Asset-Price Redistribution*

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Abstract

Over the last several decades, there has been a large increase in asset valuations across many asset classes. These rising valuations had important effects on the distribution of wealth. However, little is known regarding their effect on the distribution of *welfare*. To make progress on this question, we derive a sufficient statistic for the (money metric) welfare effect of a change in asset valuations, which depends on the present value of an individual's *net asset sales*: rising asset prices benefit prospective sellers and harm prospective buyers. We estimate this quantity using panel microdata covering the universe of financial transactions in Norway from 1994 to 2015. We find that rising asset valuations had large redistributive effects: they redistributed welfare from the young towards the old and from the poor towards the wealthy.

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1 Introduction

Over the last several decades, there has been a large increase in valuations across many asset classes.[1] These rising valuations had important effects on the distribution of wealth. This raises the question: what are the *welfare* consequences of such asset price changes? Who wins and loses from a rise in asset valuations?

One viewpoint is that these wealth gains due to rising valuations represent an actual shift of resources towards the wealthy, and should be taxed as such (e.g., Piketty and Zucman, 2014; Saez, Yagan and Zucman, 2021).[2] An opposite viewpoint is that these wealth gains are just "paper gains", with no effect on actual income and therefore welfare (e.g., Cochrane, 2020; Krugman, 2021).[3] Which (if any) of these two opposing views is correct?

To make progress on this question, we develop a sufficient statistic approach that quantifies the individual (money metric) welfare gain associated with a change in asset prices. We operationalize this approach by using Norwegian administrative panel data on asset transactions from 1994 to 2015. This method allows us to quantify the distribution of welfare gains over this time period.

Consider the observed time paths of asset prices starting from an initial date, say the year 1994. We ask the following question: how much did a given individual win or lose in terms of welfare from the realized trajectory of asset prices relative to a baseline scenario? The answer to this question is given by the following formula (here for the case of one asset – the extension to multiple assets is straightforward):

$$\text{Welfare Gain}_i = \sum_{t=0}^{T} R^{-t} \left(\text{Sales}_{it} \times \text{Price Deviation}_t \right), \tag{1}$$

where i denotes the individual, T is the length of the sample period, $R > 1$ is a discount rate, Sales_{it} are the net sales of the asset by the individual in year t, and Price Deviation_t is the deviation of the price of the asset from the baseline scenario. The welfare gain is in dollar terms and corresponds to the individual willingness to pay for the deviation in asset prices.[4] Importantly, the welfare gain is computed holding the asset's cash flows constant so that the price deviations represent a pure valuation effect: a change in the asset's price without a change in cash flows. The formula follows from an application of the envelope theorem and thus holds for small price deviations, a point we discuss in more detail below.

[1] See for example Farhi and Gourio (2018) and Greenwald, Lettau and Ludvigson (2019).

[2] For example, Piketty and Zucman (2014) write: "Because wealth is always very concentrated [... a] high [wealth-to-income ratio] implies that the inequality of wealth, and potentially the inequality of inherited wealth, is likely to play a bigger role for the overall structure of inequality in the twenty-first century than it did in the postwar period. This evolution might reinforce the need for progressive capital taxation."

[3] Cochrane (2020) writes "much of the increase in 'wealth inequality' [...] reflects higher market values of the same income flows, and indicates nothing about increases in consumption inequality". Krugman (2021) discusses the hypothetical effect of declining interest rates on large fortunes in 19th-century England and writes "So since the ownership of land, in particular, was concentrated in the hands of a narrow elite, would falling interest rates and rising land prices have meant increased inequality? Clearly not. [...] The paper value of their estates would have gone up, but so what? The distribution of income wouldn't have changed at all."

[4] Our sufficient statistic formula measures the willingness to pay for a small price deviation. Because it corresponds to a first-order approximation, it can alternatively seen as an equivalent variation or a compensating variation (see Mas-Colell, Whinston and Green, 1995).

When implementing formula (1) empirically, we isolate valuation effects by considering a price deviation relative to a baseline scenario in which asset prices grow at the same rate as dividends (i.e., a world with constant price-dividend ratios). More precisely, we compute the price deviation in (1) as the relative difference between the actual price-dividend ratio PD_t and a baseline price-dividend ratio $\overline{PD}$:

$$\text{Price Deviation}_t = \frac{PD_t - \overline{PD}}{PD_t}. \tag{2}$$

Under the assumption that dividends follow a random walk, this price deviation can be interpreted as the deviation from a world in which dividends are discounted at a constant rate (i.e., deviations due to "discount rate shocks" in the language of Campbell and Shiller, 1988).[5] For our application, we use the 1992–1996 average price-dividend ratio as the baseline (i.e., 5-year window around the beginning of the sample). Importantly, all of the variables in (1) and (2) are readily observable in our data.

The formula for the welfare gain in (1) generates two main insights. First, what matters are asset *transactions*, not asset holdings. Intuitively, higher valuations are good news for prospective sellers (those with $\text{Sales}_{it} > 0$) and bad news for prospective buyers (those with $\text{Sales}_{it} < 0$). A particularly interesting case is an individual who owns assets but does not plan to buy or sell (i.e., $\text{Sales}_{it} = 0$). For such an individual, rising asset prices are merely "paper gains", with no corresponding welfare implications.[6]

Second, asset price changes are *purely redistributive*. When asset prices rise, there is a redistribution of welfare from sellers to buyers. But since for every seller there is a buyer, summing the welfare gains in formula (1) across all parties and counterparties of financial transactions in the economy implies that these aggregate to zero.[7] This aggregation result holds across all participants in asset markets, and not just the aggregate household sector. Because households trade with other sectors of the economy, namely both foreigners and the government, the household sector as a whole may benefit, but necessarily at the expense of another sector.

It is useful to contrast these results with the two polar viewpoints described earlier. The first viewpoint posited that rising asset prices redistribute toward existing asset holders. Our formula shows that, instead, it is *sellers* that benefit, not *holders*. If an asset holder never sells, they do not benefit from the unrealized capital gains generated by the price deviation. In the data, some individuals with large asset positions buy and hence lose in welfare terms; con-

[5]While our main results compute welfare gains relative to a baseline scenario with constant price-dividend ratios, thereby capturing pure valuation effects, formula (1) can also be used to compute welfare gains relative to other baseline scenarios. For example, we may instead be interested in computing the welfare gains and losses of asset-price changes due to cash flow changes. In this case, our formula is still correct but we would also want to take into account the direct effect of cash flows on individual welfare (an additional additive term).

[6]Our baseline model and hence formula (1) abstract from a number of potentially important mechanisms. Some of these may be operational even for individuals who neither buy nor sell. One such effect is that rising asset prices loosen collateral constraints thereby allowing for welfare-improving borrowing. Section 2.4 discusses such mechanisms and how our formula can be extended to take them into account.

[7]This result arises because our measure of welfare gains is a money metric, i.e. it is measured in dollars but is silent on the value of these extra dollars to the individual or to a social planner. It is therefore also silent on the desirability of an asset price change from the point of view of social welfare. Whenever, sellers and buyers systematically have different marginal utilities of consumption (or more generally social welfare weights), then the effect of a price deviation on social welfare can be positive or negative.

versely, others with small positions sell and hence win. The second viewpoint held that all (or at least most) of rising asset prices are irrelevant for welfare. As our formula shows, this is only true if assets are not traded (e.g., in an economy with a representative household). But when heterogeneous households buy and sell assets like they do in the real world, fluctuations in asset prices do generate welfare gains and losses. Both polar viewpoints are therefore incomplete.

As we show in the paper, the formula easily extends to multiple assets including bonds and long-lived assets subject to transaction costs (e.g., housing). Our key contribution is an empirical implementation of this extended welfare formula for the Norwegian economy to compute welfare gains and losses due to observed asset price changes for the time period 1994 to 2015, relative to a baseline scenario with constant price-dividend ratios. Price deviations in Norway have been particularly large for real estate (i.e., house prices have grown much faster than rents) and debt (i.e., real interest rates have declined sharply). Note that our approach remains agnostic on the causes of price deviations: our goal is to quantify the effect of asset price deviations on welfare, not explain them.

Our main findings are as follows. First, rising asset valuations have had large redistributive effects. While the average individual-level money metric welfare gain is around \$7,000, it is $-$\$35,000 at the 10th percentile and \$80,000 at the 90th percentile (in 2011 dollars). As a fraction of total wealth (i.e., financial wealth plus human wealth), the welfare gains are -6% at the 10th percentile and 80% at the 90th percentile. Importantly, the distribution of welfare gains differs substantially from the distribution of wealth gains (defined as the discounted sum of holdings times return deviations), which is positive for almost everyone.

Second, we quantify the amount of redistribution *across cohorts*. Overall, we find a large amount of redistribution from young to old. For instance, the average welfare gain is approximately $-$\$15,000 for individuals aged 15 and younger in 1994 (Millennials), and around \$30,000 for individuals aged 50 and older in 1994 (Baby boomers). This is primarily due to the fact that the young are net buyers of housing. Declining interest rates of mortgage debt offset the welfare losses of the young due to rising house prices but do so only partially.

