a simple aggregation formula to answer two simple questions: (1) how much did the actual policy change contribute to change in aggregate efficiency (ex-post evaluation)? (2) how would aggregate efficiency have changed if the policy had been extended to all firms in the economy (scale-up)?

Beyond its simplicity, our method has multiple advantages. First, it deals with measurement error and real frictions (by comparing treated and control firms). Second, it is policy-relevant (the experiment has been implemented in practice). Third, our method does not require the potentially strenuous estimation of a structural model of firm behavior, although it is consistent with most models of firm dynamics used in the literature.

[1] See the references in the literature review below.

Finally, our method allows empiricists to avoid specifying exactly how the experiment affects firms, as long as firms' input choices belong to a (large) class of models.

This method, though intuitive and simple, presents conceptual challenges, that we describe in Section 2 using a simplified version of our baseline model. Conceptually, these challenges arise because we measure treatment effects in the experimental data, but our aggregation formula requires treatment effects in unobserved, counterfactual economies. Consider for instance the case of an econometrician wishing to measure the effect of the experiment on aggregate TFP. She faces two obstacles. First, the experiment affects the average firm in the sample through general equilibrium. This general equilibrium effect is differenced out – and therefore unknown – in a difference-in-difference setting. Second, the aggregation formula relies on estimating treatment effects in a counterfactual world where only the reform takes place. In practice, however, additional shocks may have taken place coincidentally, e.g, a shock to average firm productivity. A priori, nothing guarantees that the treatment effects estimated in the actual data apply to the counterfactual economy, i.e. that they are externally valid.

A similar external validity issue arises when the econometrician endeavors to measure how aggregate efficiency would change if the policy was extended to all firms in the economy. All the econometrician can do is measure the effect of the policy change when the policy is not at scale. However, scaling up the experiment will result in changes in equilibrium conditions. For instance, it may lead to a wage increase, and firms may respond differently to the policy treatment when the labor market is tighter. There again, nothing guarantees that the estimated treatment effects in the real data can be used in a counterfactual exercise where general equilibrium conditions have changed.

While these two obstacles are real, our paper provides a broad set of conditions under which they can be safely ignored. Section 3 shows that under broad conditions, applicable in most macroeconomic models with heterogeneous firms, the distribution of MRPKs is independent of general equilibrium conditions. This is our main Theorem 1. As we show, this invariance relies crucially on two key assumptions about technology and frictions. First, the sources of distortions (financing frictions and constraints, tax schedules, adjustment costs) are assumed to be homogeneous of degree one. Intuitively, homogeneity guarantees that frictions remain on average constant on a size-adjusted basis. Hence, a change in general equilibrium, which affects firms' size, will not affect the relative distribution of distortions. Second, firm-level production is Cobb-Douglas, with either constant or decreasing returns to scale. While these assumptions may appear restrictive, they are almost always satisfied in the structural macro-finance literature (see our extensive review of the literature in Table 1). As such, our sufficient statistics approach provides a valid alternative to structural estimation in the context of these models.

To close the analysis and provide aggregation formulas, all we need is an aggregation model that details how industries interact in equilibrium through product and labor markets. In the main text below, we provide aggregation formulas using the aggregation model of Hsieh and Klenow (2009), which has become a benchmark in the literature. These formulas show how to combine aggregation parameters with the difference-in-difference estimates of three key moments of the distribution of log-MRPK. The formulas are intuitive as we explain in Section 3.4. Importantly, our methodology easily extends to more complex market structures and additional sources of heterogeneity in technology. In Appendix A.7, we extend the baseline formulas to roundabout production and input-output linkages. In Appendix A.8, we explore an extension of the Hsieh and Klenow (2009) with variable industry output shares.

The paper concludes with an application of our method to a specific large-scale experiment: the French banking deregulation episode of the 1980s. Prior to this reform, the French banking sector was heavily controlled by the government, which fixed prices and quantities in the loan market, while channeling loans to priority industries. Even in the private sector, the profit motive was largely absent and competition was limited. The reform, implemented in the mid-1980s, organized the rapid transition of the industry into a more classically decentralized, competitive sector. Using a difference-in-difference analysis, Bertrand et al. (2007) qualitatively show that the reform led to a significant increase in capital reallocation across firms. We complement their analysis by providing a precise quantification of the resulting aggregate TFP gains. Using a similar identification strategy, our aggregation formula implies that the French banking deregulation episode led to a 2.7 % increase in TFP over the sample period, which corresponds to about one-fourth of the total aggregate TFP gains in France over this period.[2]

Our paper offers a novel way to quantify the extent of misallocation in the data. Our approach departs from the misallocation literature in three ways. First, we build on the applied microeconomic literature that produces well-identified evidence on the effect of economic policies on firm-level outcomes. A first strand of papers evaluates the effect of financial reforms (see for instance Aghion et al. (2007), Bertrand et al. (2007), Ponticelli and Alencar (2016), Larrain and Stumpner (2017)). Others analyze firms' response to the availability of subsidized credit (e.g. Lelarge et al. (2010), Banerjee and

[2]Average TFP in France between 1985 to 1992 (or post-experiment sample) is about 8.8% higher than between 1978 to 1983 (our pre-experiment sample). See https://fred.stlouisfed.org/series/RTFPNAFRA632NRUG.

Duflo (2014), Brown and Earle (2017)), or changes in monetary policy and prudential regulation (Fraisse et al. (2017), Blattner et al. (2017)). The effect of capital taxes or subsidies on firm investment and hiring is the focus of Yagan (2015), Zwick and Mahon (2015), Giroud and Rauh (2016) or Rotemberg (2017). These papers provide important causal evidence on the role of frictions for firm-level outcomes. Our methodology allows evaluating whether the policies investigated in these papers have a significant effect on allocative efficiency. In this sense, our paper offers a way to measure the importance of misallocation in the data by measuring potential reallocation gains from policies actually implemented in the real world.

Our second departure relative to the misallocation literature is that we allow capital wedges to arise from a large class of firm dynamics models. In a seminal paper building on Restuccia and Rogerson (2008), Hsieh and Klenow (2009) show how to compute TFP losses due to misallocation of inputs using a simple sufficient statistics approach. Hsieh and Klenow (2009), however, abstract from the origin of distortions and treat distortions – wedges between the marginal revenue product of factors and their cost – as primitives of the model. Similarly, Baqaee and Farhi (2017) derive non-parametric formula for aggregating microeconomic shocks in general equilibrium economies with distortions, but, in the spirit of Hsieh and Klenow (2009), they assume distortions can be represented by exogenous wedges. In contrast, our approach considers endogenous capital wedges and show that, for a large class of firm dynamics models, the distribution of these endogenous wedges is invariant to macroeconomic conditions. Put differently, we show that it is correct, in a large class of models, to consider the wedge distribution as exogenous to equilibrium conditions.

We are, of course, not the first to offer a quantification of the role of specific frictions or policies on allocative efficiency. Most contributions in the macro-economic literature, however, rely on structural estimation: researchers start from a particular general equilibrium; firms in the model face frictions when optimizing inputs; the model is usually calibrated using generic moments from macroeconomic and firm-level data; the role of policies or frictions is then analyzed through simulations of counterfactual economies. In this spirit, Asker et al. (2014) look at the contribution of capital adjustment costs on standard misallocation measures. Buera et al. (2011), Midrigan and Xu (2014), Buera et al. (2012) and Catherine et al. (2018) study the effect of financial frictions on allocative efficiency. Edmond et al. (2018) analyze the welfare losses from heterogeneous markups. David et al. (2016) and David and Venkateswaran (2019) focus on the effect of information frictions on aggregate TFP. A limitation of this approach, besides its technical complexity, is that it forces the researcher to specify the nature of frictions faced by firms (cash or collateral constraints, quadratic or linear adjustment costs, etc.) and to model explicitly the policy being studied. In contrast, our approach does not require the estimation of a structural model.

The rest of the paper proceeds as follows. Section 2 describes, in a simplified model, the method to estimate allocative effects from (quasi-) experimental evidence and presents, in the context of this simple model, the key challenge to this methodology. Section 3 shows the validity of this methodology in common models used in macro-finance; we then provide simple formula mapping reduced-form estimates from (quasi-) experimental data to aggregate TFP using a simple aggregation model. The appendix contains alternative aggregation formulas that entertain a more complex market structure and additional heterogeneity in technology and demand. Section 4 applies the method in the context of the French banking deregulation episode. Section 5 concludes.

2 Estimating Misallocation in Experimental Data: Challenges

This section uses a simplified framework to illustrate the difficulty in drawing inference about misallocation using (quasi-) experimental data. Details of the proofs are in Appendix A.1.

2.1 Set-up

We use here a simplified version of the aggregation framework in Hsieh and Klenow (2009). Consider an initial steady-state economy with a continuum of monopolies indexed by $i \in [0;1]$. Each monopoly supplies an intermediate good to a competitive final goods sector that combines them with constant elasticity of substitution (CES) technology with an elasticity of substitution (ES) $\frac{1}{1-\theta}$. Production takes place with a Cobb-Douglas function with constant returns to scale and a capital share α. z_i is firm i's log TFP. These assumptions imply that firm's i revenue is $p_i y_i = Y^{1-\theta} e^{\theta z_i} \left(k_i^{\alpha} l_i^{1-\alpha}\right)^{\theta}$, where Y is aggregate output, whose price is normalized to 1. We call $\mathcal{Z}_0 = \left(\int_i e^{\frac{\theta}{1-\theta} z_i} di\right)^{\frac{1-\theta}{\theta}}$ the aggregate TFP in the absence of distortions. Firms face frictions when optimizing their capital stock. We denote Θ_0 the structural parameters governing these frictions. Because of these frictions, firms' marginal revenue product of capital (MRPK), $\alpha\theta.\frac{p_i y_i}{k_i}$, differs from the user cost of capital. We assume that MRPKs are log-normally distributed. There is no distortion in the labor market. Aggregate TFP is then:

$$\log TFP_0 = \underbrace{\log(\mathcal{Z}_0)}_{\text{technology}} - \underbrace{\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\text{var}\left(\log\frac{p_i y_i}{k_i}|\Theta_0,\mathcal{Z}_0\right)}_{\text{misallocation}} \tag{1}$$

This equation conditions the distribution of $\frac{p_i y_i}{k_i}$ to the structural parameters governing frictions in optimizing inputs, Θ_0 and aggregate productivity $\mathcal{Z}_0$.[3] In this economy, these parameters pin down equilibrium prices, which may, in turn, affect $\frac{p_i y_i}{k_i}$. Note also that this equation does not impose structure on the capital frictions driving the distribution of MRPKs. In particular, we do *not* assume here that MRPKs are exogenous – in most cases that we explore in this paper, they are not.

2.2 Introducing an Experiment

A policy experiment takes place in this economy. To simplify exposition, assume that a random of half the firms (the treatment group, T) receives a treatment that modifies the set of frictions they face: Θ^T goes from Θ_0 to Θ_1. Frictions for the remaining firms (the control group C) remain unchanged at $\Theta^C = \Theta_0$. The new steady-state is also possibly affected by an aggregate TFP shock that may coincide with the policy experiment (e.g., a shift to z_i so that $\mathcal{Z}_0 \to \mathcal{Z}_1$). As we show in our main analysis, our results also hold in the presence of additional confounding shocks (e.g., aggregate labor supply shocks or aggregate shocks to Θ_0).

Given Equation 1, aggregate TFP in this new steady-state is:

$$\log TFP_1 \quad = \quad \log(\mathcal{Z}_1) - \frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\text{var}\left(\log\frac{p_i y_i}{k_i}|\begin{pmatrix}\Theta^C = \Theta_0\\ \Theta^T = \Theta_1\end{pmatrix},\mathcal{Z}_1\right)$$

We are interested in two policy-relevant measures of the effect of this policy change on misallocation:

(1) **ex post evaluation**: how much did the actual policy change affect aggregate efficiency?

(2) **scale-up**: how would aggregate efficiency change if the policy was extended to all firms in the economy?

[3]We focus on shocks to aggregate TFP to simplify exposition in this Section only. Our results carry out for more general shocks on the distribution of z, provided these shocks are orthogonal to the policy treatment.

2.3 Ex Post Evaluation: Effect of the Policy on Misallocation

How much did the policy change contribute to change in aggregate efficiency between the two economies? Formally, the answer to this question is:

$$\Delta \log TFP = -\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\left[\text{var}\left(\log\frac{p_i y_i}{k_i} | \begin{pmatrix}\Theta_0\\ \Theta_1\end{pmatrix}, \mathcal{Z}_0\right) - \text{var}\left(\log\frac{p_i y_i}{k_i} | \begin{pmatrix}\Theta_0\\ \Theta_0\end{pmatrix}, \mathcal{Z}_0\right)\right].$$

The change in aggregate TFP is proportional to the difference in the overall variance of log-MRPKs between the initial economy and a counterfactual economy similar in all dimensions to the initial economy except for the new structural parameters for treated firms ($\Theta_0 \to \Theta_1$). In particular, in this counterfactual economy, $\mathcal{Z}$ remains at $\mathcal{Z}_0$. We are looking to estimate this counterfactual using changes in the *observed* variance of log-MRPKs for firms in the treatment and control group.

Note $\sigma^2_C\left(\begin{pmatrix}\Theta_0\\ \Theta_1\end{pmatrix}, \mathcal{Z}\right)$ (resp. $\sigma^2_T\left(\begin{pmatrix}\Theta_0\\ \Theta_1\end{pmatrix}, \mathcal{Z}\right)$) the variance of $\log(\frac{p_i y_i}{k_i})$ for firms in the control group (resp. treatment group) when firms in the control group face frictions Θ_0, firms in the treatment group face Θ_1 and aggregate productivity is $\mathcal{Z}$. Assuming that the policy treatment is small ($|\Theta_1 - \Theta_0| \ll 1$), we obtain the following approximation:

$$\Delta \log TFP \approx -\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\left(\underbrace{\Delta\sigma^2(C)}_{\text{GE effect}} + \underbrace{\frac{1}{2}\Delta\Delta\sigma^2}_{\text{Treatment effect}}\right). \tag{2}$$

where:

- $\Delta\Delta\sigma^2 = \sigma^2_T\left(\begin{pmatrix}\Theta_0\\ \Theta_1\end{pmatrix}, \mathcal{Z}_0\right) - \sigma^2_T\left(\begin{pmatrix}\Theta_0\\ \Theta_0\end{pmatrix}, \mathcal{Z}_0\right) - \left(\sigma^2_C\left(\begin{pmatrix}\Theta_1\\ \Theta_0\end{pmatrix}, \mathcal{Z}_0\right) - \sigma^2_C\left(\begin{pmatrix}\Theta_0\\ \Theta_0\end{pmatrix}, \mathcal{Z}_0\right)\right)$ is the difference-in-difference effect of the policy change on the dispersion of log-MRPK in the counterfactual where $\mathcal{Z}$ remains at $\mathcal{Z}_0$. It measures how the variance of log-MRPKs of treated firms evolves *relative to firms in the control group*. $\Delta\Delta\sigma^2$ thus captures the causal effect of the policy change on the variance of treated firms in the counterfactual economy.

- $\Delta\sigma^2(C) = \sigma^2_C\left(\begin{pmatrix}\Theta_0\\ \Theta_1\end{pmatrix}, \mathcal{Z}_0\right) - \sigma^2_C\left(\begin{pmatrix}\Theta_0\\ \Theta_0\end{pmatrix}, \mathcal{Z}_0\right)$ is the change in the variance of log-MRPKs for firms in the control group in the counterfactual economy (i.e., the economy where $\mathcal{Z}$ remains at $\mathcal{Z}_0$). This change *only arises through general equilibrium (GE) effects*: the structural parameters for control firms are not affected (they remain equal to Θ_0), but the change affecting treated firms may modify equilibrium prices, which, in turn, may affect control firms' choices and therefore the variance of their log MRPKs. $\Delta\sigma^2(C)$ thus captures the change in the variance of log-MRPKs that would purely arise through equilibrium effects if only the experiment takes place.

Estimating Equation 2 is challenging. Consider first the GE term, $\Delta\sigma^2(C)$. In the post-experiment data, equilibrium prices are changing not only because of the policy experiment but also because of the change in $\mathcal{Z}$. Therefore, the observed change in the variance of log-MRPKs for control firms may not be a valid estimate for $\Delta\sigma^2(C)$. Our main result, Theorem 1 below, helps us overcome this challenge. This theorem shows that, in the class of models we consider, the distribution of log-MRPK for a group of firms facing similar frictions Θ solely depends on the frictions faced by these firms and not on the frictions faced by other firms in the economy, so that:

$$\sigma_C^2\left(\binom{\Theta_0}{\Theta_1}, \mathcal{Z}_0\right) = \sigma_C^2\left(\binom{\Theta_0}{\Theta_0}, \mathcal{Z}_0\right) \Rightarrow \Delta\sigma^2(C) = 0$$

In the counterfactual we consider, and for the class of models described below, the GE effect is, therefore, equal to zero.

Consider now the treatment effect, $\Delta\Delta\sigma^2$. In the data, we observe:

$$\widehat{\Delta\Delta\sigma^2} = \sigma_T^2\left(\binom{\Theta_0}{\Theta_1}, \boldsymbol{\mathcal{Z}_1}\right) - \sigma_T^2\left(\binom{\Theta_0}{\Theta_0}, \mathcal{Z}_0\right) - \left(\sigma_C^2\left(\binom{\Theta_1}{\Theta_0}, \boldsymbol{\mathcal{Z}_1}\right) - \sigma_C^2\left(\binom{\Theta_0}{\Theta_0}, \mathcal{Z}_0\right)\right),$$

while $\Delta\Delta\sigma^2$ only depends on the dispersion of log-MRPK when $\mathcal{Z} = \mathcal{Z}_0$. Again, Theorem 1 helps us solve this issue. The theorem ensures that, within the class of models we consider, the distribution of log-MRPK for a group of firms facing similar frictions Θ is independent of equilibrium prices, and therefore independent of $\mathcal{Z}$:

$$\sigma_T^2\left(\binom{\Theta_0}{\Theta_1}, \mathcal{Z}_0\right) = \sigma_T^2\left(\binom{\Theta_0}{\Theta_1}, \boldsymbol{\mathcal{Z}_1}\right) \quad \text{and:} \quad \sigma_C^2\left(\binom{\Theta_0}{\Theta_1}, \mathcal{Z}_0\right) = \sigma_C^2\left(\binom{\Theta_0}{\Theta_1}, \boldsymbol{\mathcal{Z}_1}\right)$$

This result implies that the empirical difference-in-difference, $\widehat{\Delta\Delta\sigma^2}$ is a valid estimate for the counterfactual difference-in-difference: $\widehat{\Delta\Delta\sigma^2} = \Delta\Delta\sigma^2$. Together, these results imply that the counterfactual change in aggregate efficiency is proportional to the empirical difference-in-difference estimate:

$$\Delta \log TFP = -\frac{\alpha}{2}\left(1 + \frac{\alpha\theta}{1-\theta}\right)\frac{\widehat{\Delta\Delta\sigma^2}}{2}$$

2.4 Scaling up the Experiment

Our second exercise asks: How would aggregate efficiency change if the policy was extended to all firms? Formally, the answer to this question is:

$$\Delta \log TFP = -\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\left[\text{var}\left(\log\frac{p_i y_i}{k_i}|\begin{pmatrix}\Theta_1\\ \Theta_1\end{pmatrix}, \mathcal{Z}_0\right) - \text{var}\left(\log\frac{p_i y_i}{k_i}|\begin{pmatrix}\Theta_0\\ \Theta_0\end{pmatrix}, \mathcal{Z}_0\right)\right]$$

Again, estimating this equation from the experimental data is a priori challenging. The experimental data allows us to estimate $\widehat{\Delta\Delta\sigma^2}$: the causal effect of the policy change on the variance of treated firms when, in the post-experiment data, $\mathcal{Z} = \mathcal{Z}_1$ and *only the treated firms receive the policy treatment*. In contrast, the counterfactual requires an estimate of this treatment effect in an economy with different equilibrium prices: $\mathcal{Z}$ remains unchanged at $\mathcal{Z}_0$ and *all firms receive the policy treatment*. However, thanks again to Theorem 1, we know that, for a group of firms facing similar structural parameters, the distribution of log-MRPKs solely depends on these parameters, i.e. it is independent of $\mathcal{Z}$ or the frictions that other firms in the economy face. Theorem 1 therefore implies that the causal effect of the policy treatment on treated firms observed in the experimental data, $\widehat{\Delta\Delta\sigma^2}$, would be similar even if all firms were treated and $\mathcal{Z}$ had remained at $\mathcal{Z}_0$. This implies that the counterfactual change in aggregate efficiency is again proportional to the empirical difference-in-difference $\widehat{\Delta\Delta\sigma^2}$:

$$\Delta \log TFP = -\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\widehat{\Delta\Delta\sigma^2}$$

the difference between this formula and the “ex-post evaluation formula” is the $1/2$ factor. In ex-post evaluation, only half of the firms (the randomly chosen treatment group) are affected, while in the scale-up, all of them are.

3 Set-Up and Main Result

This Section introduces a class of models of firms dynamics with frictions for which the distribution of MRPKs does *not* depend on the general equilibrium of the economy, i.e. Theorem 1 holds.