Third, we quantify the amount of redistribution *across the wealth distribution*. We rank adults according to their total initial wealth in 1994 and find that welfare gains have been concentrated at the top of the wealth distribution. The top 1% experienced on average a \$125,0000 welfare gain, while the corresponding number is nearly zero at the 10th percentile. However, and perhaps surprisingly, this inequality in welfare gains tracks total wealth inequality almost one-for-one along most of the wealth distribution: the welfare gains as a fraction of total wealth are roughly 2% from the 20th through the 80th percentile.

Finally, we quantify the amount of redistribution *across sectors* of the economy: households, the government, and foreigners. Overall, the household sector experienced a small, but positive, welfare gain of roughly \$7,000 per individual. However, this was almost entirely offset by a "welfare loss" for the consolidated government sector (i.e., government plus central bank and non-profit institutions). The reason is that households are net debtors while the government is a net creditor to an almost identical extent. As a result, declining interest rates have benefited households at the expense of the government. Using the government budget constraint, we show that the household sector will eventually have to bear the cost of this "government welfare loss" through lower net transfers.

Literature. Our paper contributes to several strands of literature. In recent decades, there has been a sustained rise in valuations across many asset classes (e.g., Piketty and Zucman, 2014, Farhi and Gourio, 2018, Greenwald, Lettau and Ludvigson, 2019). As a response to this trend, a growing literature focuses on understanding the effect of rising asset prices (and declining interest rates) on wealth inequality (e.g., Kuhn, Schularick and Steins, 2020; Gomez, 2016; Wolff, 2022; Smith, Zidar and Zwick, 2021; Catherine, Miller and Sarin, 2020; Gomez and Gouin-Bonenfant, 2020; Cioffi, 2021; İmrohoroğlu and Zhao, 2022; Greenwald, Leombroni, Lustig and Van Nieuwerburgh, 2021). Relative to this literature, our contribution is to study the heterogeneous effect of rising asset prices on *welfare*.[8]

Our focus on the heterogeneous welfare effect of asset price fluctuations connects this paper to Doepke and Schneider (2006), who study the redistributive effect of inflation episodes using data from the Survey of Consumer Finances. Similarly, Glover, Heathcote, Krueger and Ríos-Rull (2020) examine the intergenerational redistribution due to the drop in asset prices during the Great Recession using a calibrated model. Our sufficient statistic approach is related to the one developed in Auclert (2019), which measures the welfare and consumption effect of interest rate and inflation shocks. The effect of asset prices on welfare is also studied by Dávila and Korinek (2018), who emphasize the pecuniary externalities that arise in an environment with financial constraints. Relative to this literature, our contribution is to develop an empirical framework to measure a money-metric notion of welfare gains and losses and to implement it using household-level transaction data.

A key advantage of our sufficient statistic is that it only requires data on financial transactions. Therefore, we sidestep the difficult task of estimating the market value of illiquid assets such as family businesses, future labor income, and defined-benefit pensions (e.g., Lettau and Ludvigson, 2001; Catherine, Miller and Sarin, 2020; Greenwald, Leombroni, Lustig and Van Nieuwerburgh, 2021). While these illiquid forms of wealth are important determinants of the path of financial transactions over the life-cycle, they do not enter our sufficient statistic directly.

Our paper is also related to a large asset pricing literature on the role of discount rate shocks (i.e., valuation shocks). A seminal paper is Campbell and Shiller (1988), who document the importance of discount rate shocks for high-frequency asset price fluctuations. Merton (1973) and Campbell and Vuolteenaho (2004) examine the implications of discount rate shocks on portfolio holdings. Under some assumptions, our sufficient statistic precisely measures the heterogeneous effect of discount rate shocks on welfare. While our main empirical application is to study longer-run trends in wealth and welfare inequality, our methodology could also be

[8]Our analysis builds on Moll (2020) who studied a two-period model similar to that in Section 2.1. Our result that the welfare of a household who never buys or sells an asset is unaffected when the asset's price changes is related to (but different from) a result by Sinai and Souleles (2005) that a household with an infinite expected residence spell is insulated from house price risk.

used to study the consequences of higher-frequency asset price fluctuations as in this literature.

More broadly, we contribute to a large literature that uses micro data to study the heterogeneity in saving and portfolio choices over the life cycle (e.g., Feiveson and Sabelhaus, 2019; Calvet, Campbell, Gomes and Sodini, 2021; Black, Devereux, Landaud and Salvanes, 2022) and along the wealth distribution (e.g., Bach, Calvet and Sodini, 2017; Fagereng, Holm, Moll and Natvik, 2019; Mian, Straub and Sufi, 2020; Bach, Calvet and Sodini, 2020).

Finally, our argument that rising asset valuations benefit sellers and not asset holders has some historical precedent in the works of Paish (1940), Kaldor (1955) and Whalley (1979) which were, in turn, part of a debate in the public finance literature whether unrealized capital gains are a form of income and should therefore be taxed (Haig, 1921; Simons, 1938).[9]

Roadmap. The remainder of this paper is organized as follows. In Section 2, we present our sufficient statistic for the welfare effect of a deviation in asset prices and discuss model extensions. In Section 3, we implement the sufficient statistic approach using administrative data from Norway. We report our empirical results in Section 4 and discuss robustness checks in Section 5. Section 6 concludes.

2 Theory

This section presents our main theoretical result. We examine the effect of a sequence of small deviations in asset prices on individual welfare. To focus on the intuition, we first derive our main result in a two-period model with only one asset. We then generalize the result to an infinite horizon model with multiple assets and adjustment costs. Finally, we discuss a number of important extensions such as bequest, borrowing constraints, and housing in the utility function.

2.1 Intuition in two-period model

Time is deterministic with two time periods $t = 0, 1$. Households have time separable preferences with a strictly concave utility function $U(\cdot)$ and a subjective discount factor $\beta < 1$. Households receive labor income Y_0 at time 0 and Y_1 at time 1. There is one asset available for trading at time $t = 0$ with price $P_0 > 0$, which pays a dividend $D_1 > 0$ at time 1. Moll (2020) analyzes a similar two-period environment.

Household problem. Denote by C_t the consumption of the household at time t and N_t the number of shares owned at the end of period t. Given initial asset holdings N_{-1}, the problem of the household is to choose a sequence of consumption and holdings that maximize welfare

$$V = \max_{\{C_0, C_1, N_0\}} U(C_0) + \beta U(C_1), \tag{3}$$

[9]For example, Kaldor (1955) writes: "We may now turn to the other type of capital appreciation which [comes] without a corresponding increase in the flow of real income accruing from that wealth. [... I]n so far as a capital gain is realized and spent [...] the benefit derived from the gain is equivalent to that of any other casual profit. If however it is not so realized, there is clearly only a smaller benefit."

subject to the following budget constraints:

$$C_0 + (N_0 - N_{-1})P_0 = Y_0, \tag{4}$$

$$C_1 = N_0 D_1 + Y_1. \tag{5}$$

These budget constraints say that, at each period t, consumption plus net asset purchases (the left hand side) must equal income (the right hand side).[10]

Comparative static with respect to prices. What is the effect of a small rise in the price P_0 on welfare? Since the price P_0 only appears in the budget constraint at time $t = 0$ (see equation (4)), the envelope theorem states that

$$\mathrm{d}V = U'(C_0)(N_{-1} - N_0)\,\mathrm{d}P_0. \tag{6}$$

The effect of a rise in P_0 is given by the marginal utility of consumption at $t = 0$, $U'(C_0)$, times the extent to which it relaxes the budget constraint at $t = 0$, namely asset sales $N_{-1} - N_0$. Intuitively, a rise in the price of the asset benefits households who plan to sell the asset (i.e., $N_0 < N_{-1}$) and hurts households who plan to buy the asset (i.e., $N_0 > N_{-1}$). In particular, a rise in the price of the asset does not affect households who do not plan to trade (i.e., $N_0 = N_{-1}$): for those households, the rise in the price of the asset is merely a "paper gain" with no corresponding effect on consumption and thus welfare.

Importantly, the comparative static in equation (6) holds the dividend D_1 constant. The asset price change $\mathrm{d}P_0$ thus represents a pure valuation effect: a rise in the asset price without a rise in the asset's cash flows. The price change $\mathrm{d}P_0$ can hence be thought of as being generated by a decline in the rate at which the dividend D_1 is discounted (i.e., a "discount rate shock" in the language of Campbell and Shiller, 1988).[11] If instead the asset's dividend increased $\mathrm{d}D_1 > 0$ together with the asset price increase, the comparative static in equation (6) would have an extra term $\beta U'(C_1) N_0\,\mathrm{d}D_1$, capturing the intuition that higher dividends benefit asset *holders*.

Welfare versus wealth gains. The result in equation (6) may be surprising at first. How can an asset holder (i.e., $N_{-1} > 0$) not benefit from a rise in prices given that the market value of her initial wealth $N_{-1}P_0$ increases? The reason is that, while a rise in P_0 increases the initial return on the asset at time $t = 0$, it also *decreases* the future return of holding the asset until $t = 1$. As a result, only individuals whose holdings decline over time (i.e., sellers) benefit from a rise in asset valuation.

[10] Note that we implicitely assume $P_1 = 0$ (i.e., the world ends at $t = 1$). However, even with $P_1 > 0$, a rise in P_0 would have the same welfare effect as in equations (6) and (8) below. In particular, holding P_1 constant, a rise in P_0 would still *decrease* the return of holding the asset from period $t = 0$ to $t = 1$.