3.1 Set-up

Time t is discrete and there is no aggregate uncertainty. There is a continuum of firms indexed by $i \in [0;1]$. Each firm belongs to an industry $s \in [1, S]$. Firm i revenue is given by: $p_{it}y_{it} = A_{s,t}^{1-\theta_s} e^{\theta_s z_{it}} k_{it}^{\alpha_s \theta_s} l_{it}^{(1-\alpha_s)\theta_s}$. k_{it} is the capital stock and l_{it} employment. Log productivity $z_{it} \in Z$ follows a (potentially firm-specific but stationary) Markovian process and is i.i.d across firms. $A_{s,t}$ is an industry-specific revenue shifter. We discuss below several market structures that are consistent with such a revenue function. We consider the steady-state of this economy. At the steady-state, households' consumption Euler equation ties the equilibrium interest rate r_t to their discount rate so that the safe interest rate r is pinned down throughout our analysis.[4] We note $A_s = A_{s,t}$ the steady-state industry shifter, and w_s the, potentially industry-specific, steady-state wage. The aggregation models we will explore below assume a single labor market and therefore a single wage, so that we will assume here that $w_s = w$. However, this assumption is not required for Theorem 1 below to hold.

To clarify exposition, we assume no frictions in adjusting labor, so that firm i's profit writes:

$$\pi(z_{it}, k_{it}; w, A_s) = \max_l \left\{ A_s^{1-\theta_s} e^{\theta_s z_{it}} k_{it}^{\theta_s \alpha_s} l^{\theta_s(1-\alpha_s)} - wl \right\} = \Omega_s \left(\frac{A_s^{1-\Phi_s}}{w^{\Phi_s \frac{1-\alpha_s}{\alpha_s}}} \right) e^{\frac{\Phi_s}{\alpha_s} z_{it}} k_{it}^{\Phi_s} \ ,$$

where $\Phi_s = \frac{\alpha_s \theta_s}{1-(1-\alpha_s)\theta_s} < 1$, and Ω_s is an industry-specific constant. As it turns out, the assumption of frictionless labor choice is not necessary: Our main results are robust to the presence of some forms of labor adjustment costs.

Firms face several frictions in optimizing capital input. $\mathbf{\Theta}_s$ is a vector of structural parameters characterizing these frictions for firms in industry s. δ_s is the depreciation rate of capital in industry s. Investment is subject to a one period time-to-build. Firms can finance investment with short-term debt or equity. b_{it+1} is the debt payment due to creditors in period $t+1$. Debt and capital are the endogenous state variables of the firm's problem which we note: $\boldsymbol{x}_{it} = (k_{it}, k_{it+1}, b_{it}, b_{it+1})$. r_{it} is the interest rate charged by lenders, so that $\frac{b_{it+1}}{1+r_{it}}$ is the proceed from debt issued at date t. Investment and debt financing at date t can be subject to adjustment costs $\Gamma(z_{it}, \boldsymbol{x}_{it}; w, A_s; \mathbf{\Theta}_s)$. We also assume that firms pay taxes and receive subsidies: $\mathcal{T}(z_{it}, \boldsymbol{x}_{it}; w, A_s; \mathbf{\Theta}_s)$ corresponds to the net tax paid by the firm.

Finally, we also allow for generic forms of financing frictions. First, equity issuance

[4] This "exogeneity" of r results from our steady-state assumption, and holds for any additively separable utility function.

(negative net cash-flows) may be costly, and we note such costs $\mathcal{C}(z_{it}, \boldsymbol{x}_{it}; w, A_s; \boldsymbol{\Theta}_s)$. These costs are zero when the firm does not issue equity. Second, the amount of outside financing may be constrained, which we capture through a vector of constraint: $\boldsymbol{M}(z_{it}, \boldsymbol{x}_{it}; w, A_s; \boldsymbol{\Theta}_s) \leq 0$. Third, the interest rate on debt is described by a function $r()$ such that r_{it}=$r(z_{it}, \boldsymbol{x_{it}}; w, A_s; \boldsymbol{\Theta}_s)$. This function allows for risky debt and may embed costs of financial distress, such as liquidation costs.

We note e_{it} the cash-flows to equity holders, net of equity issuance costs:

$$\begin{aligned} e_{it} &= \pi(z_{it}, k_{it}; w, A_s) - (k_{it+1} - (1-\delta_s)k_{it}) + \left(\frac{b_{it+1}}{1 + r(z_{it}, \boldsymbol{x_{it}}; w, A_s; \boldsymbol{\Theta}_s)} - b_{it} \right) \\ &\quad - \Gamma\left(z_{it}, \boldsymbol{x}_{it}; w, A_s; \boldsymbol{\Theta}_s\right) - \mathcal{T}(z_{it}, \boldsymbol{x}_{it}; w, A_s; \boldsymbol{\Theta}_s) - \mathcal{C}(z_{it}, \boldsymbol{x}_{it}; w, A_s; \boldsymbol{\Theta}_s) \\ &= e(z_{it}, \boldsymbol{x}_{it}; w, A_s; \boldsymbol{\Theta}_s), \end{aligned}$$

i.e. cash-flows depend on state variables ($(z_{it}, \boldsymbol{x}_{it})$), the equilibrium wage w and industry-specific demand shifter A_s, and structural parameters governing industry-s frictions, $\boldsymbol{\Theta}_s$.

The timing is standard. At the beginning of period t, productivity z_{it} is realized. Then the firm produces, it selects the next period stock of capital k_{it+1}, pays the corresponding adjustment costs, reimburses its existing debt b_{it} and receives the proceeds from debt issuance $\frac{b_{it+1}}{1+r_{it}}$. Then, the period ends. We allow for – potentially strategic – default: the firm will operate in period $t+1$ if and only if its productivity belongs to a "survival set": $z_{it+1} \in \mathcal{Z}(k_{it+1}, b_{it+1}; w, A_s; \boldsymbol{\Theta}_s)$. When $z_{it+1} \notin \mathcal{Z}(k_{it+1}, b_{it+1}; w, A_s; \boldsymbol{\Theta}_s)$, default occurs and the continuation value to the firm's owner is assumed to be zero.[5] So that the number of firms remain constant at steady-state, we assume that each exiting firm is replace by a new firm endowed with no debt and the average capital stock of exiting firms.

Firms maximize the expected present value of cash-flows. To save on notations, let us temporarily omit the *it* index and denote with primes next-period variables. We make additional technical assumptions listed in Appendix A.2, and briefly explain why they are satisfied in most models. These assumptions ensure that the Bellman equation has a unique solution.[6] Under these assumptions, the present value of expected cash-flows

[5]This stark assumption is for the sake of exposition. Our results would carry through for a larger class of default continuation values, as long as they satisfy an homogeneity property similar to the assumptions in Theorem 1.

[6]These results are broadly used in the literature. They arise from the extension of the results in Stokey and Lucas (1989) from strictly continuous to piecewise continuous cash-flow functions. See Caballero and Leahy (1996) for a version of the proof with fixed adjustment costs and Hennessy and Whited (2007) for a version of the proof with fixed equity issuance costs. In both cases, the logic is the same and applies

$V(z,k,b;w,A_s;\mathbf{\Theta}_s)$ is uniquely defined as:

$$V(z,k,b;w,A_s;\mathbf{\Theta}_s) = \left| \begin{array}{l} \max_{k',b'} \left[e(z,k,k',b,b';w,A_s;\mathbf{\Theta}_s) + \beta \mathbb{E}_{z' \in \mathcal{Z}(k',b';w,A_s;\mathbf{\Theta}_s)} \left(V(z,k,b;w,A_s;\mathbf{\Theta}_s) | z \right) \right], \\ \boldsymbol{M}(z,k,k',b,b';w,A_s;\mathbf{\Theta}_s) \leq 0 \end{array} \right. \tag{3}$$

where $\beta < 1$ is the firm's discount rate, which is constant in this set-up with no aggregate uncertainty. This equation also uniquely defines investment and financing policy functions, which therefore depend on structural parameters $\mathbf{\Theta}_s$ and (w, Y_s).

3.2 Main Result

This section provides sufficient conditions under which the joint distribution of MRPKs $\left(\frac{py}{k}\right)$ is independent of (w, A_s). As we explain in the following Section, these conditions hold in a large class of models of firm dynamics, commonly used in macro-finance.

Theorem 1 (Distribution of wedges)**.**
Define $S = \frac{A_s}{w^{\frac{(1-\alpha_s)\Phi_s}{(1-\Phi_s)\alpha_s}}}$, the "scale" of industry s. Assume that:

1. *adjustment costs $\Gamma()$, taxes $\mathcal{T}()$, funding constraints $\boldsymbol{M}()$ and the equity issuance cost function $\mathcal{C}()$* all *satisfy the following property:*

$$\forall \ (z, \boldsymbol{x}; w, A_s; \mathbf{\Theta}_s), \quad Q(z, \boldsymbol{x}; w, A_s; \mathbf{\Theta}_s) = S \times Q\left(z, \frac{\boldsymbol{x}}{S}; 1, 1; \mathbf{\Theta}_s\right) \tag{4}$$

2. *the interest rate $r()$ satisfies the following property:*

$$\forall \ (z, \boldsymbol{x}; w, A_s; \mathbf{\Theta}_s), \quad r(z, \boldsymbol{x}; w, A_s; \mathbf{\Theta}_s) = r\left(z, \frac{\boldsymbol{x}}{S}; 1, 1; \mathbf{\Theta}_s\right) \tag{5}$$

3. *the survival set $Z()$ does not depend on aggregate conditions:*

$$Z(k', b'; w, A_s; \mathbf{\Theta}_s) = Z(k', b'; 1, 1; \mathbf{\Theta}_s) \tag{6}$$

Then, for firms in industry s, the steady-state distribution of MRPKs, $\frac{py}{k}$, does not depend on (w, A_s).

Proof. See Appendix A.3. □

whenever the cash-flow is piecewise continuous.

This theorem shows that, given structural parameters Θ_s, the steady-state distribution of MRPKs does not depend on the wage w or revenue-shifter A_s. This is the key result of the paper. It has important implications for inference in (quasi-) experimental settings. Consider an experiment that changes Θ_s for a particular industry. Because this experiment changes the frictions faced by firms, it will affect the distribution of MRPKs in this industry. Theorem 1 says that a similar change in Θ_s would similarly affect the distribution of MRPKs even if it occurred in an economy with different A_s and w. In other words, estimates of the effect of this change in policy on the distribution of MRPK in industry s are externally valid: they solely depend on the actual policy changes and not on the equilibrium where these effects are estimated. As we explained in Section 2, this theorem, combined with an aggregation model, allows us to measure the effect of a policy change on equilibrium misallocation using (quasi-) experimental data.

Theorem 1 rests on two key assumptions. The first one is that the elasticity of revenue to factors is constant. This property ensures that firm-level operating profits scale proportionally to $S = \frac{A_s}{w^{\frac{(1-\alpha_s)\Phi_s}{(1-\Phi_s)\alpha_s}}}$:

$$\pi\left(z, k; w, A_s\right) = \max_l \left\{A_s^{1-\theta_s} e^{\theta_s z} k^{\theta_s \alpha_s} l^{\theta_s(1-\alpha_s)} - wl\right\} = S\pi\left(z, \frac{k}{S}; 1, 1\right).$$

There is indeed evidence that capital and labor are more complement than suggested by Cobb-Douglas (Oberfield and Raval (2014)), but the overwhelming majority of macro-finance models use revenue functions with constant elasticity in labor and capital. The second key set of assumptions is the homogeneity of frictions in equations (4)-(6). We show in the following section that these homogeneity assumptions are satisfied in most models of firm dynamics.

3.3 Validity of Theorem 1 in Standard Models of Firm Dynamics

We now relate the assumptions in Theorem 1 to standard models of firm dynamics. We then conduct an extensive review of the recent literature on firm dynamics and find that in the overwhelming majority of models we review, these assumptions hold. Our sufficient statistics approach therefore provides a valid alternative to structural estimation when computing aggregate counterfactuals in the context of these models.

3.3.1 Adjustment Costs

Standard linear and quadratic adjustment costs to capital satisfy the assumptions in Theorem 1 since they are homogeneous of degree 1 in k. Similarly, fixed costs or asymmetric costs that scale with the capital stock, such as $k\mathbb{1}_{\{k'-(1-\delta)k<0\}}$, $\mathbb{1}_{\{k'-(1-\delta)k\neq 0\}}k$ also satisfy these assumptions. Finally, fixed costs that scale with output (e.g., $\mathbb{1}_{\{k'-(1-\delta)k\neq 0\}}py$) also work since $p_i y_i \propto S^{1-\Phi_s} e^{\frac{\Phi_s}{\alpha_s} z_i} k_s^{\Phi} = S \times e^{\frac{\Phi_s}{\alpha_s} z_i} \left(\frac{k}{S}\right)^{\Phi_s}$. Instead, if fixed costs are expressed in absolute terms (i.e. they do not scale with firms' output or capital stock), then the assumption in Theorem 1 no longer hold.

3.3.2 Financing Frictions

Standard models of financing frictions also satisfy the assumptions of Theorem 1. Consider first the interest rate function. For instance, in Michaels et al. (2016) or Gilchrist et al. (2014), debt is risky and in the event that the firm is unable to repay, the lender can seize a fraction $1-\zeta_s$ of the firm's fixed assets k. The firm's future market value cannot be used as collateral, so that a firm's access to credit is mediated by a net worth covenant, which restrains the firm's ability to sell new debt based on its current physical assets and liabilities. Concretely, default is triggered when net worth reaches 0, which defines a threshold value for productivity z_s^* such that:

$$0 = \kappa_0 \times S^{1-\Phi_s} e^{\frac{\Phi_s}{\alpha_s} z_s^*} k^{\Phi_s} - b + p_k(1-\delta_s)k, \tag{7}$$

with κ_0 a constant and $p_k < 1$ is the second-hand price of capital, which we treat as a technological parameter. As in Michaels et al. (2016), the right side of the previous equation represents the resources that the firm could raise in order to repay its debt just prior to bankruptcy. The zero-profit condition for lenders then pins down the risky interest rate:

$$\beta \left[\int_0^{z_s^*} \left(\kappa_0 S^{1-\Phi_s} e^{\frac{\Phi_s}{\alpha_s} z'} k_s'^{\Phi} + (1-\zeta_s)(1-\delta_s)k' \right) dH(z'|z) + (1-H(z^*|z))b' \right] = \frac{b'}{1+r(z,\boldsymbol{x};w,Y_s;\boldsymbol{\Theta}_s)}. \tag{8}$$

Equations (7) and (8) define the survival set $Z()$ and interest rate function $r()$. It is clear from Equation (7) that $z_s^*(k,b;w,A_s;\boldsymbol{\Theta}_s) = z_s^*(\frac{k}{S},\frac{b}{S};1,1;\boldsymbol{\Theta}_s)$. As a result, it is direct to see that Equation (8) implies that $r()$ is scale independent: $r(z,\boldsymbol{x};w,A_s;\boldsymbol{\Theta}_s) = r(z,\frac{\boldsymbol{x}}{S};1,1;\boldsymbol{\Theta}_s)$, and satisfies the assumption in Theorem 1.

Similarly, the specification of debt renegotiation in Hennessy and Whited (2007) would also satisfy our assumptions. More generally, these models make the probability of default independent of the scale of the economy S, and the loss given default proportional to S. These properties ensure our assumption about $r()$ in Theorem 1 is satisfied. Models of risk-free debt, such as Midrigan and Xu (2014), also satisfy our assumption.

Our assumption on the cost of equity is also verified in Michaels et al. (2016) and Gilchrist et al. (2014), who posit that equity issuances are subject to an underwriting fees such that there is a positive marginal cost to issue equity:

$$\mathcal{C}(z, \boldsymbol{x}; w, A_s; \boldsymbol{\Theta}_s) \propto -\min\{0, e(z, \boldsymbol{x}; w, A_s; \boldsymbol{\Theta}_s)\}$$

Given that $e(z, \boldsymbol{x}; w, A_s; \boldsymbol{\Theta}_s) = Se(z, \frac{\boldsymbol{x}}{S}; 1, 1; \boldsymbol{\Theta}_s)$, it is direct that $\mathcal{C}(z, \boldsymbol{x}; w, A_s; \boldsymbol{\Theta}_s) = \mathcal{C}e(z, \frac{\boldsymbol{x}}{S}; 1, 1; \boldsymbol{\Theta}_s)$. Thus, financing frictions specified in Gilchrist et al. (2014) and Michaels et al. (2016) satisfy the assumptions of Theorem 1. Additionally, fixed or quadratic equity issuance costs would also satisfy these assumptions as long as they scale with the size of the firm. For instance, issuance costs proportional to $\frac{\min\{0,e\}^2}{k}$ or $k\mathbb{1}_{e<0}$ belong to that category.

Finally, our formulation of financing frictions also encompasses debt constraints as in Midrigan and Xu (2014) or Catherine et al. (2018). In both models, debt is assumed to be risk-free through full collateralization: $b' \leq \xi k'$. and producers can only issue claims to a fraction χ of their future profits: $-e \leq \chi \times V$. In this case, the vector $\boldsymbol{M}()$ of funding constraints consists of the last two inequalities, which are both homogeneous of degree 0 in S and thus satisfy the assumptions of Theorem 1. Of course, any combination of the constraints in Midrigan and Xu (2014) and Hennessy and Whited (2007) would also satisfy these assumptions. Note that our model similarly encompasses debt constraints (instead of equity constraints) where debt financing is limited by existing or future cash flows.

3.3.3 Taxes

Standard specifications for the corporate income tax, $\mathcal{T}(.) = \tau \max(0, \pi(.) - \delta k - b)$, satisfy the assumption of Theorem 1 since $\pi(z, \boldsymbol{x}; w, A_s; \boldsymbol{\Theta}_s) = S\pi(z, \frac{\boldsymbol{x}}{S}; 1, 1; \boldsymbol{\Theta}_s)$. A progressive tax system could be consistent with these assumptions provided that the tax brackets are defined in terms of percentile of the firm profit distribution. Similarly, size-based regulations typically will not generically satisfy the assumption in Theorem 1. However, a specification where regulation is specified as a piece-wise linear labor tax and the thresholds are defined in terms of percentile of the output distribution would satisfy the assumptions in Theorem 1.

3.3.4 Recent literature review

The previous section discussed standard features of firm dynamics models that fall within the assumption of Theorem 1. How common are these features in the literature? We answer this question by conducting a systematic literature review. We search for all papers citing Hennessy and Whited (2007), Midrigan and Xu (2014) and Moll (2014), published within a list of twelve journals[7] and with at least 50 Google Scholar citations. This search delivers a list of 44 recent papers. We group the set of assumptions used in these models into five categories: production function, adjustment costs, borrowing constraint, equity issuance costs, and taxes. For each of the 44 papers and each of these 5 categories, we check whether the paper's assumptions satisfy the conditions of Theorem 1 for this category. Table 1 shows that modeling choices made in these papers are almost always consistent with Theorem 1's assumptions. *All* papers but one assume Cobb-Douglas production. In five of these papers, operating leverage is modeled through a non-scalable fixed cost. Non-scalable fixed costs do not fit Theorem 1's assumptions. These types of fixed costs, however, have the unpleasant feature that they become irrelevant as firms grow and are therefore not desirable features for models of firm dynamics. In almost all papers, the specification for physical adjustment costs scales linearly with S. Even when there are fixed costs of investment, these costs are assumed to be proportional to total sales and therefore also scale with S. In all but 3 papers, the borrowing constraint is scale-free. Equity issuance costs constitute the most frequent deviation from the assumptions of Theorem 1: nine papers introduce fixed equity issuance costs that do not scale with the size of the firm. Finally, all but 2 papers introduce a standard corporate tax, which naturally scales with S. Overall, the assumptions of Theorem 1 hold in most existing models in the recent literature.

3.4 Aggregation Model

Equipped with Theorem 1, we now proceed to discuss how to use difference-in-difference estimates to perform aggregate counterfactual analysis. What is needed at this stage is an aggregation model.

To fix ideas, we use here the aggregation model of Hsieh and Klenow (2009). Our approach works with more complex aggregation models as we describe below. The economy is composed of a measure 1 of firms, belonging to an industry $s \in [1, S]$. Each

[7] American economic review, Econometrica, Journal of Political Economy, Review of Economic Studies, Quarterly Journal of Economics, Journal of Finance, Journal of Financial Economics, Review of Financial Studies, one of the three American Economic Journal and Journal of Monetary Economics.

firm produces intermediary inputs y_i. Industry output Y_s^0 is produced by combining intermediate inputs y_i with a CES technology. A perfectly competitive final good market aggregates industry output using a Cobb-Douglas technology:

$$\log(Y^0) = \sum_{i=1}^{S} \phi_s \log(\left(Y_s^0\right)^{\phi_s}) \text{ and } Y_s^0 = \left(\int_{i \in s} y_i^{\theta_s}\right)^{\frac{1}{\theta_s}}$$

Production follows a Cobb-Douglas technology ($y_i = e^{z_i} k_i^{\alpha_s} l_i^{1-\alpha_s}$), so that firm i's revenue is similar to the one introduced in Section 3.1 with $A_s = Y_s \left(\frac{Y_s}{Y}\right)^{-\frac{1}{1-\theta_s}}$. Labor is supply elastically by households as described in Section 3.4.3 below. There is a perfectly elastic supply of the capital good, which is produced from the final good by a perfectly competitive market using a constant return-to-scale technology. Note that this assumption, however, is not necessary for our results: in Appendix A.9, we show, in a simplified version of our model, that even when the supply of capital is imperfectly elastic, Theorem 1 still hold, and our aggregation formula for TFP is unchanged.