[11] To put this more precisely, it is useful to adopt the perspective that the asset price is the present discounted value of future cashflows: $P_0 = D_1/\mathcal{R}$ where $\mathcal{R}$ is a discount rate (which we take as exogenously given). An increase in the price P_0 without a change in the dividend D_1 is then equivalent to a fall in the discount rate $\mathcal{R}$. Also note that this pure valuation effect results in an increase in the price-dividend ratio P_0/D_1. Appendix A.2 spells out the analogous logic in the multiperiod model of Section 2.2.

To formalize this point, denote by R_t the return of the asset at time t; that is, $R_0 = P_0/P_{-1}$ and $R_1 = D_1/P_0$. Note that a rise in P_0 increases R_0 but *decreases* R_1. Formally, we have

$$\begin{aligned} \frac{\mathrm{d}R_0}{\mathrm{d}P_0} &= 1/P_{-1} > 0, \\ \frac{\mathrm{d}R_1}{\mathrm{d}P_0} &= -R_1/P_0 < 0. \end{aligned} \tag{7}$$

The welfare effect of a change in asset prices can be equivalently expressed as marginal utility times the present value of the change in returns on wealth:

$$\begin{aligned} \mathrm{d}V &= \underbrace{U'(C_0) \times N_{-1}P_{-1} \times \mathrm{d}R_0}_{\text{Contribution of return at } t=0} + \underbrace{R_1^{-1}U'(C_0) \times N_0 P_0 \times \mathrm{d}R_1}_{\text{Contribution of return at } t=1}, \\ &= U'(C_0) \times N_{-1} \times \mathrm{d}P_0 - U'(C_0) \times N_0 \times \mathrm{d}P_0, \end{aligned} \tag{8}$$

where the second line uses equation (7). This welfare effect is the sum of two terms: the first term accounts for the (positive) effect of a rise in P_0 on today's return while the second term accounts for the (negative) effect of a rise in P_0 on tomorrow's return. For a household that does not trade, the two terms offset each other: as a result, asset prices have no effect on welfare.

The important point is that the second term, which captures the negative effect of rising asset prices on future returns, is often far from neglible, especially for long-lived assets. In our empirical exercise, we stress the fact that the *welfare* effect of a deviation in asset prices—the left-hand side of equation (8)—differs significantly from the *wealth* effect—the first term on the right-hand side of equation (8).

Graphical intuition. Building on Whalley (1979), we now provide a graphical intuition for equation (6). The household's optimization problem is equivalent to the standard problem of intertemporal choice: maximize utility (3) subject to a present-value budget constraint. Figure 1 shows the standard budget constraint and indifference curve, with the slope of the former given by (the negative of) the asset return $R_1 = D_1/P_0$.[12]

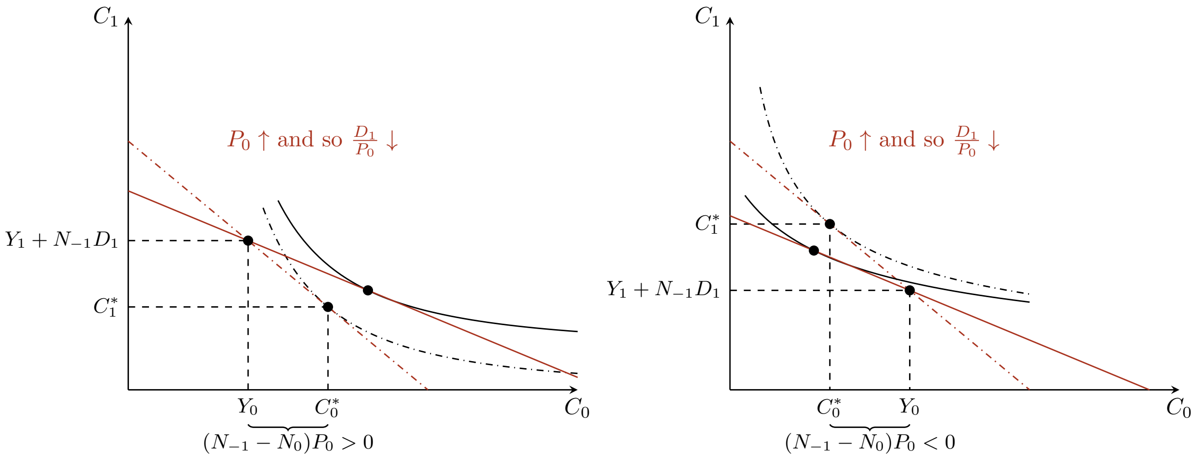
Consider the welfare consequences of a rise in the asset price P_0 for a hypothetical seller (panel a) and buyer in (panel b). In both panels, the dashed budget constraint and indifference curve correspond to the allocation at the initial asset price and the solid lines are those at the new, higher price. When the asset price P_0 rises, the budget constraint rotates through the endowment point and becomes shallower (the slope is $-D_1/P_0$). Panel (a) depicts the case of a household selling the asset at $t = 0$ (i.e., $N_{-1} - N_0 > 0$) so that optimally chosen initial consumption exceeds initial labor income $C_0 > Y_0$. Panel (b) considers the case of a buyer.

[12]To obtain the standard present-value budget constraint and to see that its slope is indeed $-D_1/P_0$, we combine the period budget constraints (4) and (5) and obtain

$$C_1 = \frac{D_1}{P_0}(Y_0 - C_0) + Y_1 + N_{-1}D_1 \qquad \text{or} \qquad C_0 + \frac{C_1}{D_1/P_0} = Y_0 + \frac{Y_1}{D_1/P_0} + N_{-1}P_0.$$

The first version also makes it clear that the endowment point is given by $C_0 = Y_0$ and $C_1 = Y_1 + N_{-1}D_1$ as in Figure 1. The second version states that the present-value of consumption must equal the present-value of income plus initial wealth.

The figure shows clearly that the seller ends up on a higher indifference curve (her welfare increases) whereas the buyer ends up on a lower indifference curve (her welfare decreases).



(a) Effect on seller

(b) Effect on buyer

Figure 1: Welfare effect of a rise in the asset price P_0 (two-period model)

Notes. Figure 1 graphically analyzes the effect of an increase in the asset price P_0 on the welfare of a seller (panel a) and that of a buyer (panel b). In both panels, the present-value budget constraint goes through the endowment point $C_0 = Y_0$ and $C_1 = Y_1 + N_{-1}D_1$ and has slope $-D_1/P_0$. See footnote 12 for a derivation. The dashed budget constraint and indifference curve correspond to the allocation at the initial asset price and the solid lines are those at the new, higher price. When the asset price P_0 increases, the budget constraint rotates through the endowment point and becomes shallower. The seller's welfare increases (panel a) and the buyer's welfare decreases (panel b).

2.2 Baseline model

For the sake of intuition, the previous section focused on the case of a two-period economy with only one asset. We now extend our formula to an infinite-horizon economy with multiple assets and adjustment costs (henceforth the "baseline model"), which is key to bringing the theory to the data. We continue to work with a deterministic environment. Section 2.4 discusses a number of important extensions, including the stochastic case.

Financial markets. There is a sequence of liquid one-period bonds B_t with a face value of one and price Q_t available for trading. Note that holding a one-period bond is equivalent to investing in a deposit account with an interest rate $R_{t+1} = 1/Q_t$ between t and $t+1$. Denote by $R_{0\to t} = R_1 \cdot R_2 \cdots R_t$ the cumulative return of the liquid asset between 0 and t.

There are also K long-lived assets available for trading (i.e., stocks, housing, private businesses). Each share of asset k is a claim to a stream of dividends $\{D_{k,t}\}_{t=0}^{\infty}$ and has price $P_{k,t}$ at the end of period t. The asset's return between t and $t+1$ is thus $R_{k,t+1} \equiv (D_{k,t+1} + P_{k,t+1})/P_{k,t}$.

We assume that trading these long-lived assets is subject to adjustment costs which may be large or small depending on the asset. Some assets, such as houses and privately-traded equity, are illiquid and the adjustment costs capture this illiquidity (see, e.g., Kaplan and Violante, 2014 and Kaplan et al., 2018). For other assets, such as publicly traded equity, the adjustment costs—which may be arbitrarily small but positive—are instead a technical assumption that is necessary in our deterministic setup. In short, they allow different assets to have different returns without generating the possibility of infinite profits via arbitrage, hence guaranteeing a unique and differentiable solution to the household's portfolio allocation problem.[13] Specifically, to buy a quantity of shares $N_{k,t} - N_{k,t-1}$ of asset k at time t, the household will have to pay $\chi_k(N_{k,t} - N_{k,t-1})$ in adjustment costs, where $\chi_k(\cdot)$ is a stricly convex function. While the particular functional form does not matter for the effect of asset price changes on welfare at the first order (i.e., for infinitesimal small price deviations), it will matter for higher-order effects, as discussed in Section 5.