Firms face no frictions when optimizing labor inputs. In contrast, firms' select their capital stock under a set of frictions described in Section 3 and that satisfy Theorem 1's assumptions. Θ_s^0 describe the vector of structural parameters that govern frictions for firms in industry s in this initial economy. These frictions generate a wedge between MRPKs, $\frac{p_i y_i}{k_i}$ and firms' user cost of capital. Following Hsieh and Klenow (2009), we assume that within each industry, the joint distribution of log-productivity z_i and log-MRPKs is normal.

With this aggregation model, aggregate efficiency, TFP, is naturally defined, as:

$$\log(TFP) = \log(Y) - \alpha^* \log(K) - (1 - \alpha^*) \log(L),$$

with K the aggregate capital stock, L is aggregate employment, and $\alpha^* = \sum_{s=1}^{S} \phi_s \alpha_s$ is the weighted-average capital share, weighted by the industry share in total output.

3.4.1 Ex Post Evaluation of Policy Experiments

We introduce a policy experiment in this economy. In industry s, firms receives a policy treatment that changes their structural parameters by $d\Theta_s$. At the same time as this policy treatment, two potentially confounding changes take place:

(1) average productivity changes so that the undistorted aggregate TFP, $\mathcal{Z}$ =

$$\prod_{s=1}^{S}\left(\int_{i\in s} e^{\frac{\theta_s}{1-\theta_s}z_{is}}di\right)^{\frac{\phi_s(1-\theta_s)}{\theta_s}}$$, goes from $\mathcal{Z}_0$ to $\mathcal{Z}_1$.[8]

(2) all firms experience a common change $d\Theta$ to their structural parameters.

Post-experiment, the structural parameters in industry s are therefore:

$$\Theta_s^1 = \Theta_s^0 + d\Theta + d\Theta_s$$

. We assume that the policy experiment and the aggregate shocks are small: $d\Theta_s \ll 1$, $d\Theta \ll 1$. We also assume that there is limited ex ante heterogeneity in frictions: for all s and s', $\left|\Theta_s^0 - \Theta_{s'}^0\right| \ll 1$.

We are looking to estimate the contribution of the policy change to change in aggregate TFP: how much would have aggregate TFP increased *if the only change taking place in the economy was the change in policy* $d\Theta_s$? We show in Proposition 1 below that the answer depends on three sufficient statistics that can be directly estimated in the (quasi-)experimental data:

- $\widehat{\Delta\Delta\sigma^2}(s)$ is the difference-in-difference estimate of the effect of the policy change on the variance of log-MRPK in industry s. It corresponds to the change in the variance of log-MRPK for firms in industry s, relative to a set of industries that are not affected by the policy change. As we explain below, this statistic can also be estimated using a heterogeneous treatment intensity approach, which is commonly used in the empirical literature.

- $\widehat{\Delta\Delta\mu}(s)$ is, similarly, the difference-in-difference estimate of the effect of the policy change on the mean log-MRPK in industry s.

- $\Delta\Delta\widehat{\sigma_{lMRPK,lpy}}(s)$ corresponds to the difference-in-difference estimate of the effect of the policy change on the covariance between log output and log sales in industry s.

Proposition 1 (Ex post evaluation of policy experiments)**.** *Assume that the the firm-level model satisfies the assumptions in Theorem 1. Then, the counterfactual change in aggregate TFP*

[8] The same result would hold in the presence of other aggregate shocks such as a shock to the elasticity of labor supply or a shock to aggregate labor supply.

purely due to the change in policy $(d\Theta_s)_{s\in[1,S]}$ *is approximated by the following expression:*

$$\Delta \log(TFP) \approx -\frac{\alpha^*}{2}\sum_{s=1}^{S}\kappa_s\left(1+\frac{\alpha\theta_s}{1-\theta_s}\right)\widehat{\Delta\Delta\sigma^2}(s) \\ -\sum_{s=1}^{S}(\phi_s\alpha_s-\kappa_s\alpha^*)\left[\widehat{\Delta\Delta\mu}(s)+\Delta\Delta\widehat{\sigma_{lMRPK,lpy}}(s)+\frac{1}{2}\frac{\alpha_s\theta_s}{1-\theta_s}\widehat{\Delta\Delta\sigma^2}(s)\right] \tag{9}$$

where κ_s *is the share of industry* s *in the total capital stock of the pre-reform economy.*

Proof. See Appendix A.4. □

This formula decomposes misallocation into two terms. The first one captures how the policy change affects within-sector misallocation. The second one measures how the policy change induces cross-industry reallocation of production, which, in turn, affects aggregate efficiency. This last term corresponds to the sum of changes in industry output (the terms in squared brackets), weighted by the difference, for each industry, between output share and capital share ($(\phi_s\alpha_s - \kappa_s\alpha^*)$). This difference is larger for more distorted industries, those with a higher sales-to-capital ratio than average. Hence, through this last term, TFP is lower if output increases more in distorted sectors.

Practical estimation There are two standard ways in the empirical literature to estimate the statistics in Equation 9. A first approach assumes that a control group, made of a subset of industries, does not receive any policy treatment. In this case, for each industry s receiving a treatment $d\Theta_s$, the statistics are estimated using the standard difference-in-difference estimate relative to this control group. For example, $\widehat{\Delta\Delta\sigma^2}(s)$ corresponds to the pre-post change in the observed variance of log-MRPKs for firms in industry s minus the same pre-post change for firms in control industries.

A second approach, more common in the literature, and which we use in our empirical application, imposes a linear structure on the effect of the reform. Industries are heterogeneous in their *exposure* to an aggregate policy change: $d\Theta^{\text{reform}}$. λ_s is the exposure of industry s to $d\Theta^{\text{reform}}$. This structure allows us to also allow for a confounding idiosyncratic shock, η_s as long as it is orthogonal to λ_s (a classic identifying assumption in this literature):

$$\Theta_s^1 = \Theta_s^0 + \lambda_s d\Theta^{\text{reform}} + d\Theta + \eta_s, \text{ with: } \eta_s \perp \lambda_s$$

Note in particular that this approach assumes that industries with $\lambda_s = 0$ are not directly exposed to the aggregate policy change $d\Theta_s$. In this setting, we can estimate $\widehat{\Delta\Delta\sigma^2(s)}$

by regressing the observed variance of log-MRPK across industries, $\widehat{\sigma^2}(s)$, against the exposure measure λ_s and a post-experiment dummy and then multiplying the resulting coefficient estimate by λ_s. $\widehat{\Delta\Delta\mu(s)}$ and $\Delta\Delta\widehat{\sigma_{lMRPK,lpy}}(s)$ can be estimated in a similar way.

3.4.2 Scaling up policy experiments

We now consider the case of an experiment that is not at scale within each industry. The model is similar to Section 3.4.1 except that, in each industry s, only a random share ν_s of firms receives the policy treatment $d\Theta_s \ll 1$. We are looking to estimate the counterfactual change in aggregate TFP that would result from extending the policy $d\Theta_s$ to all firms in each industry s. The answer can be simply expressed as a function of the three sufficient statistics introduced in Section 3.4.1:

Proposition 2 (Scaling-up policy experiments)**.** *Assume that the data-generating process belongs to a model that satisfies the assumptions in Theorem 1. Then, relative to pre-experiment economy, the counterfactual change in aggregate TFP resulting from extending the policy experiment to all firms $(d\Theta_s)_{s\in[1,S]}$ is approximated by the following expression:*

$$\Delta \log(TFP) \approx -\frac{\alpha^*}{2}\sum_{s=1}^{S}\kappa_s\left(1+\frac{\alpha\theta_s}{1-\theta_s}\right)\widehat{\Delta\Delta\sigma^2}(s) - \sum_{s=1}^{S}\left(\phi_s\alpha_s-\kappa_s\alpha^*\right)\left(\widehat{\Delta\Delta\mu}(s)+\Delta\Delta\widehat{\sigma_{lMRPK,lpy}}(s)+\frac{1}{2}\frac{\alpha_s\theta_s}{1-\theta_s}\widehat{\Delta\Delta\sigma^2}(s)\right) \tag{10}$$

where κ_s is the share of industry s in the total capital stock of the pre-reform economy.

Proof. See Appendix A.5. □

In this context, the statistics $\widehat{\Delta\Delta\sigma^2}(s)$, $\widehat{\Delta\Delta\mu}(s)$ and $\Delta\Delta\widehat{\sigma_{lMRPK,lpy}}(s)$ are directly estimated through a difference-in-difference within industry s. For instance, $\widehat{\Delta\Delta\sigma^2}(s)$ corresponds to the pre-post change in the variance of log-MRPKs for firms in the treatment group in industry s relative to firms in the control group in industry s.

3.4.3 Effect on Output

Our approach is not restricted to analyzing the effect of policy change on misallocation. We can also estimate, in similar (quasi)-experimental settings, the contribution of a policy change to change in aggregate output or employment. In order to do this, we need to introduce the household side of the economy to obtain an aggregate labor supply curve. Assume a representative household has GHH preferences (Greenwood et al. (1988)) over consumption and leisure: $u(c_t, l_t) = \frac{1}{1-\gamma}\left(c_t - \frac{l_t^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}} \frac{\bar{w}}{\bar{L}^{\frac{1}{\epsilon}}}\right)^{1-\gamma}$, where c_t is period t consumption, l_t is period t labor supply, ϵ is the Frisch elasticity and $(\bar{w}, \bar{L})$ are normalizing constants. The representative household owns all the firms in the economy, as well as a safe asset that offers real return r in unlimited supply. In the absence of aggregate uncertainty, at the steady-state, optimal consumption and labor supply decisions imply that $L_t^s = \bar{L}\left(\frac{w}{\bar{w}}\right)^{\epsilon}$, and that the firm's discount factor β is pinned down by the household's psychological discount factor.

Consider now a policy experiment similar to the one described in Section 3.4.1. As in Section 3.4.1, we are interested in estimating how aggregate output would have changed *if the only change taking place in the economy was the policy change* $d\Theta_s$. We show, in Appendix A.6, that this counterfactual change in aggregate output can be easily estimated using the same sufficient statistics:

$$\Delta \log Y = -(1+\epsilon) \sum_{s=1}^{S} \frac{\phi_s \alpha_s}{1-\alpha^*} \left(\widehat{\Delta\Delta\mu(s)} + \frac{1}{2}\frac{\alpha_s \theta_s}{1-\theta_s} \widehat{\Delta\Delta\sigma^2}(s) + \Delta\Delta\widehat{\sigma_{MRPK,py}}(s) \right), \quad (11)$$

where the sufficient statistics $\Delta\Delta\mu(s)$, $\Delta\Delta\sigma^2(s)$ and $\Delta\Delta\sigma_{MRPK,py}(s)$ are defined in Section 3.4.1. Equation 11 is intuitive. First, if the experiment results in an increase in the average log-MRPK, aggregate output will decrease as the experiment depletes the capital stock. Second, if the policy change leads to an increase in the within-industry dispersion of log-MRPKs, total output decreases since aggregate production has become less efficient. Finally, if the experiment results in large firms being more distorted, it will lead to aggregate output losses. As shown in Section 3.4.1, these statistics can be estimated in this setting using a standard difference-in-differences approach in the presence of control industries, or through linear regression in the context of heterogeneous treatment intensity.

3.4.4 Alternative aggregation models

Our approach is compatible with more complex aggregation models. In Appendix A.7, we present similar aggregation formulas in a model with roundabout production and an input-output network. The formulas described above are only marginally affected. In Appendix A.8, we explore an extension where the revenue shares of industries are allowed to vary (the Cobb-Douglas aggregator in Hsieh and Klenow (2009) forces industry output shares to be constant across the steady-states we consider). One restriction with this model, however, is that we can only define aggregate TFP when the capital share is constant across industries: $\alpha_s = \alpha$. Appendix A.8 details the aggregation formula for this model.

In all these models, we assume that workers are perfectly mobile across industries so that a single wage clears the labor market. This assumption is standard but not necessary. Our framework can be extended to account for the imperfect mobility of workers across industries (or regions). Note also that, in all these models, we assume that production has constant return to scale and imperfect competition generates decreasing returns to scale in revenues. Our approach, however, also applies when there are decreasing returns to scale in production, with appropriate, but marginal, modifications to the aggregation formula.

3.4.5 Limitations of our approach

Like all aggregation exercises, our approach relies on several key assumptions: Cobb-Douglas production technology, homogeneous frictions, and frictionless labor optimization. These assumptions are sufficient to obtain the result in Theorem 1, which ensures that the observed changes in moments of the MRPK distributions in the experimental data are externally valid. For instance, this result does not hold with CES production functions; it would apply in the presence of taxes to labor, but not labor adjustment cost.

Our approach does not apply to any aggregation model. It relies on CES aggregation across firms and industries so that firm-level output is proportional to industry-level and aggregate output. Finally, the steady-state assumption is essential. Our results do not hold in the presence of business cycle fluctuations. This has important implications for empirical applications: our approach requires that we observe a sufficient number of periods around the experiment to captures the steady-state changes in the distribution of log-MRPK.

4 Application: French Banking Reform of 1985

We now use our framework to evaluate the macro-economic impact of the French banking deregulation episode of 1984-1985. Bertrand et al. (2007) analyze this reform using cross-industry heterogeneity as a source of identification. We combine their identification strategy and the methodology described above to estimate how much the reform contributed to aggregate TFP and output growth in this period.

4.1 Brief description of the reform

In the early 1980s, the French banking system was under the firm grip of the Treasury, who controlled, directly or indirectly, nearly all of the banking system. Interest rates were kept low to encourage investment. Corporate loans were subsidized under hundreds of different programs corresponding to different priorities of the government (preserve jobs, modernize industry, support agriculture, etc.). In order to prevent banks from overlending and keep inflation under the lid, banks were subject to close monitoring from the Bank of France and to monthly lending ceilings. In the mid-1980s, the finance ministry embarked on a series of reform of the banking system. On January 24, 1984, the French Banking Act went into effect, increasing competition between banks, allowing deposit-taking banks to have investment banking activities, encouraging investment in new branches. Subsidized loans were eliminated. Bank of France regulation was removed while interest rates were raised. The bond and money markets were revived, allowing banks to raise wholesale funding. Overall, the main effect of the reform was to move to a more decentralized decision-making process on loan amounts and interest rates and to introduce a stronger for-profit motive among banks. We defer the reader to Bertrand et al. (2007) for a more detailed description.

4.2 Data and Empirical Strategy

We are looking to estimate the contribution of banking deregulation to change in aggregate TFP around the reform. As explained above, one cannot do this by simply looking at aggregate TFP changes, as the reform may have coincided with a persistent change in technology or another policy reform. In other words, we are interested in how much aggregate TFP would have increased *if the only change taking place in the economy was banking deregulation*? The answer to this question is provided by formula 9. To estimate the three sufficient statistics required in this formula, we follow the identification strategy in Bertrand et al. (2007), which relies on varying treatment exposure at the industry-level.

We use accounting information from corporate tax files. The sample covers the 1978-1992 period and is restricted to firms that have either more than 100 employees or more than 20 million euros of sales for at least three years in our sample. We exclude firms in the financial sector (banking and insurance industries). The sample contains 4,506 firm-year observations, corresponding to 3,791 unique firms. There are 539 distinct 4-digit industries in our sample. For each of these industries, we define our measure of exposure to banking deregulation, λ_s, as the average ratio of bank debt over the net value of total assets in the pre-reform period. This heterogeneous treatment intensity approach follows Bertrand et al. (2007) and is in the tradition of the literature on financial development (e.g. Rajan and Zingales (1998)). Intuitively, industries that are more financially dependent on banks prior to the reform are more affected by the deregulation. We defer the reader to Bertrand et al. (2007) for an in-depth discussion of the underlying identifying assumption.

Because of a change in accounting regulation in 1984, bank debt is only available in tax files starting in 1984. As a result, we compute λ_s using only the average leverage ratio by industry in 1984. The change in accounting regulation also changes the reporting of value-added and fixed assets starting in 1984. However, the reporting of total sales and total assets is not affected. We, therefore, measure log-MRPK at the firm-level as the log of the ratio of sales to the gross book value of total assets.[9] This ratio is trimmed at the 5% level every year to limit the influence of outliers. To compute the moments of the log-MRPK distribution at the industry-year level, we require that there are at least 10 firms in each industry-year cell. This results in a sample of 370 unique industries s, and 4,509 industry-year observations for which we can compute moments of the log-MRPK distribution: the mean and variance of log-MRPK, as well as the covariance of log-MRPK and log sales. Table 2 provides simple summary statistics for the main variables we use in our empirical analysis.

For each one of these three moments M_{st}, we evaluate the effect of the financial reform by adopting a standard difference-in-difference estimation strategy with heterogeneous treatment exposure:

$$M_{st} = \delta_t + \eta_s + b_M.\lambda_s \times POST_t + \mu_s \times t + \eta_{st}, \tag{12}$$

where δ_t is a year fixed-effect and η_s is an industry fixed-effect. λ_s is the industry-level measure of exposure to banking deregulation, and $\mu_s \times t$ are industry-specific trends. Finally, $POST_t$ is a dummy variable for the post-reform period. Since it is difficult to unambiguously assign 1984 to the pre- or post-reform period, we exclude 1984 from our regression sample. Standard errors are clustered at the industry level. To check monotonicity, we also estimate specifications where we split the exposure variable, λ_s, into quartiles of treatment intensity.

As explained in Section 3.4.1 above, $b_{\sigma^2}.\lambda_s$ – the estimated coefficient when the dependent variable is the variance of log-MRPK σ^2 – is the empirical estimate of $\Delta\Delta\sigma^2(s)$,

[9] With Cobb-Douglas production function in capital, labor, and intermediary inputs, value-added is a constant share of total sales and the share is constant over time (since we assume that output elasticities are constant over time). As a result, treatment effects on moments of the log sales-to-capital ratio are equal to the treatment effects on moments of the log value added-to-capital ratio.

one of the sufficient statistics we use in our counterfactual analysis. The same inference applies for the two other sufficient statistics – $\Delta\Delta\mu$ and $\Delta\Delta\sigma_{lpy,lMRPK}$. Importantly, this inference relies on the assumption that industries that do not use any debt pre-reform (i.e. $\lambda_s = 0$) are not directly affected by the banking deregulation.

4.3 Empirical Results

We start with graphical evidence. For each industry s and year t, we first compute the within-industry variance of log-MRPK. We then split the sample into high exposure industries (industries with bank dependence λ_s in the top quartile) and low exposure industries (λ_s in the bottom quartile). For these two groups of industries, we compute the average variance of log-MRPK relative to its pre-reform (1978-1983) average. Figure 2 reports the yearly difference between the two groups over time. This difference mimics the difference-in-difference estimator described in Equation (12). Figure 2 shows no clear evidence of differential trends between high and low-exposure industries in the period leading up to the reform. We confirm this result through regression analysis.

Table 3 estimates Equation 12. In column (1)-(3), we use the cross-sectional variance in log-MRPK at the industry-year level, $\widehat{\sigma_t^2}(s)$, as our dependent variable. Columns (1)-(2) uses bank exposure λ_s linearly, while column (3) uses instead quartiles of λ_s. Columns (2) and (3) control for industry-specific trends, while column (1) does not. Across all three specifications, we find a significant reduction in σ^2 in the post-reform period for the most exposed industries. Columns (3) confirm that the treatment effect is monotonic with industry exposure λ_s. For industries in the top quartile of exposure, we measure a statistically significant decrease in $\widehat{\sigma_t^2}(s)$ of .07 relative to industries in the bottom quartile. These results confirm the analysis in Bertrand et al. (2007): we find evidence of efficient capital reallocation between firms. While we are using the same reform, the same data and the same identification strategy, the key difference is that the present paper focuses on a different set of moments, which allows us to *quantify* the improvement in aggregate TFP through improved capital allocation.

Columns (4)-(6) investigate the effect of the reform on the average log-MRPK. A priori, the reform has an ambiguous effect on this moment. On the one hand, increased banking competition should lead to an expansion in credit supply for previously-constrained constrained firms. This effect should lower the mean log-MRPK. On the other hand, the reform severely reduced subsidized loans, which restricted credit supply. This effect would lead to an increase in mean log-MRPK. Bertrand et al. (2007) shows that, on average, the reform did lead to a reduction in corporate leverage. However, we do not find any significant effect on the average log-MRPK. While column (4) shows a marginally significant increase in mean log-MRPK, consistent with a reduction in credit supply, columns (5) and (6) find that this effect is not robust to the inclusion of industry-specific trends. Similarly, column (7) reports a marginally significant and positive effect of the reform on $\sigma_{MRPK,py}$, the covariance between firm size and distortions. However, columns (8) and (9) show that this effect becomes insignificant once industry-specific trends are included.

Taken together, these results confirm the interpretation in Bertrand et al. (2007): the French banking deregulation of the mid-80s is mostly an experiment in improved capital reallocation. In the next section, we combine these estimates with our aggregation formula to quantify the effect of banking deregulation on aggregate TFP.

4.4 The Aggregate Effect of the Reform

4.4.1 MRPKS are Log-normally Distributed

One of the key assumptions to derive our simple aggregation formulas is that log-MRPKs are normally distributed. We split our sample into 4 broad industries: manufacturing, retail, services, and construction & transport. We report in Figure 1 normal probability plots for each of these broad industries, i.e. plots of the empirical c.d.f. of the standardized log output to capital ratios against the c.d.f. of a normal distribution. Figure 1 shows graphically that the log-normality assumption is reasonable in our dataset, except perhaps in services where the empirical distribution of log-MRPK exhibits a slightly higher mass than a normal distribution.

4.4.2 Implementing the methodology

Our methodology requires to calibrate some parameters. We assume that demand elasticities and capital shares are constant across industries. We use a standard calibration (e.g., David and Venkateswaran (2019)): the capital share in production is set to .33 and the price elasticity of demand is set to 6, which corresponds to $\theta \approx .83$. This price elasticity of demand is roughly in the middle of the range of values used in the literature. We compute ϕ_s and κ_s, the pre-reform share of industry s in total sales and capital directly in our firm-level dataset. For our quantification exercise, we use the specifications that exploit the continuous treatment intensity variable (λ_s) and include industry-specific trends (i.e. columns (2), (6), and (10) in Table 3). We find that: $\Delta\Delta\sigma^2(s) = -0.5.\lambda_s$, $\Delta\Delta\mu(s) = -0.3.\lambda_s$, and $\Delta\Delta\sigma_{MRPK,py}(s) = 0.2.\lambda_s$.

Theorem 1 (and our identifying assumption) ensures that these estimates correspond to the counterfactual change in the statistics that would have been observed if banking deregulation was the unique source of change in the economy. As a result, they can be used in formulas (9) to obtain the counterfactual change in aggregate TFP:

$$
\begin{aligned}
\Delta \log(TFP) \approx & \underbrace{-\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\sum_{s=1}^{S}\kappa_s \widehat{\Delta\Delta\sigma^2}(s)}_{2.9\%} \\
& \underbrace{-\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\sum_{s=1}^{S}(\phi_s-\kappa_s)\left(\widehat{\Delta\Delta\mu}(s)+\Delta\Delta\widehat{\sigma_{MRPK,py}}(s)+\frac{1}{2}\frac{\alpha\theta}{1-\theta}\widehat{\Delta\Delta\sigma^2}(s)\right)}_{-0.2\%} \\
\approx & \; 2.7\%
\end{aligned}
$$

The reallocation gains from banking deregulation are substantial: in the post-reform period, aggregate TFP increases by 2.7 % because of this financial reform. This corresponds to about a third of aggregate TFP growth over the post-reform period (8.8% between 1978-1983 and 1985-1992). The gains from within-sector reallocation contribute to 2.9% of increased TFP over this period. Cross-industry production reallocation lowers the overall TFP gain by -0.2% since the reform leads to an increase in production in industries that are relatively more distorted in the pre-reform economy.

We can also compute the counterfactual gain in aggregate output, using Equation 11:

$$
\Delta \log Y \approx -\frac{\alpha(1+\epsilon)}{1-\alpha}\sum_{s=1}^{S}\phi_s\left(\widehat{\Delta\Delta\mu(s)}+\frac{1}{2}\frac{\alpha\theta}{1-\theta}\widehat{\Delta\Delta\sigma^2}(s)+\Delta\Delta\widehat{\sigma_{MRPK,py}}(s)\right)\approx 4.9\%
$$

The gains to aggregate output in this experiment are mostly driven by the reallocation gains documented above: the effect of the reform on $\widehat{\Delta\Delta\mu(s)}$ and $\Delta\Delta\widehat{\sigma_{MRPK,py}}(s)$ cancels out so that the overall effect on aggregate Y comes from the reduction in σ^2 resulting from the banking deregulation. In terms of aggregate output, these reallocation gains are amplified in general equilibrium: labor supply responds endogenously to the increased wage on the labor market, which further increases aggregate output.

It is interesting to compare our aggregation result with a "naive" approach that linearly aggregates empirical estimates. A typical approach in the empirical literature would simply estimate the effect of the reform on firm-level log-sales using a similar identification strategy. In this naive approach, the resulting treatment effect on log-sales corresponds to the counterfactual aggregate effect of the reform on log output. Such an approach is erroneous because it fails to account for both the general equilibrium and allocative effects induced by the reform and described above.

In Table 4, we estimate Equation 12 using firm-level log-sales as a dependent variable. We find that the reform does not affect significantly firm-level sales. A naive aggregation approach would therefore conclude that banking deregulation did not increase aggregate output. Again, such an approach would be misguided as it fails to account for the effect of banking deregulation on allocative efficiency and for general equilibrium effects.

5 Conclusion

Quantifying input misallocation has become an active topic of research in macroeconomics. In their seminal paper, Hsieh and Klenow (2009) compare the dispersion in log-marginal revenue products across countries to estimate the importance of input misallocation in India and China. This approach – inferring misallocation from dispersion in productivities in microeconomic data – has become standard in the literature. There are, however, well-known limitations to this methodology. For instance, measurement errors in inputs or adjustment costs can create cross-sectional variations in observed marginal products even when resources are efficiently allocated. This approach is also silent on the particular frictions that generate misallocation and the potential policies that may improve allocative efficiency. In contrast, a large literature in applied microeconomics exploits (quasi-) experimental settings to estimate the causal effect on firm-level outcomes of policies that have the potential to reduce misallocation. Yet, this literature has mostly evaluated their microeconomic effect without quantifying how they affect allocative efficiency.

Our paper offers a simple methodology to provide such quantification in a standard quasi-experimental setting. Our methodology is consistent with a large class of models of firm dynamics but does not require the structural estimation of a particular model. In particular, we do not have to make specific parametric assumptions on the nature of distortions nor do we need to precisely map the policy change to the model's structural parameters. Our approach thus provides a simple way to measure gains from reallocation that can be achieved with actual policies, implemented in practice.

We apply our methodology to the French banking deregulation episode of the 1980s, previously analyzed in (Bertrand et al., 2007). While (Bertrand et al., 2007) show that the banking reform led to significant capital reallocation, their analysis does not quantify the effect of this reform on aggregate efficiency. Applying our methodology, we find that the banking deregulation led to an increase in aggregate TFP of about 2.7% in the post-reform period.

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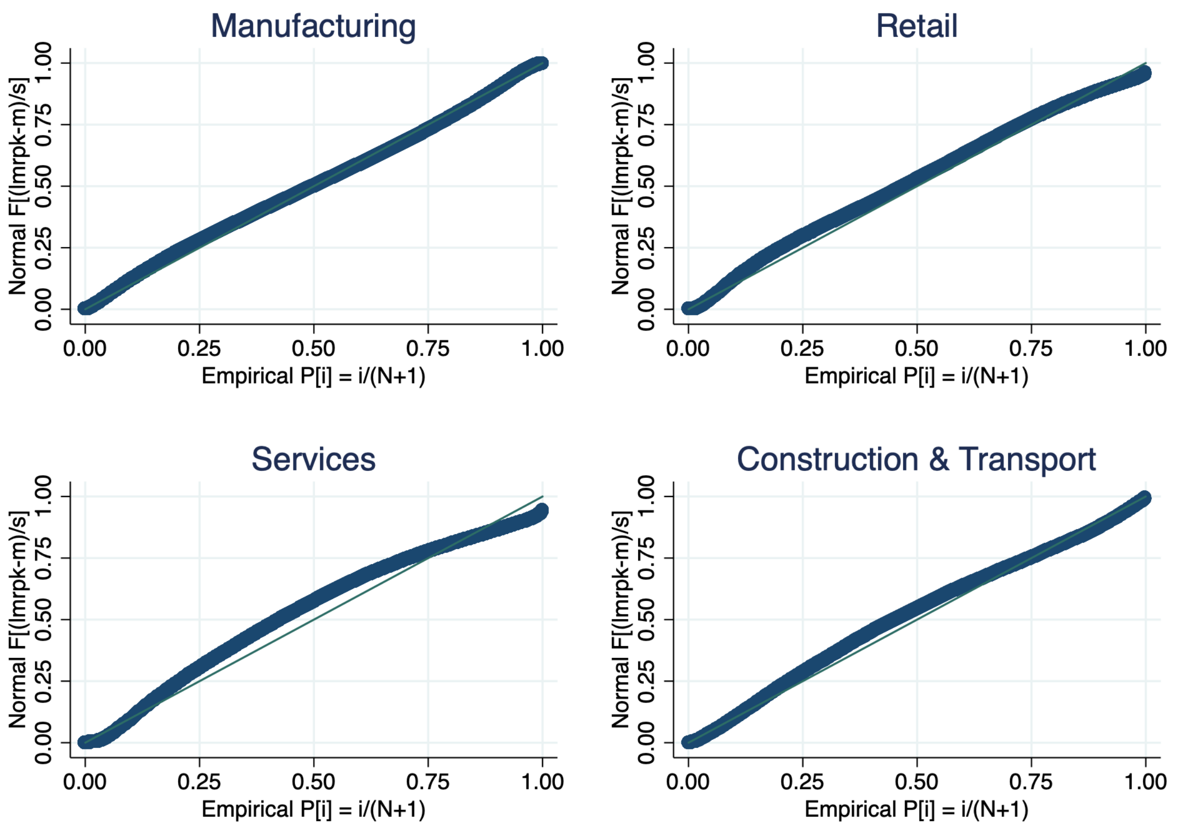
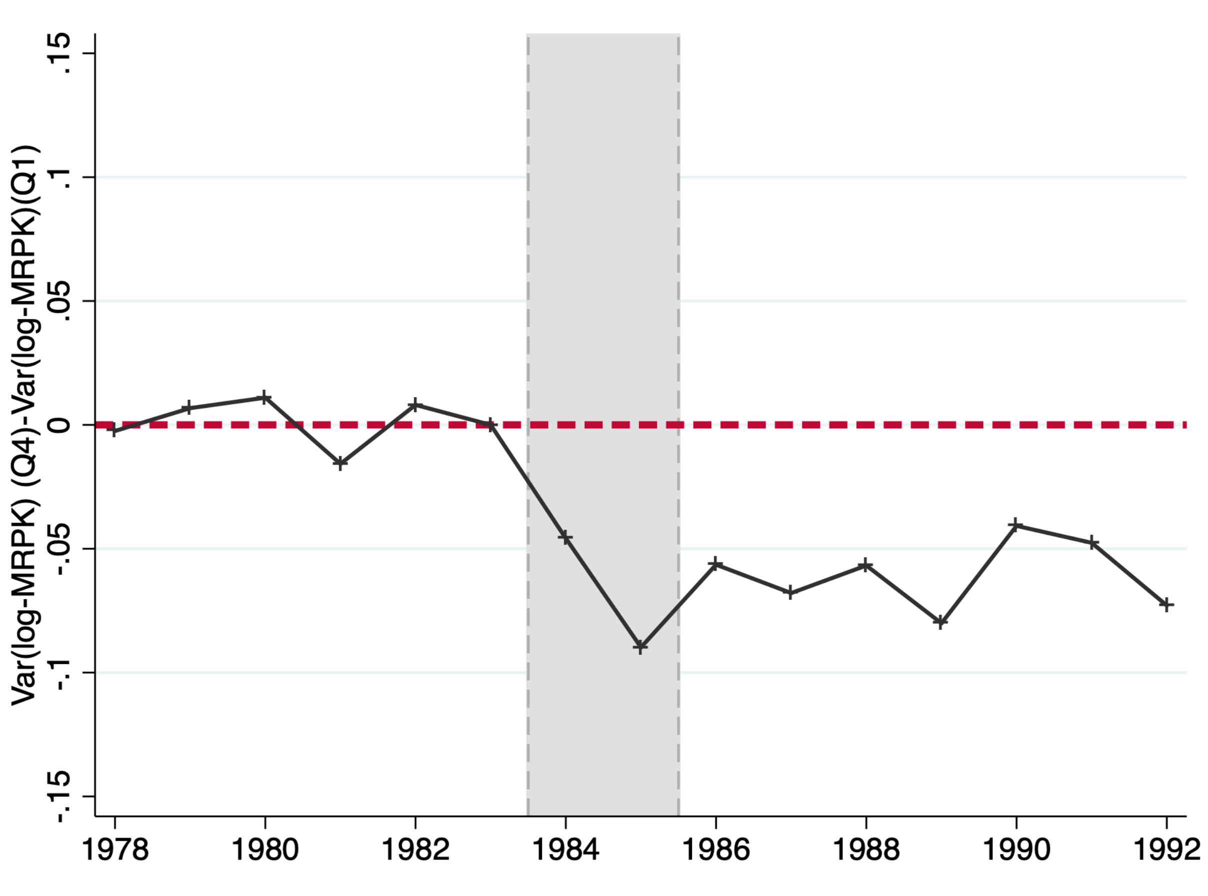
6 Tables and Figures

Figure 1: **Log-normality of MRPKs in the Data**



Note: We divide our sample into four broad industries: manufacturing, retail, services and construction&transport. The figure shows normal probability plots for these four industries for the distribution of log-MRPK. MRPK is computed as the ratio of sales to the gross book value of total assets.

Figure 2: **Variance log-MRPK distribution and banking deregulation**



Note: For each 4-digit industry-year in our sample, we compute the variance of log-MRPK. MRPK is computed as the ratio of sales to the gross book value of total assets. We split the sample into two groups: highly-exposed industries (industries with exposure λ_s in the top quartile) and low-exposure industries (λ_s in the bottom quartile). Every year, we compute the average of the variance of log-MRPK for both group and difference out the group's pre-reform (1978-1983) average. The figure shows the yearly difference between the highly-exposed group and the low-exposure group.

Table 1: Select Literature Review

Authors	Title	Date	(1) Production function function: Cobb-Douglas	(2) Adjustment costs Homogeneous	(3) Borrowing constraint: Homogeneous	(4) Equity issuance: Homogeneous	(5) Taxes schedule: Homogeneous
Panel A: Adjustment Cost papers							
Asker, Collar-Wexler, De Loecker	Dynamic inputs and resource (mis) allocation	2014	Y	Y	-	-	-
Bartelsman, Haltiwanger, Scarpetta	Cross-country differences in productivity: The role of allocation [...]	2013	N	Y	-	-	Y
Basu, Bundick	Uncertainty shocks in a model of effective demand	2017	Y*	Y	-	-	-
Bloom	The impact of uncertainty shocks	2009	Y	Y	-	-	-
Bloom, Bond, Van Reenen	Uncertainty and investment dynamics	2007	Y	Y	-	-	-
Bloom, Floetotto, Jaimovich, Saporta-Eksen, Terry	Really uncertain business cycles	2018	Y	Y	-	-	-
Cooper, Haltiwanger	On the nature of capital adjustment costs	2006	Y	Y	-	-	-
Gourio, Rudanko	Customer capital	2014	Y	Y	-	-	-
Hall	Measuring factor adjustment costs	2004	Y	Y	-	-	-
Peters, Taylor	Intangible capital and the investment-q relation	2017	Y	Y	-	-	-
Panel B: Structural & Dynamic Corporate Finance							
Bolton, Chen, Wang	A Unified Theory of Tobin's q, Corporate Investment Financing [...]	2011	Y	Y	-	Y	-
Bolton, Chen, Wang	Market timing, investment, and risk management	2013	Y	Y	-	Y	-
Cummins, Hassett, Oliner	Investment behavior, observable expectations, and internal funds	2006	Y	Y	-	-	-
DeAngelo, DeAngelo, Whited	Capital structure dynamics and transitory debt	2011	Y	Y	N	N	Y
Gamba, Triantis	The value of financial flexibility	2008	Y*	-	Y	Y	N
Gomes, Schmid	Levered returns	2010	Y*	N	Y	N	Y
Hackbarth, Mauer	Optimal priority structure, capital structure, and investment	2011	-	-	Y	N	Y
Hennessy, Whited	Debt dynamics	2005	Y	Y	Y	Y	N
Hennessy, Whited	How costly is external financing? [...]	2007	Y	-	Y	N	Y
Hennessy, Levy, Whited	Testing Q theory with financing frictions	2007	Y	Y	Y	Y	-
Li, Livdan, Zhang	Anomalies	2009	Y*	Y	-	N	-
Liu, Whited, Zhang	Investment-based expected stock returns	2009	Y	Y	-	-	-
Livdan, Sapriza, Zhang	Financially constrained stock returns	2009	Y*	Y	Y	N	-
Miao	Optimal capital structure and industry dynamics	2005	Y	-	Y	-	Y
Michaels, Page, Whited	Labor and capital dynamics under financing frictions	2018	Y	Y	Y	Y	-
Nikolov, Whited	Agency conflicts and cash: Estimates from a dynamic model	2014	Y	Y	Y	Y	Y
Riddick, Whited	The corporate propensity to save	2009	Y	Y	-	N	Y
Whited, Wu	Financial constraints risk	2006	Y	Y	N	N	-
Panel C: Macro-finance with heterogeneous firms							
Arellano, Bai, Zhang	Firm dynamics and financial development	2012	Y	Y	Y	Y	-
Buera, Shin	Financial frictions and the persistence of history [...]	2013	Y	-	Y	-	Y
Buera, Moll	Aggregate implications of a credit crunch [...]	2015	Y	-	Y	-	-
Cooley, Quadrini	Financial markets and firm dynamics	2001	Y	-	Y	Y	-
Fernandez-Villaverde, Guerron-Quintana, Rubio-ramirez, Uribe	Risk matters: The real effects of volatility shocks	2011	Y	Y	-	-	-
Gilchrist, Sim, Zakrajsek	Uncertainty, financial frictions, and investment dynamics	2014	Y	Y	Y	Y	-
Gomes	Financing investment	2001	Y	Y	Y	N	-
Gopinath, Kalemli-Ozcan, Karabarbounis, Villegas-Sanchez	Capital allocation and productivity in South Europe	2017	Y	Y	N	-	-
Gourio	Disaster risk and business cycles	2012	Y	Y	-	-	-
Gourio, Miao	Firm heterogeneity and the long-run effects of dividend tax reform	2010	Y	Y	-	Y	Y
Itskhoki, Moll	Optimal development policies with financial frictions	2018	Y	-	Y	-	Y
Khan, Thomas	Idiosyncratic shocks and the role of nonconvexities in [...] investment	2008	Y	Y	-	-	-
Khan, Thomas	Credit shocks and aggregate fluctuations[...]	2013	Y	Y	Y	Y	-
Liu, Wang, Zha	Land-price dynamics and macroeconomic fluctuations	2013	Y	Y	Y	-	-
Midrigan, Xu	Finance and misallocation: Evidence from plant-level data	2014	Y	Y	Y	Y	-
Moll	Productivity losses from financial frictions[...]	2014	Y	-	Y	-	-
Percent in line with our assumptions in all 44 papers			98%	98%	93%	79%	95%
Percent in line with our assumptions in papers that have the economic force			82%	97%	88%	59%	85%

Table 1: Literature Review (Continued)

Note: This table checks the validity of our assumption in a select review of 44 recent papers from the literature on firm dynamics. We restrict ourselves to all papers citing Hennessy and Whited (2007), Midrigan and Xu (2014) and Moll (2014), published either in the American economic review, Econometrica, Journal of Political Economy, Review of Economic Studies, Quarterly Journal of Economics, Journal of Finance, Journal of Financial Economics, Review of Financial Studies, one of the three American Economic Journal or Journal of Monetary Economics. To further restrict the scope of the review, we asked that the papers have at least 50 Google scholar citations. We end up with 44 papers, which we classify in 3 broad strands of literature: "adjustment cost papers", in which adjustment costs are the only friction, papers using dynamic corporate finance models (some of them corresponding to structural estimate, some of them being pure theoretical contributions) and macro-finance paper with financing frictions as well as competitive equilibrium modeling. For each of these papers, we then report if our core assumptions are satisfied: Cobb-Douglas production, and homogeneity of taxes, financing and real frictions. We report the results in columns (1)-(5). Y means that the assumption is satisfied, N that it is not. $-$ means that there is no such force in the model (so that our assumption is by default satisfied). Y^* means that the production function is indeed Cobb-Douglas, but the technology also includes *non-scalable fixed costs* of production. In the bottom two lines of the Table, we report the % of papers for which the assumption is satisfied. In the penultimate line, the % is computed among all papers. In the last line, it is computed only *among papers that have the force being in the model*. Hence, in column 3, 88% of the papers that have borrowing constraints satisfy the assumptions of Theorem 1, but this corresponds to 93% of the papers.

Table 2: **Summary Statistics**

	Mean (1)	Std. Dev. (2)	p10 (3)	p90 (4)	Observations (5)
Exposure (λ_s)	0.13	0.05	0.07	0.19	4,506
σ^2	0.18	0.14	0.07	0.34	4,506
μ	0.29	0.34	-0.08	0.75	4,506
$\sigma_{lMRPK,\log(py)}$	0.00	0.14	-0.13	0.15	4,506

Note: The sample period is 1978 to 1992. The sample corresponds to 370 unique industries s, and 4,509 industry-year observations. Exposure λ_s corresponds to the 1984 industry-average ratio of bank debt over total debt plus equity plus payables. σ^2 is the variance of log-MRPK computed at the industry-year level. μ is the industry-year mean of log-MRPK. $\sigma_{lMRPK,\log(py)}$ is the covariance of log-MRPK and log-sales added computed at the industry-year level.