Household problem. Households have time separable preferences with a strictly concave period utility $U(\cdot)$ and a subjective discount factor $\beta < 1$. They receive labor income Y_t at time t. Denote B_t the holdings of the one period bonds and $N_{k,t}$ the holdings of asset k at time t. Households take asset prices as given and choose an optimal path of consumption and asset holdings:

$$V = \max_{\{C_t, B_t, \{N_{k,t}\}_k\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_t), \tag{9}$$

subject to initial holdings B_{-1} and $\{N_{k,-1}\}_k\}$, as well as a sequence of budget constraints

$$C_t + \sum_{k=1}^{K} (N_{k,t} - N_{k,t-1}) P_{k,t} + B_t Q_t + \sum_{k=1}^{K} \chi_k(N_{k,t} - N_{k,t-1}) = \sum_{k=1}^{K} N_{k,t-1} D_{k,t} + B_{t-1} + Y_t, \tag{10}$$

and no-Ponzi conditions

$$\lim_{T\to\infty} R_{0\to T}^{-1} B_T Q_T = 0 \quad \text{and} \quad \lim_{T\to\infty} R_{0\to T}^{-1} N_{k,T} P_{k,T} = 0 \quad \forall k. \tag{11}$$

As in the two-period model, the budget constraint simply says that consumption plus net purchases of financial assets (the left-hand side) must equal total income in each period t (the right-hand side). Total income is given by the sum of dividend income and labor income net of adjustment costs.

Welfare gain. We are interested in the effect of a change in asset prices on welfare. Formally, we consider an arbitrary perturbation of the path of asset prices, denoted by $\{\mathrm{d}Q_t, \{\mathrm{d}P_{kt}\}_k\}_{t=0}^{\infty}$, which satisfies the following no-bubble condition

$$\lim_{T\to\infty} R_{0\to T}^{-1} \, \mathrm{d}P_{k,T} = 0 \quad \forall k. \tag{12}$$

Denote by $\mathrm{d}V$ the effect of the price deviation on welfare defined in (9). We define the *money metric welfare gain* as its effect on welfare scaled by the marginal utility of consumption at time $t = 0$

$$\text{Welfare Gain} \equiv \mathrm{d}V / U'(C_0). \tag{13}$$

[13]In the stochastic environment discussed in Section 2.4, adjustment costs can be dispensed whenever different assets have different risk profiles.

This welfare gain is in units of consumption (i.e., it is a money metric) and has the interpretation of an household's willingness to pay for this particular price deviation. For brevity we will often refer to this quantity simply as "welfare gain" but it is important to keep in mind that it is a money metric, i.e. it measures gains in dollars but is silent on the value of these extra dollars to the individual or to a social planner.

Totally differentiating the definition of welfare (9) gives the following expression for the welfare gain:

$$\begin{aligned}\text{Welfare Gain} &= \sum_{t=0}^{\infty} \beta^t \frac{U'(C_t)}{U'(C_0)} \, \mathrm{d}C_t, \\ &= \sum_{t=0}^{\infty} R_{0\to t}^{-1} \, \mathrm{d}C_t,\end{aligned} \tag{14}$$

where the second line uses the Euler equation $\beta^t R_{0\to t} U'(C_t)/U'(C_0) = 1$. Equation (14) says that our measure of welfare gain can be seen as the present value of the consumption changes caused by the price deviation.

We can now express the welfare gain in terms of the deviation in the path of asset prices.

Proposition 1 (Welfare Gain)**.** *The welfare gain implied by a price deviation* $\{\mathrm{d}Q_t, \{\mathrm{d}P_{k,t}\}_k\}_{t=0}^{\infty}$ *is*

$$\text{Welfare Gain} = \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(\sum_{k=1}^{K} (N_{k,t-1} - N_{k,t}) \, \mathrm{d}P_{k,t} - B_t \, \mathrm{d}Q_t \right). \tag{15}$$

As in the two-period model, the welfare gain depends on whether the household is a buyer or seller of assets. However, equation (15) highlights the fact that portfolio choices and the timing of sales also matter. The key insight is that the welfare gain associated with deviations in asset prices depends on financial *transactions* rather than *holdings*. Note, however, that for the liquid asset, transactions and holdings coincide given that the asset must be continuously rolled over. Thus, declining interest rates (i.e., $\mathrm{d}Q_t > 0$) benefit households holding short-term debt because lower debt payments relax their budget constraint.

Also as in the two-period model, the welfare gain in (15) is computed holding the time path of dividends $\{\{D_{kt}\}_k\}_{t=0}^{\infty}$ constant and the price deviations $\{\mathrm{d}Q_t, \{\mathrm{d}P_{k,t}\}_k\}_{t=0}^{\infty}$ thus represent pure valuation effects: changes in asset prices without a changes in the assets' cash flows.

Finally, note that the adjustment cost function does not appear in the welfare formula. This is a direct implication of the envelope theorem, which says that the changes in adjustment costs are second-order for welfare.

Aggregation. We now describe an important aggregation result. Suppose that the economy is populated by $i = 1, 2, \ldots, I$ households who trade assets with each other.

Corollary 2 (Aggregation)**.** *Suppose that initial prices* $\{Q_t, \{P_{k,t}\}_k\}_{t=0}^{\infty}$ *clear all asset markets. Welfare gains implied by a price deviation* $\{\mathrm{d}Q_t, \{\mathrm{d}P_{k,t}\}_k\}_{t=0}^{\infty}$ *aggregate to zero and thus price deviations are purely redistributive.*

$$\left.\begin{array}{l}\sum_{i=1}^{I}(N_{ik,t-1} - N_{ik,t}) = 0 \quad \forall k. \\ \sum_{i=1}^{I} B_{it} = 0\end{array}\right| \implies \sum_{i=1}^{I} \text{Welfare Gain}_i = 0$$

Corollary (2) is intuitive. For instance, when asset prices rise, sellers benefit, but market clearing implies that for every seller there is an offsetting buyer that is hurt. Hence, the welfare gains must aggregate to zero over the full population. In fact, market clearing implies that that welfare gains aggregate to zero for each asset class. This result highlights a key difference between wealth gains and welfare gains: while a rise in asset prices leads to positive wealth gains in aggregate (as long as the asset is in positive net supply), it does not lead to aggregate welfare gains. Asset price changes are therefore purely redistributive.

Our result says that welfare gains aggregate to zero. It is important to recall, however, that individual welfare gains are money metric gains as defined in (13), i.e. they are measured in dollars but are silent on the value of these extra dollars to the individual or to a social planner. The result therefore says nothing about the desirability of an asset price deviation from the point of view of total welfare as measured by a social welfare function. In particular, the effect of a price deviation on social welfare can be positive or negative, depending on whether the welfare weights assigned to individuals covary positively or negatively with the individual-speficic money metric gains.[14] What our aggregation result says is simply that the social planner could, in principle, undo the effect of asset price changes on social welfare, by redistributing resources from individuals with money metric gains to those with losses.

The assumption that households only trade with each other is key for the aggregation result in Corollary 2. In reality, however, households also trade with other entities such as the government and foreigners. In this case, the corollary can be modified to say that the welfare gain of the household, government, and foreign sectors sum up to zero in aggregate.[15]

2.3 Implementation and Sufficient Statistic

We now discuss how we bring the theory to the data in order to estimate the distribution of welfare gains (caused by asset price changes) across households.

First-order approximation. Proposition 1 gives a formula for the welfare gain associated with an arbitrary infinitesimal deviation in prices $\{\mathrm{d}Q_t, \{\mathrm{d}P_{k,t}\}_k\}_{t=0}^{\infty}$. We use this formula to obtain a first-order approximation of the welfare effect of a non-infinitesimal deviation in asset prices

[14] More precisely, the change in social welfare is $\sum_{i=1}^{I} \lambda_i U'(C_{i0}) \times \text{Welfare Gain}_i$, where λ_i is the Pareto weight assigned to household i. The term $\lambda_i U'(C_{i0})$ can be seen as a marginal welfare weight (see, e.g., Saez and Stantcheva, 2016; Dávila and Schaab, 2021).

[15] We discuss this point more precisely in Section 2.4.

$\{\Delta Q_t, \{\Delta P_{k,t}\}_k\}_{t=0}^{\infty}$:

$$\text{Welfare Gain} = \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(\sum_{k=1}^{K} (N_{k,t-1} - N_{k,t}) \Delta P_{k,t} - B_t \Delta Q_t \right). \tag{16}$$

This formula is the generalization to multiple assets of formula (1) in the introduction. The approximation is accurate for small price deviations. However, in Section 5.1, we argue that the approximation error is small in practice.

Price deviations. To arrive at the welfare formula (16) we consider price deviations ($\Delta P_{k,t}$ while, at the same time, holding dividends unchanged (i.e., $\Delta D_{k,t} = 0$). In real-world data, dividends typically change over time, in particular they display trend growth. To isolate the welfare effects of asset price changes that are orthogonal (in an appropriate sense) to changes in dividends, we construct the empirical price deviations $\Delta P_{k,t}$ as deviations of asset prices from those that would arise under a constant price-dividend ratio. Intuitively, while the theory above considers price deviations $\Delta P_{k,t}$ *without* changing dividends, in our empirical application we consider price deviations *relative* to changing dividends (i.e., the valuation effects emphasized in the introduction).