Table 3: **Moments of log-MRPK distribution and banking deregulation**

	Var(log-MRPK)			Mean(log-MRPK)			Cov(log-MRPK, log VA)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Exposure × Post	-.4***	-.5***		.4*	-.32		.25*	.2	
	(.12)	(.16)		(.21)	(.21)		(.14)	(.23)	
Q2 Exposure × Post			-.058**			-.017			.035
			(.025)			(.032)			(.032)
Q3 Exposure × Post			-.058**			-.025			.012
			(.024)			(.031)			(.03)
Q4 Exposure × Post			-.07***			-.035			.041
			(.024)			(.031)			(.031)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry-trends	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes
Observations	4,506	4,506	4,506	4,506	4,506	4,506	4,506	4,506	4,506
Adj R^2	0.63	0.72	0.72	0.90	0.94	0.94	0.46	0.53	0.53

Note: We estimate the following model:

$$X_{st} = \delta_t + \eta_s + b_X.\lambda_s \times POST_t + \mu_s \times t + \epsilon_{st},$$

where X_{st} is one of the three moments of the log-MRPK distribution mentioned above. δ_t is a year fixed-effect and η_s is an industry fixed-effect. λ_s is the industry-level measure of exposure to banking deregulation, and $\mu_s \times t$ are industry-specific trends. Finally, $POST_t$ is a dummy variable for the post-reform period (1985-1992). Columns (1) to (3) (resp. (4) to (6), and (7) to (9)) use the cross-sectional variance of log-MRPK in an industry year as the dependent variable (resp. the cross-sectional mean of log-MRPK, and the cross-sectional covariance between log-MRPK and log sales). All columns include year and industry fixed-effects. Columns (2) and (3), (5) and (7), and (8) and (9) include industry-specific trends. Standard errors are clustered at the industry level.

Table 4: **Banking deregulation and log sales**

	Average $\log(py)$		
	(1)	(2)	(3)
inter	-.61	.078	
	(.49)	(.59)	
Q2 Exposure × Post			.042
			(.082)
Q3 Exposure × Post			.077
			(.073)
Q4 Exposure × Post			.032
			(.066)
Year FE	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes
Industry-trends	No	Yes	Yes
Observations	203,143	203,143	203,143
Adj R^2	0.32	0.33	0.33

Note: We estimate the following model:

$$\log(py)_{ist} = \delta_t + \eta_s + b_X.Z_s \times POST_t + \mu_s \times t + \epsilon_{ist},$$

where $\log(py)_{ist}$ is the log sale of firm i in industry s in year t. δ_t is a year fixed-effect and η_s is an industry fixed-effect. Z_s is the industry-level measure of exposure to banking deregulation, and $\mu_s \times t$ are industry-specific trends. Finally, $POST_t$ is a dummy variable for the post-reform period (1985-1992). All columns include year and industry fixed-effects. Columns (2) and (3) include industry-specific trends. Standard errors are clustered at the industry level.

APPENDIX

A Additional Algebra and Proofs

A.1 Proof of Section 2

We first show how to define aggregate TFP in the model of Section 2. Let $R = r + \delta$ be the user cost of capital. In an efficient economy with an aggregate capital stock K and aggregate employment L:

$$Y^*(K,L) = \left| \begin{array}{l} \max\limits_{(k_i),(l_i)} \left(\int_i y_i^\theta di\right)^{\frac{1}{\theta}}, \quad y_i = e^{z_i} k_i^\alpha l_i^{1-\alpha} \\ \int_i k_i di \le K \quad (\mathcal{L}_1), \qquad \int_i l_i di \le L \quad (\mathcal{L}_2) \end{array} \right.$$

where $\mathcal{L}_1$ and $\mathcal{L}_2$ are Lagrange multipliers. The first-order condition (FOC) in capital delivers $\alpha y_i^\theta (Y^*)^{1-\theta} = \mathcal{L}_1 k_i$ and in labor $(1-\alpha) y_i^\theta (Y^*)^{1-\theta} = \mathcal{L}_2 l_i$. Aggregate these FOCs across all firms:$\alpha \frac{Y^*}{K} = \mathcal{L}_1$ and $(1-\alpha)\frac{Y^*}{K} = \mathcal{L}_2$. Now, combine the expression of k and l in the FOCs and the production function: $y_i^\theta = (Y^*)^\theta e^{z_i \frac{\theta}{1-\theta}} \left(\frac{\alpha}{\mathcal{L}_1}\right)^{\frac{\alpha\theta}{1-\theta}} \left(\frac{1-\alpha}{\mathcal{L}_2}\right)^{\frac{(1-\alpha)\theta}{1-\theta}}$. Aggregate across firms: $1 = \left(\int_i e^{z_i \frac{\theta}{1-\theta}} di\right) \left(\frac{\alpha}{\mathcal{L}_1}\right)^{\frac{\alpha\theta}{1-\theta}} \left(\frac{1-\alpha}{\mathcal{L}_2}\right)^{\frac{(1-\alpha)\theta}{1-\theta}}$. Define: $\mathcal{Z} = \left(\int_i e^{z_i \frac{\theta}{1-\theta}} di\right)^{\frac{1-\theta}{\theta}}$. Substitute $\mathcal{L}_1$ and $\mathcal{L}_2$ with their expression above to find the efficient output: $Y^* = \mathcal{Z} K^\alpha L^{1-\alpha}$. $\mathcal{Z}$ is aggregate TFP in an efficient, undistorted, economy. A natural definition for aggregate TFP in an economy with distortions producing Y with K and L is therefore:

$$\log(TFP) = \underbrace{\log(\mathcal{Z})}_{\text{efficient TFP}} + \underbrace{\log(Y) - \log(Y^*)}_{\text{output loss from distortions}} = \log(Y) - \alpha\log(K) - (1-\alpha)\log(L).$$

We now calculate aggregate TFP in the distorted economy. Define τ_i the capital wedge faced by firm i: $\alpha\theta \frac{p_i y_i}{k_i} = R(1+\tau_i)$. As we explain in Section 3, τ_i results from the firm's optimal investment decision in the presence of frictions. In particular, τ_i is not an exogenous wedge – it depends on the history of firm i's productivity shocks and the friction faced by the firm (adjustment costs, financing frictions, etc. summarized by Θ).

The first-order condition in labor is: $(1-\alpha)\theta p_i y_i = w l_i$. Combining the definition of the capital wedge and this first-order condition, we can express firm i's output: $p_i y_i = Y e^{\frac{\theta}{1-\theta} z_i} \left(\frac{(1-\alpha)\theta}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}} \left(\frac{\alpha\theta}{R(1+\tau_i)}\right)^{\frac{\alpha\theta}{1-\theta}}$. Aggregate this equation across all firms: $1 = \underbrace{\left(\int_i \frac{e^{\frac{\theta}{1-\theta} z_i}}{(1+\tau_i)^{\frac{\alpha\theta}{1-\theta}}} di\right)}_{=I} \left(\frac{(1-\alpha)\theta}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}} \left(\frac{\alpha\theta}{R}\right)^{\frac{\alpha\theta}{1-\theta}}$, so that $\frac{w}{(1-\alpha)\theta} = \left(\frac{\alpha\theta}{R}\right)^{\frac{\alpha}{1-\alpha}} I^{\frac{1-\theta}{(1-\alpha)\theta}}$. Now, aggregate the FOC in labor across firms: $(1-\alpha)\theta Y = wL$, so that $\log(\frac{Y}{L}) = \frac{1-\theta}{(1-\alpha)\theta}\log(I) + \frac{\alpha}{1-\alpha}\log\left(\frac{\alpha\theta}{R}\right)$. Use the definition of τ_i and the expression for $p_i y_i$ to express firm i's capital stock: $k_i = Y e^{\frac{\theta}{1-\theta} z_i} \left(\frac{1-\alpha}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}} \left(\frac{\alpha\theta}{R(1+\tau_i)}\right)^{1+\frac{\alpha\theta}{1-\theta}}$. Aggregate across firms: $K = Y \underbrace{\left(\int_i \frac{e^{\frac{\theta}{1-\theta}z_i}}{(1+\tau_i)^{1+\frac{\alpha\theta}{1-\theta}}} di\right)}_{=J} \left(\frac{1-\alpha}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}} \left(\frac{\alpha\theta}{R}\right)^{1+\frac{\alpha\theta}{1-\theta}} = Y\frac{J}{I}\left(\frac{\alpha\theta}{R}\right)$, so that: $\log(\frac{Y}{K}) = \log(I) - \log(J) - \log\left(\frac{\alpha\theta}{R}\right)$. This allows to find aggregate TFP in the distorted economy:

$$\log(TFP) = \alpha\log(\frac{Y}{K}) + (1-\alpha)\log(\frac{Y}{L}) = \left(\alpha + \frac{1-\theta}{\theta}\right)\log(I) - \alpha\log(J)$$

Recall that z and $\log(1+\tau)$ are joint-normally distributed: $\begin{pmatrix}\log(z_i) \\ \log(1+\tau_i)\end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix}\mu_z \\ \mu\end{pmatrix}, \begin{pmatrix}\sigma_z^2 & \sigma_{\tau z} \\ \sigma_{\tau z} & \sigma^2\end{pmatrix}\right]$. This implies that $\log(J) = \frac{1}{2}(1+\frac{\alpha\theta}{1-\theta})\sigma^2 + \left(1+\frac{1-\theta}{\alpha\theta}\right)\log(I) - \frac{\log(\mathcal{Z})}{\alpha}$, so that $\log(TFP) = \log(\mathcal{Z}) - \frac{\alpha}{2}(1+\frac{\alpha\theta}{1-\theta})\sigma^2$, which is Equation 1.

Assume now that firms are randomly allocated in two groups of size 1/2: firms in the control group face Θ_0, firms in the treatment group Θ_1. We can use the law of total variance to decompose σ_τ^2 in this economy:

$$\begin{aligned}\text{var}\left(\log\frac{p_i y_i}{k_i} | \begin{pmatrix}\Theta_0 \\ \Theta_1\end{pmatrix}, \mathcal{Z}\right) &= \tfrac{1}{2}\text{var}\left(\log\tfrac{p_i y_i}{k_i}|\text{treatment}, \begin{pmatrix}\Theta_0 \\ \Theta_1\end{pmatrix}, \mathcal{Z}\right) + \tfrac{1}{2}\text{var}\left(\log\tfrac{p_i y_i}{k_i}|\text{control}, \begin{pmatrix}\Theta_0 \\ \Theta_1\end{pmatrix}, \mathcal{Z}\right) \\ &+ \tfrac{1}{4}\left(\mathbb{E}\left[\log\tfrac{p_i y_i}{k_i}|\text{treatment}, \begin{pmatrix}\Theta_0 \\ \Theta_1\end{pmatrix}, \mathcal{Z}\right] - \mathbb{E}\left[\log\tfrac{p_i y_i}{k_i}|\text{control}, \begin{pmatrix}\Theta_0 \\ \Theta_1\end{pmatrix}, \mathcal{Z}\right]\right)^2\end{aligned}$$

We assume that the experiment is small: $|\Theta_1 - \Theta_0| \ll 1$. As a result, $\left|\mathbb{E}\left[\log\frac{p_i y_i}{k_i}|\text{treatment}, \begin{pmatrix}\Theta_0 \\ \Theta_1\end{pmatrix}, \mathcal{Z}\right] - \mathbb{E}\left[\log\frac{p_i y_i}{k_i}|\text{control}, \begin{pmatrix}\Theta_0 \\ \Theta_1\end{pmatrix}, \mathcal{Z}\right]\right| \ll 1$ so that, using the notations defined in Section 2, we can approximate the previous expression at the first-order with:

$$\text{var}\left(\log\frac{p_i y_i}{k_i} | \begin{pmatrix}\Theta_0 \\ \Theta_1\end{pmatrix}, \mathcal{Z}\right) \approx \frac{1}{2}\left(\sigma_T^2\left(\begin{pmatrix}\Theta_0 \\ \Theta_1\end{pmatrix}, \mathcal{Z}\right) + \sigma_C^2\left(\begin{pmatrix}\Theta_0 \\ \Theta_1\end{pmatrix}, \mathcal{Z}\right)\right),$$

We now consider the counterfactual change in aggregate TFP from an initial economy where both groups face similar structural parameters Θ_0 and $\mathcal{Z} = \mathcal{Z}_0$ to an economy where firms in the treatment group face structural parameters Θ_1, firms in the control group face Θ_0 and $\mathcal{Z} = \mathcal{Z}_0$. Given the previous expression, this counterfactual change in aggregate TFP is:s

$$\Delta\log(TFP) \approx -\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\frac{\left(\sigma_T^2\left(\begin{pmatrix}\Theta_0 \\ \Theta_1\end{pmatrix}, \mathcal{Z}_0\right) - \sigma_T^2\left(\begin{pmatrix}\Theta_0 \\ \Theta_0\end{pmatrix}, \mathcal{Z}_0\right)\right) + \left(\sigma_C^2\left(\begin{pmatrix}\Theta_0 \\ \Theta_1\end{pmatrix}, \mathcal{Z}_0\right) - \sigma_C^2\left(\begin{pmatrix}\Theta_0 \\ \Theta_0\end{pmatrix}, \mathcal{Z}_0\right)\right)}{2},$$

which can be trivially rewritten as: $\Delta\log(TFP) \approx -\frac{\alpha}{2}\left(1+\frac{\alpha\theta}{1-\theta}\right)\left(\Delta\sigma^2(C) + \frac{1}{2}\Delta\Delta\sigma^2\right)$, i.e. Equation 2.

A.2 Technical Assumptions

Here are the additional technical assumptions we need to make sure that the Bellman problem is well-behaved. To save on notations, we omit here the *it* index and denote with primes next-period variables.

Assumption 1 (Technical assumptions)**.**

1. *The cash-flow function $e(.; w, A_s; \boldsymbol{\Theta}_s)$ is a piecewise continuous function of $(z, \boldsymbol{x})$*
2. *Log-productivity $z \in Z$, where Z is compact and convex. The Markovian transition function underlying z is strictly positive, has no atom and satisfies the Feller property.*
3. *Capital takes values in a set K which is assumed to be convex and compact.*
4. *Debt values b are restricted to a compact and convex set B.*
5. *Conditionally on past (k, b) and current capital choice k', debt values b' are restricted to a correspondence set $\mathcal{B}$:*

$$\mathcal{B}(z, k, k', b; w, A_s; \boldsymbol{\Theta}_s) = B \cap \{b' | \boldsymbol{M}(z, \boldsymbol{x}; w, A_s; \boldsymbol{\Theta}_s) \leq 0\}$$

We assume that the financial constraint $\boldsymbol{M}$ is such that $\mathcal{B}$ is compact, convex and non-empty.

The first item only assumes piecewise continuity because models may include (1) fixed costs of equity issuance (e.g. Hennessy and Whited (2007)) or (2) fixed capital adjustment costs (e.g. Caballero and Leahy (1996)). Most papers typically prove the third item of the above assumption by picking a maximum capital stock that the firm will never find optimal to choose. Decreasing returns to scale ensure that all levels of capital above this maximum cannot be optimal either (e.g. Hennessy and Whited (2007)). The fourth item is typically obtained in corporate finance models through assuming that the return on cash is low compared to the shareholder's discount rate, which bounds debt b from below. b is bounded above through financing frictions (cost of financial distress, collateral or cash-flow constraints).

A.3 Proof of Theorem 1

Without loss of generality, we drop the industry s subscript from A, α, Φ and Θ. Cash-flows to equity are given by the following formula:

$$\begin{aligned} e(z, \boldsymbol{x}; w, A; \boldsymbol{\Theta}) \quad = \quad & \frac{\alpha}{\alpha + (1-\alpha)\Phi} \left(\frac{(1-\alpha)\Phi}{\alpha + (1-\alpha)\Phi} \right)^{\frac{1-\alpha}{\alpha}\Phi} S^{1-\Phi} e^{\frac{\Phi}{\alpha} z} k^{\Phi} - (k' - (1-\delta)k) - \Gamma(z, \boldsymbol{x}; w, A; \boldsymbol{\Theta}) \\ & + \left(\frac{b'}{1 + r(z, \boldsymbol{x}; w, A; \boldsymbol{\Theta})} - b' \right) - \mathcal{T}(z, \boldsymbol{x}; w, A; \boldsymbol{\Theta}) - \mathcal{C}(z, \boldsymbol{x}; w, A; \boldsymbol{\Theta}), \end{aligned}$$

where $S = \frac{A}{w^{\frac{(1-\alpha)\Phi}{\alpha(1-\Phi)}}}$. By combining the different assumptions in Theorem 1, we get that:

$$\begin{aligned} e(z, \boldsymbol{x}; w, A; \boldsymbol{\Theta}) \quad = \quad & S \left(\frac{\alpha}{\alpha + (1-\alpha)\Phi} \left(\frac{(1-\alpha)\Phi}{\alpha + (1-\alpha)\Phi} \right)^{\frac{1-\alpha}{\alpha}\Phi} e^{\frac{\Phi}{\alpha} z} \left(\frac{k}{S} \right)^{\Phi} - \left(\frac{k'}{S} - (1-\delta)\frac{k}{S}\right) \right. \\ & \left. - \Gamma\left(z, \frac{\boldsymbol{x}}{S}; 1, 1; \boldsymbol{\Theta}\right) + \left(\frac{\frac{b'}{S}}{1 + r(z, \frac{\boldsymbol{x}}{S}; \Theta, 1, 1)} - \frac{b}{S} \right) - \mathcal{T}(z, \frac{\boldsymbol{x}}{S}; 1, 1; \boldsymbol{\Theta}) - \mathcal{C}(z, \frac{\boldsymbol{x}}{S}; 1, 1; \boldsymbol{\Theta}) \right) \end{aligned}$$

Therefore, $e(z, \boldsymbol{x}; w, A; \boldsymbol{\Theta}) = Se(z, \frac{\boldsymbol{x}}{S}; 1, 1; \boldsymbol{\Theta})$.

The firm's investment problem is given by the Bellman equation, Equation 3:

$$V(z,k,b;w,A;\mathbf{\Theta}) = \left| \begin{array}{l} \max\limits_{k',b'} e(z,\boldsymbol{x};w,A;\mathbf{\Theta}) + \beta \mathbb{E}_{z'}[V(z',k',b';w,A;\mathbf{\Theta})|z] \\ \boldsymbol{M}(z,\boldsymbol{x};w,A;\mathbf{\Theta}) \leq 0 \end{array} \right.$$

Let T be the corresponding Bellman operator. Consider the set of functions $\mathcal{F}$ that satisfy $f(z,k,b;w,A;\mathbf{\Theta}) = S \times f(z,\frac{k}{S},\frac{b}{S};1,1;\mathbf{\Theta})$, for all $(z,k,b;w,A;\mathbf{\Theta})$. Then, we can show T maps functions of $\mathcal{F}$ into functions of $\mathcal{F}$:

$$\begin{aligned}
Tf(z,k,b;w,Y;\mathbf{\Theta}) \quad &= \quad \left| \begin{array}{l} \max\limits_{k',b'} S \times \left(e(z,\frac{\boldsymbol{x}}{S};1,1;\mathbf{\Theta}) + \beta \mathbb{E}_{z'}[f(z',\frac{k'}{S},\frac{b'}{S};1,1;\mathbf{\Theta})|z] \right) \\ S \times \boldsymbol{M}(z,\frac{\boldsymbol{x}}{S};1,1;\mathbf{\Theta}) \leq 0 \end{array} \right. \\
&= S \times \left| \begin{array}{l} \max\limits_{k',b'} \left\{ e(z,\frac{k}{S},\frac{k'}{S},\frac{b}{S},\frac{b'}{S};1,1;\mathbf{\Theta}) + \beta \mathbb{E}_{z'}[f(z',\frac{k'}{S},\frac{b'}{S};1,1;\mathbf{\Theta})|z] \right\} \\ \boldsymbol{M}(z,\frac{k}{S},\frac{k'}{S},\frac{b}{S},\frac{b'}{S};1,1;\mathbf{\Theta}) \leq 0 \end{array} \right. \\
&= S \times \left| \begin{array}{l} \max\limits_{k',b'} \left\{ e(z,\frac{k}{S},k',\frac{b}{S},b';1,1;\mathbf{\Theta}) + \beta \mathbb{E}_{z'}[f(z',k',b';1,1;\mathbf{\Theta})|z] \right\} \\ \boldsymbol{M}(z,\frac{k}{S},k',\frac{b}{S},b';1,1;\mathbf{\Theta}) \leq 0 \end{array} \right. \\
&= \quad S \times Tf(z,\frac{k}{S},\frac{b}{S};1,1;\mathbf{\Theta})
\end{aligned}$$

while we note that by assumption the survival set $\mathcal{Z}$ is scale invariant too, so rescaling by S does not affect the calculation of the conditional expectation term. Since $\beta < 1$, the contraction mapping theorem applies. Since $\mathcal{F}$ is a compact space, the value function V also belongs to $\mathcal{F}$:

$$V(z,k,b;w,A;\mathbf{\Theta}) = S \times V(z,\frac{k}{S},\frac{b}{S};1,1;\mathbf{\Theta}).$$

Given this property, the Bellman problem of the firm from Equation 3 rewrites:

$$V(z,\frac{k}{S},\frac{b}{S};1,1;\mathbf{\Theta}) = \left| \begin{array}{l} \max\limits_{\frac{k'}{S},\frac{b'}{S}} e(z,\frac{\boldsymbol{x}}{S};1,1;\mathbf{\Theta}) + \beta \mathbb{E}_{z'}[V(z',\frac{k'}{S},\frac{b'}{S};1,1;\mathbf{\Theta})|z] \\ \boldsymbol{M}(z,\frac{\boldsymbol{x}}{S};1,1;\mathbf{\Theta}) \leq 0 \end{array} \right.$$

which shows that the optimum policy function satisfies:

$$\begin{aligned}
\tilde{k}\,(z,k,b;w,A,\mathbf{\Theta}) &= S\tilde{k}\left(z,\frac{k}{S},\frac{b}{S};1,1,\mathbf{\Theta}\right) \\
\tilde{b}\,(z,k,b;w,A,\mathbf{\Theta}) &= S\tilde{b}\left(z,\frac{k}{S},\frac{b}{S};1,1,\mathbf{\Theta}\right)
\end{aligned}$$

Let us note $\underline{z}$ the entire past history of past productivities at the firm level. Also, define $k^*(\underline{z};w,A,\mathbf{\Theta})$, the function that maps the history and macro conditions into the current capital stock. Then, given the above property for the policy functions, it is straightforward to show that the following property holds:

$$k^*(\underline{z}; w, A, \mathbf{\Theta}) = S k^*(\underline{z}; 1, \mathbf{\Theta})$$

i.e. the capital stock in an economy of size S and given a particular history is equal to S times the capital stock given the same history and an ecnomy of size 1.

Finally, we compute the MRPK given history:

$$\frac{p^* y^*}{k^*}(\underline{z}; w, A, \mathbf{\Theta}) \propto S^{1-\Phi} e^{\frac{\Phi}{\alpha} z} \left(k^*(\underline{z}; w, A, \mathbf{\Theta})\right)^{\Phi-1} = e^{\frac{\Phi}{\alpha} z} k^*(\underline{z}; 1, \mathbf{\Theta})^{\Phi-1}$$

which does not depend on macro conditions (w, A). This proves Theorem 1. QED

A.4 Proof of Proposition 1

We first show how to define aggregate TFP in the model of Section 3.4. Let $R = r + \delta$ be the user cost of capital. In an efficient economy with an aggregate capital stock K and aggregate employment L:

$$Y^*(K, L) = \left| \begin{array}{l} \max\limits_{(k_{is}),(l_{is})} \prod\limits_{s=1}^{S} \left(\left(\int_{i \in s} y_{is}^{\theta_s} di \right)^{\frac{1}{\theta_s}} \right)^{\phi_s}, \quad y_{is} = e^{z_{is}} k_{is}^{\alpha_s} l_{is}^{1-\alpha_s} \\ \sum\limits_{s=1}^{S} \left(\int_{i \in s} k_{is} di \right) \leq K \quad (\mathcal{L}_1), \qquad \sum\limits_{s=1}^{S} \left(\int_{i \in s} l_{is} di \right) \leq L \quad (\mathcal{L}_2) \end{array} \right.$$

where $\mathcal{L}_1$ and $\mathcal{L}_2$ are Lagrange multipliers. The first-order condition (FOC) in capital delivers $\alpha_s \phi_s \left(\frac{y_{is}}{Y_s}\right)^{\theta_s} Y^* = \mathcal{L}_1 k_{is}$ and in labor $(1-\alpha_s)\phi_s \left(\frac{y_{is}}{Y_s}\right)^{\theta_s} Y^* = \mathcal{L}_2 l_{is}$. Aggregate these FOCs across all firms in industry s, and then across all industries s: $\alpha^* Y^* = K\mathcal{L}_1$ and $(1-\alpha^*)Y^* = L\mathcal{L}_2$, where $\alpha^* = \sum_{s=1}^{S} \alpha_s \phi_s$. Now, combine the expression of k and l in the FOCs and the production function: $y_{is}^{\theta_s} = (Y^*)^{\frac{\theta_s}{1-\theta_s}} Y_s^{-\theta_s \frac{\theta_s}{1-\theta_s}} e^{z_i \frac{\theta_s}{1-\theta_s}} \left(\frac{\alpha}{\mathcal{L}_1}\right)^{\frac{\alpha_s \theta_s}{1-\theta_s}} \left(\frac{1-\alpha_s}{\mathcal{L}_2}\right)^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}}$. Aggregate across firms in industry s: $Y_s^{\frac{\theta_s}{1-\theta_s}} = (Y^*)^{\frac{\theta_s}{1-\theta_s}} \left(\int_{i \in s} e^{z_{is} \frac{\theta_s}{1-\theta_s}} di\right) \left(\frac{\alpha_s}{\mathcal{L}_1}\right)^{\frac{\alpha_s \theta_s}{1-\theta_s}} \left(\frac{1-\alpha_s}{\mathcal{L}_2}\right)^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}}$. Define: $\mathcal{Z}_s = \left(\int_{i \in s} e^{z_{is} \frac{\theta_s}{1-\theta_s}} di\right)^{\frac{1-\theta_s}{\theta_s}}$. The previous expression simplifies to: $Y_s = Y^* Z_s \left(\frac{\alpha_s}{\mathcal{L}_1}\right)^{\alpha_s} \left(\frac{1-\alpha_s}{\mathcal{L}_2}\right)^{(1-\alpha_s)}$. Taking the power ϕ_s and multiplying across all industries deliver: $Y^* = \left(\prod_{s=1}^{S} Z_s^{\phi_s}\right) \left(\frac{\prod_{s=1}^{S} \alpha_s^{\alpha_s \phi_s} (1-\alpha_s)^{(1-\alpha_s)\phi_s}}{(\alpha^*)^{\alpha^*} (1-\alpha^*)^{1-\alpha^*}}\right) K^{\alpha^*} L^{1-\alpha^*}$.

Define $\mathcal{Z} = \left(\prod_{s=1}^{S} Z_s^{\phi_s}\right) \left(\frac{\prod_{s=1}^{S} \alpha_s^{\alpha_s \phi_s} (1-\alpha_s)^{(1-\alpha_s)\phi_s}}{(\alpha^*)^{\alpha^*} (1-\alpha^*)^{1-\alpha^*}}\right)$. $\mathcal{Z}$ is aggregate TFP in an efficient, undistorted, economy. A natural definition for aggregate TFP in an economy with distortions producing Y with K and L is therefore:

$$\log(TFP) = \underbrace{\log(\mathcal{Z})}_{\text{efficient TFP}} + \underbrace{\log(Y) - \log(Y^*)}_{\text{output loss from distortions}} = \log(Y) - \alpha^* \log(K) - (1-\alpha^*) \log(L).$$

We now calculate the change in aggregate TFP between two economies with different distortions. Define τ_i the capital wedge faced by firm i: $\alpha_s \theta_s \frac{p_{is} y_{is}}{k_{is}} = R(1+\tau_{is})$. The first-order condition in labor is: $(1-\alpha_s)\theta p_{is} y_{is} = w l_{is}$. Combining the definition of the capital wedge and this first-order condition, we can express firm i's output: $p_i y_i = P_s^{\frac{1}{1-\theta_s}} Y_s e^{\frac{\theta_s}{1-\theta_s} z_{is}} \left(\frac{(1-\alpha_s)\theta_s}{w}\right)^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}} \left(\frac{\alpha_s \theta_s}{R(1+\tau_{is})}\right)^{\frac{\alpha_s \theta_s}{1-\theta_s}}$. Aggregate this equation across all firms in industry s: $P_s^{-\frac{\theta_s}{1-\theta_s}} = \underbrace{\left(\int_{i \in s} \frac{e^{\frac{\theta_s}{1-\theta_s} z_{is}}}{(1+\tau_{is})^{\frac{\alpha_s \theta_s}{1-\theta_s}}} di \right)}_{=I_s} \left(\frac{(1-\alpha_s)\theta_s}{w} \right)^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}} \left(\frac{\alpha_s \theta_s}{R} \right)^{\frac{\alpha_s \theta_s}{1-\theta_s}}$, so that $P_s = \left(\frac{(1-\alpha_s)\theta_s}{w} \right)^{-(1-\alpha_s)} \left(\frac{\alpha_s \theta_s}{R} \right)^{-\alpha_s} I_s^{-\frac{1-\theta_s}{\theta_s}}$. Because final good production is Cobb-Douglas, $P_s Y_s = \phi_s Y$ so that: $Y_s = \phi_s Y \left(\frac{(1-\alpha_s)\theta_s}{w} \right)^{(1-\alpha_s)} \left(\frac{\alpha_s \theta_s}{R} \right)^{\alpha_s} I_s^{\frac{1-\theta_s}{\theta_s}}$. Taking the power ϕ_s and multiplying across industries provides the equilibrium wage:

$$1 = \prod_{s=1}^{S} \phi_s^{\phi_s} \left(\frac{(1-\alpha_s)\theta_s}{w} \right)^{(1-\alpha_s)\phi_s} \left(\frac{\alpha_s \theta_s}{R} \right)^{\alpha_s \phi_s} I_s^{\frac{(1-\theta_s)\phi_s}{\theta_s}}$$

Note $w(1)$ the equilibrium wage in an economy with distortions $\left(\Theta_s^0 + d\Theta_s\right)_{s\in[1,S]}$, and $\mathcal{Z} = \mathcal{Z}_0$. Similarly, let $w(0)$ be the equilibrium wage in an economy with distortions $\left(\Theta_s^0\right)_{s\in[1,S]}$, and $\mathcal{Z} = \mathcal{Z}_0$. Let $\Delta \log(w) = \log(w(1)) - \log(w(0))$. Since α_s, θ_s and ϕ_s are assumed to be constant across the two economies, the previous equation implies:

$$(1-\alpha^*)\Delta \log(w) = \sum_{s=1}^{S} \phi_s \frac{1-\theta_s}{\theta_s} \Delta \log(I_s),$$

where $\Delta \log(I_s) = \log(I_s(1)) - \log(I_s(0))$ is the log-difference in the value of I_s across the two economies.

Now, aggregate the FOC in labor across firms in industry s: $(1-\alpha_s)\theta_s P_s Y_s = (1-\alpha_s)\theta_s \phi_s Y = w L_s$. Aggregating across industries: $\frac{Y}{L} = \frac{w}{\sum_{s=1}^{S}(1-\alpha_s)\theta_s\phi_s}$, so that $\Delta \log(\frac{Y}{L}) = \Delta \log(w) = \frac{1}{1-\alpha^*} \sum_{s=1}^{S} \phi_s \frac{1-\theta_s}{\theta_s} \Delta \log(I_s)$.

Use the definition of τ_i and the expression for $p_i y_i$ to express firm i's capital stock: $k_{is} = P_s^{\frac{1}{1-\theta_s}} Y_s e^{\frac{\theta_s}{1-\theta_s} z_{is}} \left(\frac{(1-\alpha_s)\theta_s}{w} \right)^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}} \left(\frac{\alpha_s \theta_s}{R(1+\tau_{is})} \right)^{1+\frac{\alpha_s \theta_s}{1-\theta_s s}}$. Aggregate across firms in industry s: $K_s = P_s^{\frac{1}{1-\theta_s}} Y_s \underbrace{\left(\int_{i \in s} \frac{e^{\frac{\theta_s}{1-\theta_s} z_{is}}}{(1+\tau_{is})^{1+\frac{\alpha_s \theta_s}{1-\theta_s}}} di \right)}_{=J_s} \left(\frac{1-\alpha_s}{w} \right)^{\frac{(1-\alpha_s)\theta_s}{1-\theta_s}} \left(\frac{\alpha_s \theta_s}{R} \right)^{1+\frac{\alpha_s \theta_s}{1-\theta_s}}$. Combining the expression for P_s and $P_s Y_s = \phi_s Y$, we obtain: $P_s^{\frac{1}{1-\theta_s}} Y_s = \phi_s Y \left(\frac{(1-\alpha_s)\theta_s}{w} \right)^{-(1-\alpha_s)\frac{\theta_s}{1-\theta_s}} \left(\frac{\alpha_s \theta_s}{R} \right)^{-\alpha_s \frac{\theta_s}{1-\theta_s}} I_s^{-1}$, so that: $K_s = \phi_s Y \left(\frac{\alpha_s \theta_s}{R} \right) \frac{J_s}{I_s}$, and: $\frac{K}{Y} = \sum_{s=1}^{S} \phi_s \left(\frac{\alpha_s \theta_s}{R} \right) \frac{J_s}{I_s}$. Note that $\frac{J_s(1)}{I_s(1)} = \frac{J_s(0)}{I_s(0)} . e^{\Delta \log(J_s) - \Delta \log(I_s)}$. We can write: $\Delta \log(\frac{K}{Y}) = \log \left(\frac{\sum_{s=1}^{S} \phi_s \left(\frac{\alpha_s \theta_s}{R} \right) \frac{J_s(0)}{I_s(0)} e^{\Delta \log(J_s) - \Delta \log(I_s)}}{\sum_{s=1}^{S} \phi_s \left(\frac{\alpha_s \theta_s}{R} \right) \frac{J_s(0)}{I_s(0)}} \right) = \log \left(\sum_{s=1}^{S} \frac{K_s(0)}{K(0)} e^{\Delta \log(J_s) - \Delta \log(I_s)} \right)$.

Our assumption that the experiment is small ($|\Theta_s^1 - \Theta_s^0| \ll 1$) implies that we can neglect second-order terms in $\Delta \log(J_s)$ and $\Delta \log(I_s)$ so that the previous expression can be approximated at the first-order by: $\Delta \log(\frac{K}{Y}) \approx \sum_{s=1}^{S} \kappa_s \left(\Delta \log(J_s) - \Delta \log(I_s) \right)$, where $\kappa_s = \frac{K_s(0)}{K_s}$ is the share of industry s' capital stock in total capital in economy 0.

Recall that z_{is} and $\log(1+\tau_{is})$ are joint-normally distributed: $\begin{pmatrix} \log(z_{is}) \\ \log(1+\tau_{is}) \end{pmatrix} \sim \mathcal{N} \left[\begin{pmatrix} \mu_z(s) \\ \mu(s) \end{pmatrix}, \begin{pmatrix} \sigma_z^2(s) & \sigma_{\tau z}(s) \\ \sigma_{\tau z}(s) & \sigma^2(s) \end{pmatrix} \right]$. Combining this parametric assumptions with the fact that $\mu_z(s)$ and $\sigma_z^2(s)$ is similar in the two economies, we obtain that

$\Delta\log(J_s)-\Delta\log(I_s)=\frac{1}{2}(1+\frac{\alpha_s\theta_s}{1-\theta_s})\Delta\sigma^2(s)+\frac{1-\theta_s}{\alpha_s\theta_s}\Delta\log(I_s)$.
This leads to a first-order approximation for the change in aggregate TFP across the two economies:

$$\begin{aligned}\Delta\log(TFP) &\approx \alpha^*\Delta\log(\frac{Y}{K})+(1-\alpha^*)\Delta\log(\frac{Y}{L})\\ &\approx -\frac{\alpha^*}{2}\sum_s\kappa_s(1+\frac{\alpha_s\theta_s}{1-\theta_s})\Delta\sigma^2(s)+\sum_{s=1}^{S}\left(\phi_s-\frac{\alpha^*}{\alpha_s}\kappa_s\right)\frac{1-\theta_s}{\theta_s}\Delta\log(I_s)\end{aligned}$$

We know that $\Delta\log(I_s)=-\frac{\alpha_s\theta_s}{1-\theta_s}\Delta\mu(s)+\frac{1}{2}\left(\left(\frac{\alpha_s\theta_s}{1-\theta_s}\right)^2\Delta\sigma^2(s)-2\alpha_s\left(\frac{\theta_s}{1-\theta_s}\right)^2\Delta\sigma_{\tau,z}(s)\right)$. Additionally, we can relate $\sigma_{\tau,z}(s)$ to the covariance of $\log(p_iy_i)$ and $\log(\frac{p_iy_i}{k_i})$, $\sigma_{lMRPK,lpy}(s)$, which we can observe in the data: $\Delta\sigma_{lMRPK,lpy}(s)=\frac{\theta_s}{1-\theta_s}\Delta\sigma_{\tau,z}-\frac{\alpha_s\theta_s}{1-\theta_s}\Delta\sigma^2(s)$. Combining these two insights:

$$\frac{1-\theta_s}{\theta_s}\Delta\log(I_s)=-\alpha_s\left(\Delta\mu(s)+\Delta\sigma_{lMRPK,lpy}(s)+\frac{1}{2}\frac{\alpha_s\theta_s}{1-\theta_s}\Delta\sigma^2(s)\right),$$

which leads to our formula for the change in aggregate TFP between economy 0 and economy 1:

$$\Delta\log(TFP)\approx-\frac{\alpha^*}{2}\sum_s\kappa_s(1+\frac{\alpha_s\theta_s}{1-\theta_s})\Delta\sigma^2(s)-\sum_{s=1}^{S}(\alpha_s\phi_s-\alpha^*\kappa_s)\left(\Delta\mu_(s)+\Delta\sigma_{lMRPK,lpy}(s)+\frac{1}{2}\frac{\alpha_s\theta_s}{1-\theta_s}\Delta\sigma^2(s)\right) \tag{13}$$

We now move on to estimating the previous equation using the (quasi-) experimental data. We are interested in a counterfactual where the only change in the economy is the change in structural parameters due to the policy reform: $d\Theta_s$. In particular, we wish to estimate the change in aggregate TFP resulting from this policy reform in the absence of an aggregate change in productivity (i.e. $\mathcal{Z}$ remains at $\mathcal{Z}_0$) and in the absence of the aggregate change in structural parameters $d\Theta$.

Let $\sigma^2(\Theta_s;(\Theta_{s'})_{s'},\mathcal{Z})$ be the variance of log-MRPK for firms in industry s facing structural parameters Θ_s when the undistorted aggregate productivity is $\mathcal{Z}$ and other industries face structural parameters $(\Theta_{s'})_{s'\in[1,S]}$. Similarly, $\mu(\Theta_s;(\Theta_s)_{s'},\mathcal{Z})$ and $\sigma_{MRPK,py}(\Theta_s;(\Theta_{s'})_{s'},\mathcal{Z})$ represent the mean log-MRPK and the covariance of log-MRPK and log output for firms in industry s facing structural parameters Θ_s when aggregate productivity is $\mathcal{Z}$. We can use these notations to express the sufficient statistics in Equation 13:

- $\Delta\sigma^2(s)=\sigma^2(\Theta_s^0+d\Theta_s;(\Theta_{s'}^0+d\Theta_{s'})_{s'},\mathcal{Z}_0)-\sigma^2(\Theta_s^0;(\Theta_{s'}^0)_{s'},\mathcal{Z}_0)$. This represents the change in the variance of log-MPRK in industry s between the initial economy $((\Theta_{s'}^0)_{s'\in[1,S]}$, $\mathcal{Z}=\mathcal{Z}_0$ and a counterfactual economy where the policy change is the only change, $((\Theta_{s'}^1)_{s'\in[1,S]}=(\Theta_{s'}^0)_{s'\in[1,S]}+(d\Theta_{s'})_{s'\in[1,S]}$ and $\mathcal{Z}=\mathcal{Z}_0$, i.e. there is no aggregate shock to z_i or to the structural parameters Θ.
- $\Delta\mu(s)=\mu(\Theta_s^0+d\Theta_s;(\Theta_{s'}^0+d\Theta_{s'})_{s'},\mathcal{Z}_0)-\mu(\Theta_s^0;(\Theta_{s'}^0)_{s'},\mathcal{Z}_0)$. This is a similar counterfactual change in the mean log-MPRK.
- $\Delta\sigma_{lMRPK,lpy}(s)=\sigma_{lMRPK,lpy}(\Theta_s^0+d\Theta_s;(\Theta_{s'}^0+d\Theta_{s'})_{s'},\mathcal{Z}_0)-\sigma_{MRPK,lpy}(\Theta_s^0;(\Theta_{s'}^0)_{s'},\mathcal{Z}_0)$. This is a similar counterfactual change in the covariance of log-MRPK and log output.

As explained in Section 2, the challenge in estimating Equation 13 is that the experimental data does not allow us to directly observe the sufficient statistics introduced above. For instance, in the case of the variance of log-MRPK:

- the counterfactual requires: $\sigma^2(\Theta_s^0+d\Theta_s;(\Theta_{s'}^0+d\Theta_{s'})_{s'},\mathcal{Z}_0)$

- but, in the data, we observe: $\sigma^2(\underbrace{\Theta_s^0 + \boldsymbol{d\Theta} + d\Theta_s}_{=\Theta_s^1}; \left(\underbrace{\Theta_{s'}^0 + \boldsymbol{d\Theta} + d\Theta_{s'}}_{=\Theta_{s'}^1}\right)_{s'}, \boldsymbol{\mathcal{Z}_1})$

However, thanks to Theorem 1, we know that for a group of firms facing the same set of frictions, the distribution of MRPK depends solely on the frictions faced by this group of firms, and not on the macroeconomic conditions in the economy. As a result:

$$\begin{aligned}\sigma^2(\Theta_s^1; (\Theta_{s'}^1)_{s'}, \mathcal{Z}_1) &= \sigma^2(\Theta_s^1; \left(\Theta_{s'}^0 + d\Theta_{s'}\right)_{s'}, \mathcal{Z}_0) \qquad \text{(Theorem 1)} \\ &\approx \sigma^2(\Theta_s^0 + d\Theta_s; \left(\Theta_{s'}^0 + d\Theta_{s'}\right)_{s'}, \mathcal{Z}_0) + (d\Theta)' \frac{\partial \sigma^2}{\partial \Theta}(\Theta_s^0; (\Theta_{s'}^0)_{s'}, \mathcal{Z}_0),\end{aligned}$$

where the approximation exploits the assumption that the aggregate shock $d\Theta$ and the policy shock $d\Theta_s$ are small.