Put differently, we choose as our baseline scenario one where asset prices grow at the same rate as dividends, as in the Gordon growth model (i.e., a world in which dividends follow a random walk and discount rates are constant). We then construct price deviations as deviations from this baseline scenario. As we discuss in more detail in Appendix A.2, when log dividends follow a random walk, these price deviations isolate variations in asset prices due to variations in discount rates. Intuitively, when discount rates are constant, so are price-dividend ratios. In contrast, when discount rates fall, price-dividend ratios increase.

Formally, we denote by $PD_{k,t} \equiv P_{k,t}/D_{k,t}$ the price-dividend ratio for asset k. Given a baseline value $\overline{PD}_k$, we consider the following price deviation

$$\Delta P_{k,t} = P_{k,t} - \overline{PD}_k \times D_{k,t} \tag{17}$$

As a motivating example, Figure 2 plots the price index of houses in Norway together with the price index for rents. Notice that, starting around the mid-1990s, the price of housing has grown faster than rents. In this case, the price deviation corresponds to the difference between realized prices $\{P_{Ht}\}_{t=0}^{\infty}$ and the counterfactual price path associated with a constant price-to-rent ratio $\{\overline{PD}_H \times D_{Ht}\}_{t=0}^{\infty}$. Equation (17) can also be written as

$$\frac{\Delta P_{k,t}}{P_{k,t}} = \frac{PD_{k,t} - \overline{PD}_k}{PD_{k,t}},$$

i.e. the price deviation in relative terms equals the relative difference between the actual price-dividend ratio PD_t and a baseline price-dividend ratio $\overline{PD}$. This is equation (2) in the introduction. For the liquid asset, we consider a deviation of the price of one-period bonds from a baseline value $\overline{Q}$ (i.e., $\Delta Q_t = Q_t - \overline{Q}$).

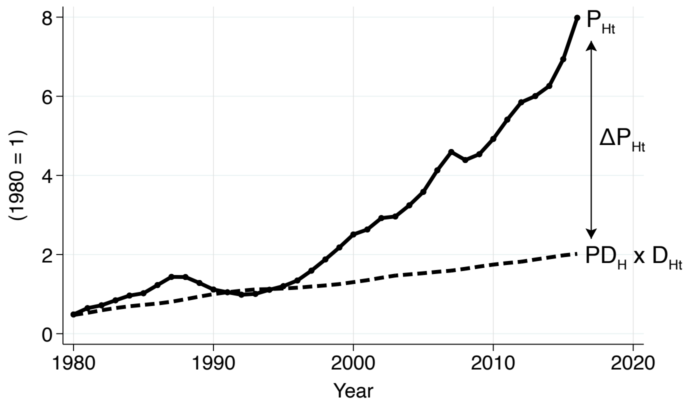


Figure 2: Graphical representation of the price deviation ΔP_{Ht}

Notes. Figure 2 plots the housing price index in Norway from Norges Bank's project on Historical Monetary Statistics (solid line) as well as the rental price index from *Statistics Norway* (dashed line). Both are normalized to one in 1980. The difference between the two can be interpreted as a deviation ΔP_{Ht} between the realized price path P_{Ht} and a counterfactual price path with constant price-to-rent ratio $\overline{PD}_h \times D_{Ht}$.

Finite time horizon. While the formula (16) depends on an infinite sum of transactions, in any empirical applications we only observe financial transactions over a finite sample period of some length T.

Our solution to this issue will be to replace the summation from $t = 0$ to $t = \infty$ with a summation from $t = 0$ to $t = T$, where T denotes the length of the sample period. Note that this truncation is inconsequential if there is no trade after year T, or, alternatively, if the price deviation stops in year T (i.e., the price deviation reverts to zero after T). In general, the quantitative effect of this truncation depends on the growth of asset prices relative to the discount rate $R_{0\to t}^{-1}$ in the welfare gain formula. An alternative solution, which we explore in Section 5, is to construct hypothetical financial transactions as well as price deviations after year T.

Sufficient statistic. Combining the first-order approximation of welfare gains (16) with the empirical price deviations (17) and truncating the formula at time horizon T, we obtain a sufficient statistic for the individual-level welfare gain of realized price deviations:

$$\text{Welfare Gain} = \sum_{t=0}^{T} R_{0\to t}^{-1} \left(\sum_{k=1}^{K} (N_{k,t-1} - N_{k,t}) P_{k,t} \times \frac{PD_{k,t} - \overline{PD}_k}{PD_{k,t}} - B_t Q_t \times \frac{Q_t - \overline{Q}}{Q_t} \right) \tag{18}$$

This formula is the generalization to multiple assets of the combination of formulas (1) and (2) in the introduction and forms the core of our empirical implementation using administrative data. It is a sufficient statistic in the sense that it depends only on data on financial transactions $(N_{k,t-1} - N_{k,t})P_{k,t}$ and $B_t Q_t$, as well as valuation ratios PD_{kt} and Q_t, which are observable.

2.4 Discussions and extensions

The baseline model is deliberately stylized and abstracts from a number of potentially important features of the real world. Before we bring our theory to the data, we consider a number of model extensions. In each case, we discuss how the extension affects our welfare gain formula (15) as well as its interpretation. Appendix A.3 provides a rigorous treatment of each model extension. In this section, we summarize the key insights.

Stochastic environment. In the baseline model, we made the simplifying assumption that asset prices are fully deterministic. In Appendix A.3.1, we provide an interpretation of our welfare gain formula in the presence of risk. We consider an extension of the baseline model where asset prices and dividends follow stochastic processes.

Using a small-noise expansion, we derive a first-order approximation for the welfare effect of *realized* shocks.[16] The resulting welfare gain formula is the same as in the baseline model, but the price perturbation $\mathrm{d}P_t$ is replaced by a stochastic innovation. The formula is an application of a more general result that may prove useful in other context as well: an envelope theorem for ex-post welfare in stochastic environments which we prove in Appendix A.4. The key takeaway is that our main theoretical result does not necessarily require households to have perfect foresight over future asset prices. Instead, our welfare gain formula (15) can be interpreted as (an approximation of) the cumulative welfare effect of a sequence of small and unexpected asset price shocks.

Borrowing and collateral constraints. In the baseline model, households can take unrestricted positions in any asset (i.e., long and short). In reality, there are limits on negative holdings, for instance on how much uncollateralized credit that a household can obtain. In Appendix A.3.2, we consider an extension of the baseline model with a borrowing constraint. We show that it affects our welfare gain formula in two ways.

First, whenever the borrowing constraint binds, the Euler equation does not hold (i.e., the marginal utility of consumption today exceeds discounted marginal utility tomorrow). Hence, the rate at which future net asset purchases must be discounted in the welfare gain formula (15) is higher than the cumulative return $R_{0\to t}$ on the liquid asset. In this case, our welfare measure will tend to overestimate the contribution of future price deviations on welfare. In our empirical implementation, we do not attempt to measure individual-specific welfare-relevant discount rates, but our choice of discount rate is meant to be conservative (i.e., higher than the interest rate on bank deposits).

Second, when the borrowing constraint depends directly on the price of an asset P_t, as in collateral constraint models, asset price deviations will directly affect the tightness of the borrowing constraint (e.g., Kiyotaki and Moore, 1997; Miao and Wang, 2012; Mian et al., 2013). In this case, the welfare gain formula (15) has an additional term that accounts for the effect

[16] See Samuelson (1958a), or more recently Bhandari et al. (2021), for applications of the small-noise expansion. It consists of estimating a function of random variables using a first-order Taylor expansion around a zero-variance baseline.

of asset prices on the tightness of the borrowing constaint. In collateral constraint models, higher asset prices tend to be welfare-improving because they allow constrained households to increase their consumption today at the expense of tomorrow (see Dávila and Korinek, 2018 for a theoretical treatment of such models). In our empirical implementation, we do not attempt to quantify the collateral channel and instead focus on the direct, purely-redistributive effect of asset prices that occur through trading.

An interesting case is wealthy households borrowing against their assets to consume, for example as part of a "buy, borrow, die" tax avoidance strategy. When rising asset prices and falling interest rates allow such households to borrow more or at better conditions, they benefit in welfare terms even without selling any of their assets.[17]

Bequests. In the baseline model, we abstract from intergenerational linkages and bequests. In practice, bequests have been shown to be an important determinant of consumption and saving decisions (De Nardi, 2004). In Appendix A.3.3, we consider an extension of the baseline model where households receive utility from giving assets to their heirs via a "warm glow bequest function". We do not specify the functional form of the bequest function, hence nesting both altruistic models and other ad-hoc specifications. Compared to the case without inheritance, the formula is modified in two ways.

First, the effect of a price deviation $\mathrm{d}P_t$ on welfare matters through the number of shares *sold* $(N_{k,t-1} - N_{k,t} + \text{Net inheritance}_{k,t})$, not the decrease in holdings $(N_{k,t-1} - N_{k,t})$ alone. Intuitively, if a household inherits a house and immediately sells it – such that holdings of housing are unchanged $N_{k,t-1} = N_{k,t}$ but housing sales equal Net inheritance$_{k,t}$ – higher house prices benefit the household. Conversely, if a household inherits a house and plans to live in it forever – such that holdings increase from $N_{k,t-1}$ to $N_{k,t} = N_{k,t-1} + \text{Net inheritance}_{k,t}$ but housing sales are zero – higher house prices are irrelevant for the household's welfare (the inheritance itself of course still benefits the household in absolute terms, just not in a way that is dependent on house prices).[18] This distinction is easy to deal with empirically, since we observe financial transactions directly, not just changes in holdings.