Note $\widehat{\Delta\sigma^2(s)}$ the pre-post difference in the observed variance of log-MRPK in the experimental data. The previous equation implies that:

$$\begin{aligned}\widehat{\Delta\sigma^2(s)} &= \sigma^2(\Theta_s^1; \left(\Theta_{s'}^1\right)_{s'}, \mathcal{Z}_1) - \sigma^2(\Theta_s^0; \left(\Theta_{s'}^0\right)_{s'}, \mathcal{Z}_0) \\ &\approx \Delta\sigma^2(s) + (d\Theta)' \frac{\partial \sigma^2}{\partial \Theta}(\Theta_s^0; (\Theta_{s'}^0)_{s'}, \mathcal{Z}_0)\end{aligned}$$

This empirical difference does not provide a valid estimate of the relevant counterfactual change $\Delta\sigma^2(s)$. This is because the variance of log-MRPK in industry s does not only change because of the experiment: the concomitant aggregate shock $d\Theta$ also leads to a change in the variance of log-MRPK for firms in industry s.

We consider two different ways to estimate $\Delta\sigma^2(s)$.

Difference-in-Difference First, assume there is an industry, industry 1, which is not affected by the policy change: $\Delta\Theta_1 = 0$. Because this control industry is not exposed to a change in policy, the variance of log-MRPK of firms in this industry should not change in a counterfactual without any aggregate shock: $\Delta\sigma^2(1) = 0$. In other words, the observed change in the variance of log-MRPK for firms in this industry simply reflects the aggregate shock to structural parameters:

$$\widehat{\Delta\sigma^2}(1) \approx (d\Theta)' \frac{\partial \sigma^2}{\partial \Theta}(\Theta_0^0; (\Theta_{s'}^0)_{s'}, \mathcal{Z}_0) \approx (d\Theta)' \frac{\partial \sigma^2}{\partial \Theta}(\Theta_s^0; (\Theta_{s'}^0)_{s'}, \mathcal{Z}_0),$$

where the last approximation exploits the limited ex ante heterogeneity in frictions ($|\Theta_s - \Theta_s'| \ll 1$). As a result, the counterfactual change in the variance of log-MRPK in industry s, $\Delta\sigma^2(s)$ corresponds to the standard difference-in-difference estimate:

$$\Delta\sigma^2(s) \approx \widehat{\Delta\sigma^2}(s) - \widehat{\Delta\sigma^2}(1) = \widehat{\Delta\Delta\sigma^2}(s)$$

Using a similar logic, the counterfactual changes in mean log-MRPK or covariance of log-MRPK and log output are approximated by their difference-in-difference estimates.

Heterogeneous treatment intensity Assume now industries have heterogeneous exposures to an aggregate policy shock $d\Theta^{\text{reform}}$ so that:

$$\Theta_s^1 = \Theta_s^0 + \lambda_s d\Theta^{\text{reform}} + d\Theta + \eta_s, \text{ with: } \eta_s \perp \lambda_s$$

We can express the counterfactual sufficient statistics, $\Delta\sigma^2(s)$ as:

$$\begin{aligned}
\Delta\sigma^2(s) &= \sigma^2(\Theta_s^0 + \lambda_s d\Theta^{\text{reform}}; \left(\Theta_{s'}^0 + d\Theta_{s'}\right)_{s'}, \mathcal{Z}_0) - \sigma^2(\Theta_s^0; \left(\Theta_{s'}^0\right)_{s'}, \mathcal{Z}_0) \\
&\approx \lambda_s d\Theta^{\text{reform}} \frac{\partial\sigma^2}{\partial\Theta}(\Theta_s^0, \left(\Theta_{s'}^0\right)_{s'}, \mathcal{Z}_0) \\
&\approx \lambda_s d\Theta^{\text{reform}} \frac{\partial\sigma^2}{\partial\Theta}(\Theta_1^0, \left(\Theta_{s'}^0\right)_{s'}, \mathcal{Z}_0),
\end{aligned}$$

where the last approximation exploits the limited ex ante heterogeneity in friction to take the derivative in an arbitrary industry, e.g., industry 1. Using a similar logic, we can approximate the empirical change in the variance of log-MRPK in industry s:

$$\begin{aligned}
\widehat{\Delta\sigma^2(s)} &= \sigma^2(\Theta_s^1; (\Theta_{s'}^1)_{s'}, \mathcal{Z}_1) - \sigma^2(\Theta_s^0; \left(\Theta_{s'}^0\right)_{s'}, \mathcal{Z}_0) \\
&\approx \underbrace{\lambda_s \times d\Theta^{\text{reform}} \frac{\partial\sigma^2}{\partial\Theta}(\Theta_1^0; (\Theta_{s'}^0)_{s'}, \mathcal{Z}_0)}_{=\Delta\sigma^2(s)} + (d\Theta + \eta_s)' \frac{\partial\sigma^2}{\partial\Theta}(\Theta_1^0; (\Theta_{s'}^0)_{s'}, \mathcal{Z}_0)
\end{aligned}$$

Since $\eta_s \perp \lambda_s$, we can estimate $\Delta\sigma^2(s)$ by regressing the observed variance of log-MRPK across industries, $\widehat{\sigma^2}(s)$, against the exposure measure λ_s and a post-experiment dummy. Let $\widehat{\beta}$ be the coefficient estimates. $\widehat{\Delta\Delta\sigma^2}(s) = \lambda_s \widehat{\beta}$ provides an unbiased estimate of $\Delta\sigma^2(s)$ A similar logic applies to the estimation of $\Delta\mu(s)$ and $\Delta\sigma_{lMRPK,lpy}(s)$.

A.5 Proof of Proposition 2

We are looking for the change in aggregate TFP between:

- economy 0: all firms in industry s have structural parameters (Θ_s^0), and $\mathcal{Z} = \mathcal{Z}_0$.
- economy 1: all firms in industry s receive the treatment $(\Theta_s^0 + d\Theta_s)$ and $\mathcal{Z} = \mathcal{Z}_1$.

However, in the data, only a share ν_s of firms receive the treatment. In addition, in the post-experiment data, $\mathcal{Z} = \mathcal{Z}_1$ and there may be an additional aggregate shock to $d\Theta$ so that $\Theta_s^1 = \Theta_s^0 + d\Theta_s + d\Theta$

We know from Equation 13 in Section A.4 that the change in aggregate TFP between two economies is simply given by:

$$\Delta \log(TFP) \approx -\frac{\alpha^*}{2} \sum_s \kappa_s (1 + \frac{\alpha_s\theta_s}{1-\theta_s}) \Delta\sigma^2(s) - \sum_{s=1}^{S} (\alpha_s\phi_s - \alpha^*\kappa_s) \left(\Delta\mu_(s) + \Delta\sigma_{lMRPK,lpy}(s) + \frac{1}{2}\frac{\alpha_s\theta_s}{1-\theta_s}\Delta\sigma^2(s) \right)$$

Using the notations from Section A.4, $\Delta\sigma^2(s)$ in this counterfactual corresponds to:

$$\Delta\sigma^2(s) = \sigma^2(\Theta_s^0 + d\Theta_s; \left(\Theta_{s'}^0 + d\Theta_{s'}\right)_{s'}, \mathcal{Z}_0) - \sigma^2(\Theta_s^0; \left(\Theta_{s'}^0\right)_{s'}, \mathcal{Z}_0)$$

Let us expand the notations from Section 2. Let $\sigma_T^2 \left(\begin{pmatrix} \Theta_s(C) \\ \Theta_s(T) \end{pmatrix}, \begin{pmatrix} \Theta_{s'}(C) \\ \Theta_{s'}(T) \end{pmatrix}_{s'}, \mathcal{Z} \right)$ be the variance of treated firms in industry s when: (1) in industry s, treated firms operate with $\Theta_s(T)$ and control firms with $\Theta_s(C)$ (2) in all other industry s', treated firms operate with $\Theta_{s'}(T)$ and control firms with $\Theta_{s'}(C)$ (3) $\mathcal{Z}$. Write $\widehat{\Delta\Delta\sigma^2}(s)$ the empirical difference-in-difference estimate in industry s:

$$\begin{aligned}\widehat{\Delta\Delta\sigma^2}(s) &= \left(\sigma_T^2\left(\begin{pmatrix}\Theta_s^0+d\Theta\\ \Theta_s^0+d\Theta_s+d\Theta\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0+d\Theta\\ \Theta_{s'}^1+d\Theta+d\Theta_{s'}\end{pmatrix}_{s'},\mathcal{Z}_1\right)-\sigma_T^2\left(\begin{pmatrix}\Theta_s^0\\ \Theta_s^0\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0\\ \Theta_{s'}^0\end{pmatrix}_{s'},\mathcal{Z}_0\right)\right)\\ &\quad-\left(\sigma_C^2\left(\begin{pmatrix}\Theta_s^0+d\Theta\\ \Theta_s^0+d\Theta_s+d\Theta\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0+d\Theta\\ \Theta_{s'}^1+d\Theta+d\Theta_{s'}\end{pmatrix}_{s'},\mathcal{Z}_1\right)-\sigma_C^2\left(\begin{pmatrix}\Theta_s^0\\ \Theta_s^0\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0\\ \Theta_{s'}^0\end{pmatrix}_{s'},\mathcal{Z}_0\right)\right)\end{aligned}$$

Define the empirical difference on the treated firms, $\widehat{\Delta\sigma^2}(T)$:

$$\begin{aligned}\widehat{\Delta\sigma^2}(T) &= \sigma_T^2\left(\begin{pmatrix}\Theta_s^0+d\Theta\\ \Theta_s^0+d\Theta+d\Theta_s\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0+d\Theta\\ \Theta_{s'}^0+d\Theta+d\Theta_s\end{pmatrix}_{s'},\mathcal{Z}_1\right)-\sigma_T^2\left(\begin{pmatrix}\Theta_s^0\\ \Theta_s^0\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0\\ \Theta_{s'}^0\end{pmatrix}_{s'},\mathcal{Z}_0\right)\\ &= \sigma_T^2\left(\begin{pmatrix}\Theta_s^0+d\Theta\\ \Theta_s^0+d\Theta+d\Theta_s\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0+d\Theta\\ \Theta_{s'}^0+d\Theta+d\Theta_s\end{pmatrix}_{s'},\boldsymbol{\mathcal{Z}_0}\right)-\sigma_T^2\left(\begin{pmatrix}\Theta_s^0\\ \Theta_s^0\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0\\ \Theta_{s'}^0\end{pmatrix}_{s'},\mathcal{Z}_0\right)\quad\text{(Theorem 1)}\\ &= \sigma_T^2\left(\begin{pmatrix}\Theta_s^0+d\Theta\\ \Theta_s^0+d\Theta_s+d\Theta\end{pmatrix},\begin{pmatrix}\boldsymbol{\Theta_{s'}^0+d\Theta_{s'}}\\ \boldsymbol{\Theta_{s'}+d\Theta_{s'}}\end{pmatrix}_{s'},\mathcal{Z}_0\right)-\sigma_T^2\left(\begin{pmatrix}\Theta_s^0\\ \Theta_s^0\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0\\ \Theta_{s'}^0\end{pmatrix}_{s'},\mathcal{Z}_0\right)\quad\text{(Theorem 1)}\\ &= \sigma_T^2\left(\begin{pmatrix}\Theta_s^0+\boldsymbol{d\Theta_s}\\ \Theta_s^0+d\Theta_s+d\Theta\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0+d\Theta_{s'}\\ \Theta_{s'}+d\Theta_{s'}\end{pmatrix}_{s'},\mathcal{Z}_0\right)-\sigma_T^2\left(\begin{pmatrix}\Theta_s^0\\ \Theta_s^0\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0\\ \Theta_{s'}^0\end{pmatrix}_{s'},\mathcal{Z}_0\right)\quad\text{(Theorem 1)}\\ &\approx \sigma_T^2\left(\begin{pmatrix}\Theta_s^0+d\Theta_s\\ \Theta_s^0+d\Theta_s\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0+d\Theta_{s'}\\ \Theta_{s'}+d\Theta_{s'}\end{pmatrix}_{s'},\mathcal{Z}_0\right)-\sigma_T^2\left(\begin{pmatrix}\Theta_s^0\\ \Theta_s^0\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0\\ \Theta_{s'}^0\end{pmatrix}_{s'},\mathcal{Z}_0\right)+(d\Theta)'\frac{\partial\sigma_T^2}{\partial\Theta}\left(\begin{pmatrix}\Theta_s^0+d\Theta_s\\ \Theta_s^0+d\Theta_s\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0+d\Theta_{s'}\\ \Theta_{s'}+d\Theta_{s'}\end{pmatrix}_{s'},\mathcal{Z}_0\right)\\ &\approx \underbrace{\sigma^2(\Theta_s^0+d\Theta_s;\left(\Theta_{s'}^0+d\Theta_{s'}\right)_{s'},\mathcal{Z}_0)-\sigma^2(\Theta_s^0;\left(\Theta_{s'}^0\right)_{s'},\mathcal{Z}_0)}_{=\Delta\sigma^2(s)}+(d\Theta)'\frac{\partial\sigma^2}{\partial\Theta}\left(\Theta_s^0+d\Theta_s;\left(\Theta_{s'}^0+d\Theta_{s'}\right)_{s'},\mathcal{Z}_0\right)\\ &\approx \Delta\sigma^2(s)+(d\Theta)'\frac{\partial\sigma^2}{\partial\Theta}\left(\Theta_s^0;\left(\Theta_{s'}^0\right)_{s'},\mathcal{Z}_0\right)\end{aligned}$$

This calculation makes extensive use of Theorem 1. To go from line 1 to 2, we know the variance of log-MRPKs for a group of firms is independent of $\mathcal{Z}$. To go from line 2 to 3, we know that the variance of log-MRPKs for a group of firms does not depend on equilibrium prices, and therefore is independent of frictions in other industries. To go from line 3 to 4, we similarly use the fact that the variance of log-MRPKs of firms in the treatment group is independent of the frictions faced by firms in the control group. Line 5 is a first-order approximation. Line 6 simply recognizes that: $\sigma_T^2\left(\begin{pmatrix}\Theta_s\\ \Theta_s\end{pmatrix},\begin{pmatrix}\Theta_{s'}\\ \Theta_{s'}\end{pmatrix}_{s'},\mathcal{Z}_0\right)=\sigma^2(\Theta_s;(\Theta_{s'})_{s'},\mathcal{Z}_0)$. Line 7 uses the fact that $d\Theta_s\ll 1$

Using a similar logic, the empirical difference on the control firms, $\widehat{\Delta\sigma^2}(C)$:

$$\begin{aligned}\widehat{\Delta\sigma^2}(C) &= \sigma_C^2\left(\begin{pmatrix}\Theta_s^0+d\Theta\\ \Theta_s^0+d\Theta+d\Theta_s\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0+d\Theta\\ \Theta_{s'}^0+d\Theta+d\Theta_s\end{pmatrix}_{s'},\mathcal{Z}_1\right)-\sigma_C^2\left(\begin{pmatrix}\Theta_s^0\\ \Theta_s^0\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0\\ \Theta_{s'}^0\end{pmatrix}_{s'},\mathcal{Z}_0\right)\\ &= \sigma_C^2\left(\begin{pmatrix}\Theta_s^0+d\Theta\\ \boldsymbol{\Theta_s^0}\end{pmatrix},\begin{pmatrix}\boldsymbol{\Theta_{s'}^0}\\ \boldsymbol{\Theta_{s'}^0}\end{pmatrix}_{s'},\boldsymbol{\mathcal{Z}_0}\right)-\sigma_C^2\left(\begin{pmatrix}\Theta_s^0\\ \Theta_s^0\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0\\ \Theta_{s'}^0\end{pmatrix}_{s'},\mathcal{Z}_0\right)\quad\text{(Theorem 1)}\\ &\approx \underbrace{\sigma_C^2\left(\begin{pmatrix}\Theta_s^0\\ \Theta_s^0\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0\\ \Theta_{s'}\end{pmatrix}_{s'},\mathcal{Z}_0\right)-\sigma_C^2\left(\begin{pmatrix}\Theta_s^0\\ \Theta_s^0\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0\\ \Theta_{s'}^0\end{pmatrix}_{s'},\mathcal{Z}_0\right)}_{=0}+(d\Theta)'\frac{\partial\sigma_C^2}{\partial\Theta}\left(\begin{pmatrix}\Theta_s^0\\ \Theta_s^0\end{pmatrix},\begin{pmatrix}\Theta_{s'}^0\\ \Theta_{s'}\end{pmatrix}_{s'},\mathcal{Z}_0\right)\\ &\approx (d\Theta)'\frac{\partial\sigma^2}{\partial\Theta}\left(\Theta_s^0;\left(\Theta_{s'}^0\right)_{s'},\mathcal{Z}_0\right)\end{aligned}$$

Therefore, to a first-order approximation: $\widehat{\Delta\Delta\sigma^2} = \widehat{\Delta\sigma^2}(T) - \widehat{\Delta\sigma^2}(C) \approx \Delta\sigma^2(s)$. A similar logic applies for $\Delta\mu(s)$ and $\Delta\sigma_{lMRPK,lpy}$, which achieves the proof of Proposition 2.

A.6 Proof of Formula 11

From the FOC in labor, we know that: $(1-\alpha_s)\theta_s p_{is} y_{is} = w l_{is}$, which aggregates into $\left(\sum_{s=1}^{S} \phi_s \theta_s (1-\alpha_s)\right) Y = wL = \frac{\bar{L}}{\bar{w}} w^{1+\epsilon}$. As a result, the steady-state difference in log-output between two economies is simply:

$$\Delta \log(Y) = (1+\epsilon)\Delta \log(w)$$

We have shown in Appendix A.4 that: $(1-\alpha^*)\Delta\log(w) = \sum_{s=1}^{S} \phi_s \frac{1-\theta_s}{\theta_s} \Delta\log(I_s)$ and $\frac{1-\theta_s}{\theta_s}\Delta\log(I_s) = -\alpha_s \left(\Delta\mu(s) + \Delta\sigma_{lMRPK,lpy}(s) + \frac{1}{2}\frac{\alpha_s\theta_s}{1-\theta_s}\Delta\sigma^2(s)\right)$. Combining these two equations:

$$\Delta \log(Y) = -(1+\epsilon)\sum_{s=1}^{S} \frac{\alpha_s \phi_s}{1-\alpha^*}\left(\Delta\mu(s) + \Delta\sigma_{lMRPK,lpy}(s) + \frac{1}{2}\frac{\alpha_s\theta_s}{1-\theta_s}\Delta\sigma^2(s)\right)$$

A.7 Input-Output Linkages

This Appendix describes a model with roundabout productions and input-output linkages. Firms in each industry consume inputs produced by all other industries. As in our baseline model, industries are heterogeneous in terms of price elasticity (θ_s), capital shares (α_s) and treatment effects (Θ_s). We still assume Cobb-Douglas aggregation across industries. This assumption efffectively shuts down inter-industry reallocation of output. We explore a more general industry aggregator in Appendix A.8.

There are S industries indexed by s. Final good production combines intermediate goods through a Cobb-Douglas technology:

$$Y = \Pi_{s=1}^{S} Y_s^{\phi_s} \text{ and } \sum_{s=1}^{S} \phi_s = 1$$

where Y_s is the quantity of industry good s used for final good production.

There is now a second layer of intermediate production in the economy. Intermediate *good* (s) is produced by combining the intermediate *inputs* produced by firms in industry s, $q_{i,s}$, using a CES technology:

$$Q_s = \left(\int (q_{is})^{\theta_s} di\right)^{1/\theta_s}$$

where Q_s is the quantity of intermediate *good* s produced. However, only a fraction of this production of intermediate good s is used for producing final good: Y_s. The rest, $M_s = Q_s - Ys$, is used in the production process of intermediate input producers in all sectors $q_{i',s'}$, as described below.

Input-output linkages are modeled the following way. Production of *input* (i,s) combines capital, labor and all intermediate *goods* $s \in [1,S]$:

$$q_{i,s} = e^{z_i} k_i^{\alpha_s} l_i^{\beta_s} \Pi_{u=1}^{S} (m_{i,s,u})^{\gamma_{su}}$$

where we assume for convenience constant returns to scale, i.e. $\alpha_s + \beta_s + \sum_{u=1}^{S} \gamma_{su} = 1$. $m_{i,s,u}$ corresponds to the quantity of intermediate *good* produced by sector u that is used for production by firm i in sector

s. $\Gamma = (\gamma_{su})_{(s,u)\in[1,S]^2}$ corresponds to the input-output matrix. There is one single labor market for all industries, and labor supply elasticity is constant with elasticity ϵ.

Finally, as in the rest of the paper, we assume that the within-industry distribution of log MRPKs and log-productivity z_i is normal.

We consider here an ex post evaluation exercise. The scale-up exercise follows a similar logic. In the pre-experiment economy, distortions are $(\Theta_s^0)_{s\in[1,S]}$, and $\mathcal{Z} = \mathcal{Z}_0$ – this is the initial economy, economy 0. In economy 1, distortions are $(\Theta_s^0 + d\Theta_s)_{s\in[1,S]}$, and $\mathcal{Z} = \mathcal{Z}_0$ – this is our counterfactual economy as, in the actual post-experiment economy, $\mathcal{Z} = \mathcal{Z}_1$ and $\Theta_s^1 = \Theta_s^0 + d\Theta_s + d\Theta$.

The following proposition shows how to evaluate the effect of this policy change on equilibrium allocation.