Second, the welfare gain formula has an additional term that accounts for the change in net inheritance as a result of asset prices. The idea is that households may decide to adjust the quantity of assets that they give to their heirs in response to an asset price change $\mathrm{d}P_t$. In our empirical implementation, we assume that this term is zero. In the context of housing, our assumption implies that parents choose the physical quantity of real estate (e.g., square

[17]One main reason households use a "buy, borrow, die" strategy is step-up in basis at death. This feature of the U.S. tax system (and some other countries) means that dying without ever having sold an asset and passing it on to an heir greatly reduces the heir's capital gains tax bill if she sells the inherited asset. Rising asset prices therefore also have an effect on the *relative* welfare of asset sellers who use the strategy relative to those who do not. When asset prices rise, asset sellers who do not use the "buy, borrow, die" strategy pay higher capital gains taxes which attenuates their welfare gain. In contrast, this attenuation effect is smaller (or non-existent) for households who use the strategy because they pay less (or no) capital gains taxes in the first place.

[18]It is also worth noting that higher house prices hurt households who did not inherit a house but are planning to buy one in the future. Thus, even if a household inherits a house and plans to live in it forever, this household still benefits from higher house prices *relative* to these other households (the inheritance may mean that the household no longer needs to buy an expensive house).

meters) they want to leave to their children, and that changes in asset prices do not affect their decision. Note, however, that a rise in the price of housing means that the *value* of the inheritance is necessarily higher, but our assumption implies that the quantity of housing inherited is unchanged.

General equilibrium. So far, we have considered a partial equilibrium environment, where asset prices are determined outside of the model. In general equilibrium, asset prices are determined by supply and demand forces. Whether we consider a partial or general equilibrium model, our sufficient statistic always has the interpretation of a willingness to pay: i.e., how much would a household be willing to pay in order to experience a small deviation of asset prices from their current path.

However, in general equilibrium, the fundamental shock that generates the price deviation will also have a direct effect on household welfare.[19] Hence, our welfare gain formula does not capture the full welfare effect associated with the fundamental shock, but rather the welfare effect that is caused by the resulting change in asset prices.

To clarify the interpretation of our welfare gain formula in a general equilibrium model, we solve a simple overlapping generation model with a single asset (as in Samuelson, 1958b), which can be solved analytically (see Appendix A.3.4). We simulate a demand shock (rise in patience) as well as a supply shocks (rise in labor endowment) that both yield an equilibrium increase in the price of the asset. We show that the total welfare effect associated with these shocks is the sum of the baseline welfare gain formula (15) (which aggregates to zero across households), plus the direct contribution of the shock itself on welfare.

Government sector. When households only trade assets with each other, the aggregate household welfare gain of asset price deviations is zero (see Corollary 2). The logic is that for every household selling an asset, there is an offsetting household purchasing it. In practice, however, households routinely trade assets with non-household entities, such as the government. For example, if households are net buyers of government bonds, a change in the interest rate on government debt leads to a redistribution of resources from the government towards households.

In Appendix A.3.5, we study an extension of the baseline model with a government that taxes and makes transfers and is allowed to run surpluses and deficits (subject to a no-Ponzi condition as in the household problem). We do not assume that the government maximizes a social welfare function and instead make a weaker assumption on cost minimization (i.e., the marginal return of saving/borrowing in the different assets is equalized). We obtain two main results.

First, relative to the individual welfare gain formula in the baseline model, there is an additional term that accounts for the present value of changes in net government transfers. The idea is that, in general, the government will adjust the path of taxes in transfers in response

[19] In practice, the trend of declining interest rates and rising asset prices amongst developed economies is often thought to be the result of an increased demand for saving from abroad (Bernanke, 2005), population aging (Auclert et al., 2020), or inequality (Mian et al., 2021).

to a change in asset prices. In our empirical exercise, we will not estimate how a deviation in asset prices affects household-specific net transfers.

Second, summing over all households, we show that aggregate present value of changes in net government transfers is precisely equal to the "welfare gain of the government" (i.e., equation (15) in the baseline model). This is intuitive and follows directly from the government budget constraint. For instance, if the government is a borrower and its cost of borrowing increases (i.e., negative government welfare gain), then it means that there are less resources available for doing net transfers to households.

Housing and wealth in the utility function. In the baseline model, households only get utility from consumption and thus care about asset ownership only indirectly. In reality, households may also care about asset ownership per se. An important example is owning a house and living in it which generates a direct utility flow. Other examples include preferences for social status or power. In Appendix A.3.6, we consider an extension of the baseline model where assets enter the utility function directly. We show that if only the quantity of assets enters the utility function (as is natural in the housing case), this "joy of ownership" channel does not affect our welfare gain formula.

However, if households care about the *market value* of their assets, for instance due to social status (Smith, 1759; Weber and Kalberg, 1958; Bakshi and Chen, 1996; Carroll, 1998; Roussanov, 2010) or political power (Piketty et al., 2013), the welfare gain formula has an additional term. In this case, rising asset prices would have a direct and heterogeneous effect on welfare. In our empirical implementation, we do not attempt to quantify this channel. We instead focus on the effect of asset price changes on welfare that operate through changes in consumption.

Relation to duration mistmatch. Proposition 1 is related to Theorem 1 in Auclert (2019), who expresses the sensitivity of welfare to a shift in the yield curve in terms of the mismatch between consumption and income. We show the equivalence between the two results in Appendix A.3.7. An interesting application is the case of a permanent decline in interest rates, in which case welfare gains depend on the *duration* mismatch between consumption and income (see Greenwald et al., 2021).

For our application, our sufficient statistic has two advantages. First, it expresses the welfare gain in terms of financial transactions, which we observe directly, rather than in terms of the path of consumption, which is harder to observe. Second, it allows us to consider the welfare effect of arbitrary valuation changes in different asset classes, rather than only the ones implied by a shift in the yield curve.

3 Data

We use a combination of administrative and publicly-available data from Norway to quantify our sufficient statistic formula (18). In this section, we briefly describe the data. A more detailed description can be found in Appendix B.

3.1 Sample and asset classes

We estimate equation (18) using data for the 1994–2015 period, i.e. our sample spans $T = 20$ years, with the year 1994 corresponding to the initial period $t = 0$. Our data covers the universe of individuals in Norway who were at least 18 years old for at least one year in the 1994–2015 period.

We consider four asset classes: housing, debt, deposits, and equity, which correspond to the four main asset classes traded by Norwegian households. Note that we do not need to account for fully illiquid forms of wealth such as human wealth and defined-benefit pensions since they are not traded (i.e., they have no market price).

Given this, we estimate our sufficient statistic as follows:

$$
\begin{aligned}
\text{Welfare Gain} &= \sum_{k \in \{\text{housing,debt,deposit,equity}\}} \text{Welfare Gain}_k, \\
\text{Welfare Gain}_{\text{housing}} &= \sum_{t=0}^{20} R^{-t} (N_{H,t-1} - N_{H,t}) P_{H,t} \times \frac{PD_{H,t} - \overline{PD}_H}{PD_{H,t}}, \\
\text{Welfare Gain}_{\text{debt}} &= \sum_{t=0}^{20} R^{-t} (-B_{M,t} Q_{M,t}) \times \frac{Q_{M,t} - \overline{Q}_M}{Q_{M,t}}, \\
\text{Welfare Gain}_{\text{deposit}} &= \sum_{t=0}^{20} R^{-t} (-B_{D,t} Q_{D,t}) \times \frac{Q_{D,t} - \overline{Q}_D}{Q_{D,t}}, \\
\text{Welfare Gain}_{\text{equity}} &= \sum_{t=0}^{20} R^{-t} (N_{E,t-1} - N_{E,t}) P_{E,t} \times \frac{PD_{E,t} - \overline{PD}_E}{PD_{E,t}},
\end{aligned}
\tag{19}
$$

where $\overline{PD}_H, \overline{Q}_M, \overline{Q}_D$, and $\overline{PD}_E$ represent the average valuation of housing, mortgage debt, deposits, and equity (respectively) over 1992–1996.

Note that our empirical implementation (19) also assumes that the discount rate in equation (18) is constant, $R_t = R$ and hence $R_{0 \to t}^{-1} = R^{-t}$. We set this discount rate to 5% (i.e., $R = 1.05$), which roughly corresponds to the average of the deposit and mortgage rates over 1992–1996 (i.e., five-year window around the start of our sample).

Computing these welfare gains requires data on (i) valuation ratios (to compare the actual valuations to a baseline) (ii) market value of financial transactions (at the individual level). We now discuss each of these data.