Proposition 3 (Formulas with Input-output Linkages)**.** *Denote ϕ_s^* the linkage-adjusted industry share, the s^{th} element of the vector defined by $(I-\Gamma)^{-1}\boldsymbol{\phi}$, with $\boldsymbol{\phi}$ being the vector of input shares in final production, and Γ the input-output matrix (so $(I-\Gamma)^{-1}$ is the Leontieff inverse). $\alpha^* = \sum_s \alpha_s \phi_s^*$ is the linkage-adjusted capital share. Define aggregate TFP in this economy as $TFP = \frac{Y}{K^{\alpha^*}L^{1-\alpha^*}}$.*

Then, the difference in outputs and TFPs between economy 1 and economy 0 is:

$$\begin{cases} \Delta \log Y = -(1+\epsilon)\sum_{s=1}^{S}\frac{\alpha_s\phi_s^*}{1-\alpha^*}\left(\widehat{\Delta\Delta\mu}(s) + \frac{1}{2}\frac{\alpha_s\theta_s}{1-\theta_s}\widehat{\Delta\Delta\sigma^2}(s) + \Delta\Delta\widehat{\sigma_{lMRPK,lpy}}(s)\right) \\ \Delta \log TFP = -\frac{\alpha^*}{2}\sum_s \kappa_s\left(1+\frac{\alpha_s\theta_s}{1-\theta_s}\right)\Delta\sigma^2(s) - \sum_{s=1}^{S}(\alpha_s\phi_s^* - \kappa_s\alpha^*)\left(\widehat{\Delta\Delta\mu}(s) + \frac{1}{2}\frac{\alpha_s\theta_s}{1-\theta_s}\widehat{\Delta\Delta\sigma^2}(s) + \Delta\Delta\widehat{\sigma_{lMRPK,lpy}}(s)\right) \end{cases}$$

where κ_s is the capital share of sector s in the initial economy and the statistics $\widehat{\Delta\Delta\sigma^2}(s)$, $\widehat{\Delta\Delta\mu}(s)$ and $\Delta\Delta\widehat{\sigma_{lMRPK,lpy}}(s)$ are similar to the ones introduced in Section 3.4.1.

Proof. Available in the Online Appendix. □

A.8 Heterogeneous Mark-ups and Reallocation

In this section, we allow for output reallocation across sectors. We depart from the model in Section A.7 by assuming that industries are aggregated via a CES final goods production function. One limitation of this aggregation model is that aggregate TFP is well-defined only when the undistorted capital share is constant across industries, so we assume that for all industry s, $\alpha_s = \alpha$.

Industry output Y_s is produced by combining intermediate inputs y_{is} with a CES technology:

$$Y_s = \left(\int_{i\in s} y_{is}^{\theta_s} di\right)^{\frac{1}{\theta_s}}$$

The elasticity of substitution θ_s is industry-specific and implies industry-specific markups. Production is: $y_{is} = e^{z_{is}}k_{is}^{\alpha}l_{is}^{1-\alpha}$

The final good Y is produced by combining industry output Y_s using a CES technology:

$$Y = \left(\sum_{s=1}^{S}\phi_s Y_s^{\psi}\right)^{\frac{1}{\psi}}, \text{ with } \sum_{s=1}^{S}\chi_s = 1$$

$\psi = 0$ corresponds to Cobb-Douglas aggregation. When $\psi \neq 0$, however, intermediate input shares are no longer constant and heterogeneous markups across industries create additional misallocation.

As in Section A.7, we consider here only the case of an ex post evaluation exercise – the logic for a scale up exercise is similar. We consider two economies: in economy 0, distortions are $\left(\Theta_s^0\right)_{s\in[1,S]}$, and $\mathcal{Z} = \mathcal{Z}_0$ – this is the initial economy; in economy 1, distortions are $\left(\Theta_s^0 + d\Theta_s\right)_{s\in[1,S]}$, and $\mathcal{Z} = \mathcal{Z}_0$ – this is our counterfactual economy as, in the actual post-experiment economy, $\mathcal{Z} = \mathcal{Z}_1$ and $\Theta_s^1 = \Theta_s^0 + d\Theta_s + d\Theta$.

The following proposition shows how to calculate the change in aggregate TFP and aggregate output between these two economies using only simple statistics from the experimental data.

Proposition 4. *Define* $\omega_s = \left(\frac{\gamma_s}{1-\psi} - \kappa_s(1 + \frac{\alpha\psi}{1-\psi}) - \frac{(1-\alpha)\psi}{1-\psi}\chi_s\right)$, *where* γ_s *is the output share in the initial economy,* κ_s *the capital share and* χ_s *the labor share. Define aggregate TFP in this economy as* $TFP = \frac{Y}{K^\alpha L^{1-\alpha}}$.

Then, the change in outputs and TFPs due to the reform is:

$$\begin{cases} \Delta \log Y = -\sum_{s=1}^{S} \left(\frac{\alpha\gamma_s}{1-\alpha}(1+\epsilon) + \frac{\psi}{1-\psi}\left(\gamma_s - \chi_s\right)\alpha\right)\left(\widehat{\Delta\Delta\mu}(s) + \frac{1}{2}\frac{\alpha\theta_s}{1-\theta_s}\widehat{\Delta\Delta\sigma^2}(s) + \Delta\Delta\widehat{\sigma_{IMRPK,lpy}}(s)\right) \\ \Delta \log(TFP) = -\frac{\alpha}{2}\sum_{s=1}^{S}\kappa_s(1+\frac{\alpha\theta_s}{1-\theta_s})\widehat{\Delta\Delta\sigma^2}(s) - \alpha\sum_{s=1}^{S}\omega_s\left(\widehat{\Delta\Delta\mu}(s) + \frac{1}{2}\frac{\alpha\theta_s}{1-\theta_s}\widehat{\Delta\Delta\sigma^2}(s) + \Delta\Delta\widehat{\sigma_{IMPRK,lpy}}(s)\right) \end{cases}$$

where the statistics $\widehat{\Delta\Delta\sigma^2}(s)$, $\widehat{\Delta\Delta\mu}(s)$ *and* $\Delta\Delta\widehat{\sigma_{IMRPK,lpy}}(s)$ *are similar to the ones introduced in Section 3.4.1.*

Proof. Available in the Online Appendix. □

A.9 Elastic Supply of the Capital Good

This Appendix explores the case where the capital good is produced out of the final good with an elasticity smaller than 1. We show that our results (the invariance Theorem and the aggregation formula for TFP) are unchanged in this extension. Put differently, our methodology is robust to adding this effect.

A.9.1 Extension of Theorem 1 with Imperfectly Elastic Capital Supply

We consider a simplified version of the baseline model described in Section 3.1. The main difference with the baseline model is that the price of capital is now q, and can be different from 1 at the steady-state. Mostly to clarify exposition, we make additional assumptions. There is only one industry. Firms face three frictions: (1) quadratic adjustment costs to capital: $c_k\frac{(qk'-(1-\delta)qk)^2}{qk}$ (2) a collateral constraint $b' \leq \zeta qk'$ and (3) a no-equity issuance constraint $\tilde{e} \geq 0$, where $\tilde{e}$ represents the cash-flow to equity. Note that the constraints and costs are now all expressed in terms of the \$ value of the capital stock, which is an important assumption. With this assumption, it is straightforward to generalize the proof to more generic frictions as in Theorem 1.

Let $\Omega = \frac{\alpha}{\alpha+(1-\alpha)\Phi}\left(\frac{(1-\alpha)\Phi}{\alpha+(1-\alpha)\Phi}\right)^{\frac{1-\alpha}{\alpha}\Phi}$. Cash-flows to equity in this new problem are given by:

$$\begin{aligned} \tilde{e}(z,k,k',b,b';q,w,A,\Theta) &= \Omega S^{1-\Phi}e^{\frac{\Phi}{\alpha}z}k^\Phi - q(k'-(1-\delta)k) - \frac{c_k}{2}q\frac{(k'-(1-\delta)k)^2}{k} + \left(\frac{b'}{1+r} - b'\right) \\ &= \tilde{e}(z,qk,qk',b,b';1,w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}},\Theta), \end{aligned}$$

where $S = \frac{A}{w^{\frac{(1-\alpha)\Phi}{\alpha(1-\Phi)}}}$. The new Bellman equation when $q \neq 1$ becomes:

$$W(z,k,b;q,w,A,\Theta) = \begin{vmatrix} \max_{k',b'} \left\{ \tilde{e}(z,k,k',b,b';q,w,A;\Theta) + \frac{1}{1+r}\mathbb{E}[W(z,k',b';q,w,A;\Theta)] \right\} \\ b' \leq \zeta q k' \\ \tilde{e}(z,k,k',b,b';q,w,A;\Theta) \geq 0 \end{vmatrix}$$

Let T be the Bellman operator of this Bellman equation. Let $f(z,k,b;q,w,A;\Theta)$ be a function such that: $f(z,k,b;q,w,A;\Theta) = f(z,qk,b;1,w,\frac{A}{q^{\frac{\phi}{1-\phi}}});\Theta$. We show that Tf then follows the same property:

$$\begin{aligned}
Tf(z,k,b;q,w,A;\Theta) &= \begin{vmatrix} \max_{k',b'} \left\{ \tilde{e}(z,k,k',b,b';q,w,A;\Theta) + \frac{1}{1+r}\mathbb{E}[f(z,k',b';q,w,A;\Theta)] \right\} \\ b' \leq \zeta q k' \\ \tilde{e}(z,k,k',b,b';q,w,A;\Theta) \geq 0 \end{vmatrix} \\
&= \begin{vmatrix} \max_{k',b'} \left\{ \tilde{e}(z,qk,qk',b,b';1,w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}},\Theta) + \frac{1}{1+r}\mathbb{E}[f(z,qk',b';1,w,\frac{A}{q^{\frac{\phi}{1-\phi}}};\Theta)] \right\} \\ b' \leq \zeta q k' \\ \tilde{e}(z,qk,qk',b,b';1,w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}},\Theta) \geq 0 \end{vmatrix} \\
&= \begin{vmatrix} \max_{\vec{k}',b'} \left\{ \tilde{e}(z,qk,\vec{k},b,b';1,w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}};\Theta) + \frac{1}{1+r}\mathbb{E}[f(z,\vec{k}',b';1,w,\frac{A}{q^{\frac{\phi}{1-\phi}}};\Theta)] \right\} \\ b' \leq \zeta \vec{k}' \\ \tilde{e}(z,qk,\vec{k},b,b';1,w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}};\Theta) \geq 0 \end{vmatrix} \\
&= Tf(z,qk,b;1,w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}}';\Theta)
\end{aligned}$$

Thanks to the contraction mapping theorem, this proves that $W(z,k,b;q,w,A;\Theta) = W(z,qk,b;1,w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}};\Theta)$. Note that $W(z,k,b;1,w,A) = V(z,k,b;w,A)$, the value in the baseline setting, since $W(z,k,b;1,w,A)$ corresponds to the problem when $q=1$, i.e. the initial Bellman equation. As a result:

$$W(z,k,b;q,w,A) = V(z,qk,b;w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}})$$

Finally, note also that the cash-flow to equity when $q \neq 1$ can be expressed as a function of the cash-flow to equity when $q = 1$:$\tilde{e}(z,k,k',b,b';q,w,A,\Theta) = \tilde{e}(z,qk,qk',b,b';1,w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}},\Theta) = e(z,qk,qk',b,b';w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}};\Theta)$, where $e(z,k,k',b,b';w,A;\Theta) = \Omega S^{1-\Phi} e^{\frac{\Phi}{\alpha}z} k^{\Phi} - (k'-(1-\delta)k) - \frac{c_k}{2}\frac{(k'-(1-\delta)k)^2}{k} + \left(\frac{b'}{1+r} - b'\right)$ is the cash-flow to equity in our original Bellman problem $V()$ when $q=1$.

Now, let $\tilde{k}(z,k,b;q,w,A;\Theta)$ be the optimal capital stock selected by a firm with (z,k,b) when the price of capital is q, wage is w and demand shifter is A

$$\tilde{k}(z,k,b;q,w,A;\Theta) = \left| \begin{array}{l} \text{argmax}_{k'} \left\{ \tilde{e}(z,k,k',b,b';q,w,A;\Theta) + \frac{1}{1+r}\mathbb{E}[W(z,k',b';q,w,A;\Theta)] \right\} \\ b' \leq \zeta q k' \\ \tilde{e}(z,k,k',b,b';q,w,A;\Theta) \geq 0 \end{array} \right.$$

$$= \left| \begin{array}{l} \text{argmax}_{k'} \left\{ e(z,qk,qk',b,b';w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}};\Theta) + \frac{1}{1+r}\mathbb{E}[V(z,qk',b';w,\frac{A}{q^{\frac{\phi}{1-\phi}}};\Theta)] \right\} \\ b' \leq \zeta q k' \\ e(z,qk,qk',b,b';w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}};\Theta) \geq 0 \end{array} \right.$$

$$= \left| \begin{array}{l} \frac{1}{q}\text{argmax}_{k'} \left\{ e(z,qk,k',b,b';w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}};\Theta) + \frac{1}{1+r}\mathbb{E}[V(z,\vec{k}',b';w,\frac{A}{q^{\frac{\phi}{1-\phi}}})] \right\} \\ b' \leq \zeta \vec{k}' \\ e(z,qk,\vec{k}',b,b';w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}};\Theta) \geq 0 \end{array} \right.$$

$$= \frac{\tilde{k}(z,qk,b;1,w,\frac{A}{q^{\frac{\Phi}{1-\Phi}}})}{q},$$

Define $\tilde{S} = \frac{S}{q^{\frac{\Phi}{1-\Phi}}}$. Thanks to Theorem 1, we know that:

$$\tilde{k}(z,k,b;w,A,q;\Theta) = \frac{\tilde{S}}{q}k^*(z,\frac{qk}{\tilde{S}},\frac{b}{\tilde{S}};1,1,1;\Theta)$$
$$\tilde{b}(z,k,b;w,A,q;\Theta) = \tilde{S}b^*(z,\frac{qk}{\tilde{S}},\frac{b}{\tilde{S}};1,1,1;\Theta)$$

As in the proof of Theorem 1, we note $\underline{z}$ the entire history of past productivities of the firm and $k^*\tilde{k}(\underline{z};w,A,q;\Theta)$ the function that maps this history and macro conditions to the current capital stock. Then, the two policy functions above imply that the following property must hold:

$$k^*(\underline{z};w,A,q;\Theta) = \frac{S'}{q}k^*(\underline{z};1,1,1;\Theta)$$

We conclude the proof by computing the revenue per $ of capital:

$$\frac{p^*y^*}{qk^*}(\underline{z};w,A,q;\Theta) \propto e^{\frac{\Phi}{\alpha}z}\frac{S^{1-\Phi}}{q}\left(k^*(\underline{z};w,A,q;\Theta)\right)^{\Phi-1} = e^{\frac{\Phi}{\alpha}z}\left(k^*(\underline{z};1,1,1;\Theta)\right)^{\Phi-1}$$

Therefore, the distribution of MRPK when $q \neq 1$ depends again solely on the distribution of productivity histories, and, in particular, it is independent of q, w and Y. QED

A.9.2 Aggregation Formula when $q \neq 1$

We assume an elasticity of capital supply to the price of capital q of η: $K = q^{\eta}$. This arises if the capital stock is produced from the final good in a competitive sector with decreasing returns to scale $\frac{\eta}{1+\eta}$. Our baseline model therefore corresponds to $\eta \to \infty$. The definition of the capital wedge now takes into account the price of the capital good: $\alpha\theta p_i y_i = qR(1+\tau_i)k_i$. The FOC in labor remains unchanged: $(1-\alpha)\theta p_i y_i = wl_i$. Combining these two equations delivers firm i's output:$p_i y_i = Ye^{\frac{\theta}{1-\theta}z_i}\left(\frac{(1-\alpha)\theta}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}}\left(\frac{\alpha\theta}{qR(1+\tau_i)}\right)^{\frac{\alpha\theta}{1-\theta}}$ Aggregating across firms ties the price of the capital good with the price of labor:

$$1 = \left(\frac{(1-\alpha)\theta}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}}\left(\frac{\alpha\theta}{qR}\right)^{\frac{\alpha\theta}{1-\theta}} I, \quad \text{with } I = \int_i \frac{e^{\frac{\theta}{1-\theta}z_i}}{(1+\tau_i)^{\frac{\alpha\theta}{1-\theta}}}di \tag{14}$$

After input optimization, firm i's capital stock is: $k_i = Ye^{\frac{\theta}{1-\theta}z_i}\left(\frac{(1-\alpha)\theta}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}}\left(\frac{\alpha\theta}{qR(1+\tau_i)}\right)^{1+\frac{\alpha\theta}{1-\theta}}$, which, after aggregating across all firms and exploiting the previous expression, delivers aggregate capital demand:

$$K = Y\left(\frac{(1-\alpha)\theta}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}}\left(\frac{\alpha\theta}{qR}\right)^{1+\frac{\alpha\theta}{1-\theta}} J = Y\frac{\alpha\theta}{qR}\frac{J}{I}, \quad \text{with: } J = \int_i \frac{e^{\frac{\theta}{1-\theta}z_i}}{(1+\tau_i)^{1+\frac{\alpha\theta}{1-\theta}}}di$$

Aggregating firm-level labor across all firms, and introducing aggregate labor supply leads to: $(1-\alpha)\theta Y = wL \propto w^{1+\epsilon}$. Since aggregate capital supply is q^{η}, we can equate aggregate capital supply and demand to obtain:

$$q^{1+\eta} = Y\frac{\alpha\theta}{R}\frac{J}{I} = \frac{w^{1+\epsilon}}{R}\frac{\alpha}{1-\alpha}\frac{J}{I}$$

Equation 14 implies that $w \propto q^{-\frac{\alpha}{1-\alpha}} I^{\frac{1-\theta}{(1-\alpha)\theta}}$, so that:

$$q^{1+\eta+\frac{\alpha}{1-\alpha}(1+\epsilon)} \propto J \times I^{\frac{1-\theta}{(1-\alpha)\theta}(1+\epsilon)-1}$$

We are now ready to compute the change in aggregate TFP between two economies with different distortions Θ:

$$\begin{aligned}
\Delta\log(TFP) &= \alpha\Delta\log\left(\frac{Y}{K}\right) + (1-\alpha)\Delta\log\left(\frac{Y}{L}\right) \\
&= \alpha\left(\Delta\log(q) - \Delta\log(\frac{J}{I})\right) + (1-\alpha)\Delta\log(w) \\
&= \alpha\left(\Delta\log(q) - \Delta\log(\frac{J}{I})\right) + (1-\alpha)\left(-\frac{\alpha}{1-\alpha}\Delta\log(q) + \frac{1-\theta}{(1-\alpha)\theta}\Delta\log(I)\right) \\
&= \frac{1-\theta}{\theta}\Delta\log(I) - \alpha\Delta\log\left(\frac{J}{I}\right) \\
&= -\frac{\alpha}{2}(1+\frac{\alpha\theta}{1-\theta})\Delta\sigma^2,
\end{aligned}$$

where we use the parametric assumption on $(z, \log(1+\tau))$. Importantly, this formula for the change in aggregate TFP between two economies with different distortions is similar to the formula when $q = 1$, i.e. the capital stock is inelastic.

Finally, we can easily compute the change in aggregate output between these two economies. We know that $\Delta\log(Y) = (1+\epsilon)\Delta\log(w)$. Since $w \propto q^{-\frac{\alpha}{1-\alpha}} I^{\frac{1-\theta}{(1-\alpha)\theta}}$ and $q^{1+\eta+\frac{\alpha}{1-\alpha}(1+\epsilon)} \propto J \times I^{\frac{1-\theta}{(1-\alpha)\theta}(1+\epsilon)-1}$, the change in equilibrium wage between the two economies is:

$$\begin{aligned}
\Delta\log(w) &= \frac{1-\theta}{(1-\alpha)\theta}\Delta\log(I) - \frac{\alpha}{1+(1-\alpha)\eta+\alpha\epsilon}\left(\Delta\log(J) + \left((1+\epsilon)\frac{1-\theta}{(1-\alpha)\theta} - 1\right)\Delta\log(I)\right) \\
&= \frac{\alpha}{1+(1-\alpha)\eta+\alpha\epsilon}\Delta\log(\frac{I}{J}) + \frac{1-\theta}{\theta}\frac{1+\eta}{1+(1-\alpha)\eta+\alpha\epsilon}\Delta\log(I) \\
&= -\frac{\alpha}{2}(1+\frac{\alpha\theta}{1-\theta})\frac{\Delta\sigma^2}{1+(1-\alpha)\eta+\alpha\epsilon} + \frac{1-\theta}{\theta}\frac{\eta}{1+(1-\alpha)\eta+\alpha\epsilon}\Delta\log(I) \\
&= -\frac{\alpha}{2}(1+\frac{\alpha\theta}{1-\theta})\frac{\Delta\sigma^2}{1+(1-\alpha)\eta+\alpha\epsilon} - \alpha\frac{\eta}{1+(1-\alpha)\eta+\alpha\epsilon}\left(\Delta\mu + \Delta\sigma_{lMRPK,lpy} + \frac{1}{2}\frac{\alpha\theta}{1-\theta}\Delta\sigma^2\right)
\end{aligned}$$

So that:

$$\Delta\log(Y) = -(1+\epsilon)\frac{\alpha}{2}(1+\frac{\alpha\theta}{1-\theta})\frac{\Delta\sigma^2}{1+(1-\alpha)\eta+\alpha\epsilon} - (1+\epsilon)\frac{\alpha\eta}{1+(1-\alpha)\eta+\alpha\epsilon}\left(\Delta\mu + \Delta\sigma_{lMRPK,lpy} + \frac{1}{2}\frac{\alpha\theta}{1-\theta}\Delta\sigma^2\right)$$

Note that for $\eta \to \infty$ (i.e. perfectly elastic capital supply, our baseline assumption), this formula converges to our baseline formula