3.2 Data on valuations

We rely on publicly available data sources for asset prices. For interest rates on household debt and deposits (i.e., the inverse of the price of one-period bonds Q in the theory), we use *Statistics Norway*'s database on interest rates on loans and deposits offered by banks and mortgage companies.[20] Note that more than 90 percent of Norwegian mortgage debt has adjustable interest rates in our sample period, so that the year-to-year variation in bank-level interest rates

[20]The interest rate data are available on *Statistics Norway*'s web site https://www.ssb.no/en/statbank/table/08175/.

immediately affects households' interest costs.[21]

For the price-to-rent ratio (i.e., the price-dividend ratio for housing $PD_{H,t} = P_{H,t}/D_{H,t}$ in the theory) in the Norwegian housing market, we combine data from different sources. The best existing data is produced by *Eiendomsverdi* (EV), a private company that collects data on the housing market. Their data comes from registries of housing transactions, rental brokers, and the main Norwegian housing rental market place, `Finn.no`. However, EV's price-to-rent ratio is only available starting in 2012. We therefore combine two other indices, one for house prices and one for housing rents, to obtain our price-to-rent series in the years before 2012. The rental index comes from *Statistics Norway*, and is part of the official Consumer Price Index. The house price series comes from Norges Bank's project on Historical Monetary Statistics Eitrheim and Erlandsen (2005).[22] As these two series are indices, we scale their ratio so that in 2012, it equals EV's measure of the price-to-rent ratio.[23] In the results that follow, we use our constructed series for the years prior to 2012 and EV's series after 2012.

For equity valuation (i.e., the price-dividend ratio for equity $PD_{E,t} = P_{E,t}/D_{E,t}$ in the theory), we use an aggregate measure of cash flows over enterprise value (i.e., market value of equity plus debt) amongst publicly-listed Norwegian firms using data from *Worldscope*.[24] Note that, unlike the price-dividend ratio, our equity valuation ratio (i.e., an equity yield) is capital structure neutral: it does not depend on leverage. We handle the fact that firms have financial liabilities besides equity (such as debt for most firms and deposits for private banks) by allocating these indirectly-held assets to the equity holders (see Appendix B.2 for more details). Hence, our methodology implies that equity holders are exposed to asset price changes not only through changes in the valuation of firm (i.e., change in equity yield) but also through changes in interest rates, which affects how much of firm cash flows can distributed to equity holders (through dividends and stock buybacks) or invested (through retained earnings).

Figure 3 plots the yield of each asset class over time (i.e., $R_t - 1$ for debt and deposits and $D_{k,t}/P_{k,t}$ for long-lived assets $k = H$ and $k = E$), which are the inverse of the valuation ratios in Equation (19). All yields decline substantially over time (i.e., valuations increase), except for the equity yield, which remains roughly constant. The housing yield declines by 7 pp., the yield on mortgage debt declines by 4 pp., and the yield on deposits declines by 3pp. Note that the fact that equity valuations have remained stable in Norway is in sharp contrast with rising equity valuations in the U.S. (Greenwald et al., 2021).

[21] Mortgage contracts in Norway typically are annuity loans with 25-year repayment schedules. When interest rates change, the payment schedule adjusts so that the sum of monthly debt repayment and interest costs remains constant at a new level throughout the remaining period of the contract. Such adjustments happen frequently, normally whenever the Central Bank policy rate changes.

[22] This house price index is in turn obtained from combining data by the Norwegian Real Estate Broker's Association, the private consulting firm Econ Poyry, and listings at the main platform for house transactions `Finn.no`. Norges Bank updates these data regularly and provides them online, currently at `https://www.norges-bank.no/en/topics/Statistics/Historical-monetary-statistics/House-price-indices/`.

[23] Importantly, because all these three data series exist after 2012, we can use this most recent period to validate that our constructed price-to-rent series for the years before 2012 tracks the high-quality EV series after 2012. Indeed, we find no substantial difference between using EV's price-to-rent ratio or using our constructed alternative based on publicly available data for the years after 2012.

[24] The series is similar to other series of price-to-earnings series ratio, such as those produced by *Global Financial Data*.

To compute the welfare gains of asset price deviations, Equation (19) requires a measure of the relative difference between valuations at time t and their average baseline value (i.e., their averages over the 1992–1996 period). Figure 18 in Appendix B visualizes these price deviations.

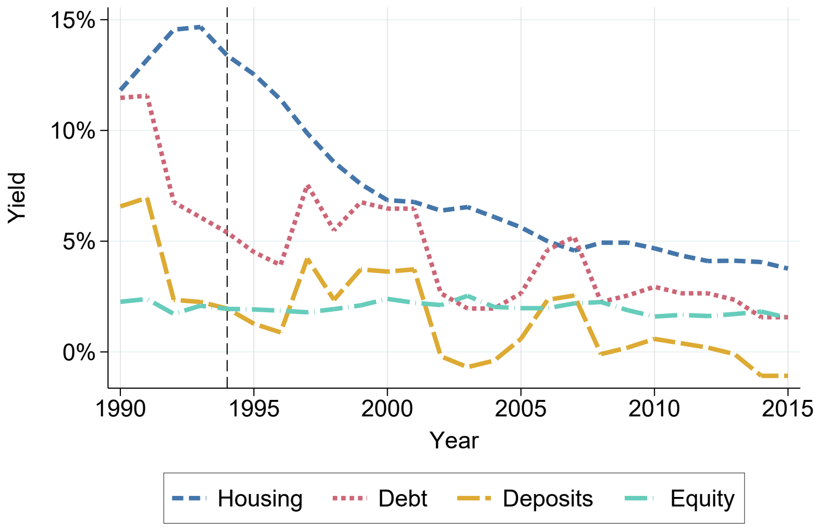


Figure 3: Evolution of yields in Norway

Notes. Figure 3 contains the yield of each asset class over time. For debt and deposit, the yield corresponds to the average real interest rate on mortgages and debt, respectively, as estimated by *Statistics Norway*. The housing yield corresponds to the rent-to-price ratio (see text for details). The equity yield corresponds to the aggregate ratio of cash flows to enterprise value amongst publicly-listed Norwegian firms from *Worldscope*.

3.3 Microdata on holdings and transactions

We combine data from a variety of Norwegian administrative registries that cover the universe of Norwegian households and the private businesses that they own from 1993 to 2015. These data come with identifiers at the individual, household, and firm level, as well as information on parent-children links. In particular, we use registries for individual tax payments, holdings of equity shares (listed and unlisted corporations), private business balance sheets, and housing transactions. Flow variables are measured annually, whereas assets and liabilities are valued at the end of the year. The data are uncensored (i.e., no top coding), and the only sources of attrition are mortality and emigration. The income and wealth data are largely third-party reported (i.e., employers and financial intermediaries) and scrutinized by the tax authority as they are used for tax purposes.

Holdings. On household balance sheets, we separately observe bank deposits, bond holdings (corporate, sovereign, mutual, and money market funds), debt, vehicles (cars and boats), stock mutual funds, listed stocks, private businesses, housing and other forms of estate holdings. With the stockholder registry we observe individual ownership shares in every corporation (including private businesses). In principle, we observe each individual's holdings, yet we aggregate the holdings data at the household level because this is the unit subject to wealth taxation in Norway.[25]

We construct five main variables that cover most of financial wealth: "debt" (mortgages, student loans, and unsecured credit); "deposits" (bank deposits and bonds); "housing" (principal residence, secondary homes, and recreational estates); "private equity" (equity in private businesses); "public equity" (listed stocks and stock funds). All of these variables are recorded at market value at the end of the year, except for private equity, which is a tax assessed value (i.e., value reported to the tax authority, which is typically higher than the book value of equity). For housing, we use a valuation approach that combines transaction data and registered housing characteristics to estimate a value for each house over our sample period (see Fagereng et al., 2020).

Some households own private businesses. These firms hold assets and liabilities directly, but in many cases also own shares in other firms. To properly account for households' ownership, we must therefore include their indirect asset positions held through private businesses. Our procedure is as follows. For each household, we compute their direct and indirect ownership of private businesses. For instance, if a household owns 80% of firm A, which in turn holds 50% of firm B, then the household owns 80% of firm A and 40% of firm B. Moreover, firm B might hold 25% of firm C, which then implies that the household owns 10% of C. We compute each household's indirect ownership by going through ten such layers of firm holdings.[26] Next, we allocate private business holdings to households by combining these ownership shares with firm balance sheet data. See Appendix B.2 for details.

Our notion of welfare gain can be interpreted as the present value of the deviation in consumption due to the deviation in asset prices (see Equation 14). It would therefore be natural to express it as a share of the present value of consumption. However, we do not observe consumption directly in our sample. Instead, in some exercises, we will scale the welfare gain by "total wealth", which is defined as the sum of financial wealth (i.e., debt, deposits, housing, and equity) and human wealth (i.e., the present value of future labor income plus net government transfers received between 1994 and 2015, discounted at 5 pp. annually).

Table 1 summarizes the data. Throughout the paper, we express all values in real terms (2011 Norwegian Krone using the CPI) and then convert them to US dollars using a fixed exchange rate of 5.607. In Appendix B.4, we show that our aggregated microdata matches publicly-available data on household wealth by asset almost exactly.

Transactions. Equation (19) highlights the fact that we need data on *holdings* for debt and deposits, and *net transactions* for housing and equity. For housing, we observe the annual value of market transactions in the housing market at the individual level. Thus, net transactions in housing are directly observed.

For public equities, we observe holdings at the beginning and end of the year and a price

[25] While financial holdings are *registered* at the individual level, they are *taxed* at the household level. Hence, the reported allocation of assets between individuals *within* the household can be somewhat arbitrary, as the tax authority does not scrutinize it.

[26] It is possible to compute ownership through more than ten layers. However, after going through four layers of firm holdings, further layers contribute only minuscule amounts to household equity.

Table 1: Household wealth in 1993 (thousands of USD)

	Mean	S.D.	P10	P50	P90	P99
Total wealth	556.12	725.57	190.72	522.98	894.13	1,633.28
Financial and real wealth	125.63	649.84	-9.76	88.75	276.37	765.50
Housing	130.57	218.21	0.00	109.78	272-24	625.35
Debt	43.79	131.81	0.00	20.54	110.81	264.52
Deposits	22.62	93.97	0.05	6.66	56.39	211.83
Public equity	2.70	431.41	0.00	0.00	0.18	24.42
Private equity	9.17	419.39	0.00	0.00	0.00	121.35
Human wealth	430.49	280.82	98.91	429.01	722.89	1,220.19
Present value of labor income	323.29	310.89	0.00	314.90	673.86	1,169.33
Present value of net transfers	107.20	98.84	13.20	75.59	245.09	411.20

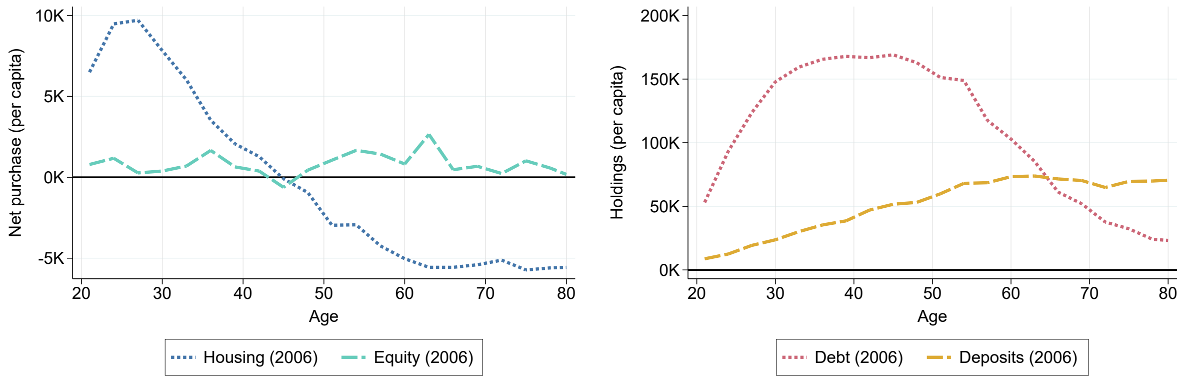
Notes. The total number of observations is 3,270,273. Values are reported in thousands of 2011 US dollars. Each statistic is computed for each variable separately.

index. We then compute a measure of unrealized capital gains by assuming that all transactions are in the same direction and uniformly distributed within a year. Net transactions are thus constructed as the change in market value minus imputed capital gains. The price index used for imputation differs between assets. For listed stocks, the method differs depending on the available information. Starting in 2005, we have information on individual stock ownership and use market prices on individual stocks to impute capital gains. Before 2005, we lack information on individual stock ownership and use capital gains from the financial accounts to impute capital gains on listed stocks at the individual level. We also use capital gains from the financial accounts to impute individual capital gains for mutual funds.

For equity in private businesses, we assume that it is completely illiquid. In principle, we observe how individual-level ownership shares in private businesses evolve over time, which allows us to back out an measure of net equity transactions. In practice, however, these transactions are extremely rare and we do not observe their market value. As a result, our assumption implies that private business owners are not directly exposed to private equity valuation changes. However, they are very much exposed to changes in interest rates through their firm's balance sheet. Indeed, for both public- and private-equity holders, we carefully quantify their exposure to changes in interest rates on debt and deposits, as well as changes in public equity and housing valuations. This is particularly, important for individuals at the top of the wealth distribution, who hold a lot of assets through their private firms. See Appendix B.2 for more details.

Bequest events pose two challenges when computing net transactions. First, housing transactions may be problematic at the time of death. In most cases, when an individual dies, the estate is transferred to heirs. In this case, heirs sell the property and net transactions are computed correctly. But in a few cases, parts of the estate is sold after death but before it is transferred to the heirs. In this case, we allocate the transaction to the deceased's living children, in accordance with the Norwegian inheritance law.[27]

Second, because our imputation of net transactions in equity is based on changes in holdings net of imputed capital gains, a bequest event may be problematic because transfers of wealth may be counted as transactions. For example, if one household gives 100 equity shares to another household, this should not be reported as a purchase by the recipient nor as a sale by the giver. To address this issue, we allocate all imputed equity transactions of givers to recipients when there is a bequest event. A bequest event is defined as any transfer reported in the inheritance tax registry (both inter vivo and at death).[28]



(a) Housing and equity

(b) Debt and deposits

Figure 4: Financial holdings and transactions by age group (2006)

Notes. Figure 14 plots net transactions per capita data by age in 2006 (thousands of 2011 US dollars).

Figure 14 plots the transaction data that enters the sufficient statistic (19) for the year 2006, which roughly represents the mid-point of our sample. Because this will be useful for understanding our empirical results regarding redistribution across cohorts in Section 4 below, we plot asset transactions versus age. Importantly (though unsurprisingly), young households are on average net buyers of housing whereas old households are net sellers.

4 Empirical findings

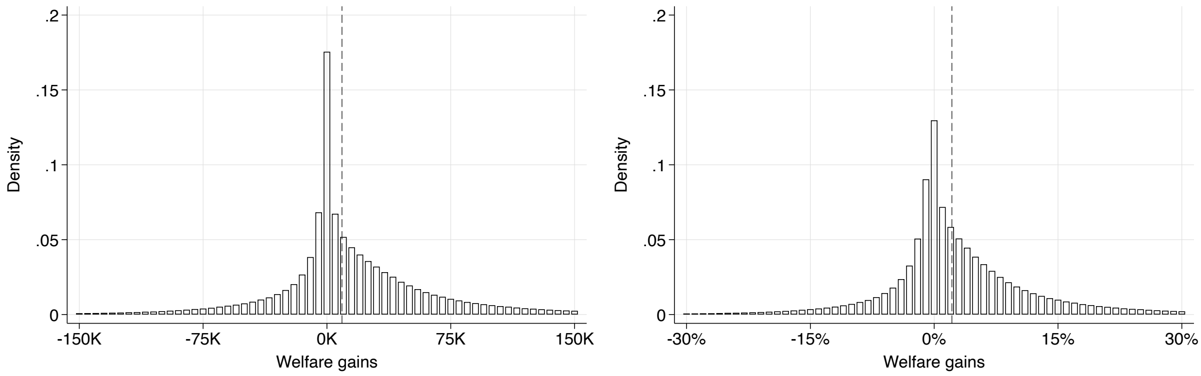
We now estimate our sufficient statistic (19) for all Norwegians who were at least 18 years old at some point between 1994 and 2015. We first describe the heterogeneity in welfare gains across individuals in Section 4.1. We then quantify redistribution across cohorts (Section 4.2) and across the wealth distribution (Section 4.3). Finally, we quantify the redistribution across sectors (i.e. households, government and foreigners) in Section 4.4.

[27] By the letter of the law, inheritance is split equally between all direct descendants unless otherwise is explicitly specified in a will.

[28] Before 2014, there was an inheritance tax in Norway and the tax authority collected information on sender, receiver, and the amount transacted. However, this register does not contain information on the types of assets transferred.

4.1 Redistribution across individuals

Welfare gains. We start by documenting the heterogeneity in welfare gains in the full population. Figure 5 reports the histogram for total welfare gains. As predicted by Section 2, the average welfare gain is close to zero. However, there is substantial heterogeneity: the welfare gain is $-\$35{,}000$ at the 10th percentile and \$80,000 at the 90th percentile, with a standard deviation of \$58,000. Similarly, it is $-\$280{,}000$ at the 1st percentile, \$1,000,000 at the 99th percentile, and $\$5,000{,}000$ at the 99.9th percentile (i.e., for the top 0.1%). Note that there is a large mass around zero, consistent with a large fraction of households having consumption approximately equal to income.



(a) Welfare gains in dollar terms

(b) Welfare gains as percent of total wealth

Figure 5: Distribution of welfare gains

Notes. Figure 5 plots the density of individual welfare gains, as defined in (19), across individuals in Norway. Panel (a) plots welfare gains in levels (in 2011 US dollars) while panel (b) plots welfare gains normalized by initial wealth, where initial wealth is defined as the sum of financial wealth and human capital (i.e., the present value of labor income earned and government benefits received from 1994 to 2015) in 1994.

To understand which asset class contributes the most to redistribution, Figure 6 decomposes the welfare gain at each percentile of the welfare gain distribution. Housing is by far the asset class that generates the most redistribution. However, notice that debt is also an important (and always positive) contributor, with a relatively large magnitude both at the top and at the bottom of the welfare gain distribution. In contrast, welfare gains due to equity are small, reflecting the fact that there is very little time variation in the Norwegian equity yield in this period (see Figure 3). Similarly, deposits make a very small (and always negative) contribution. Note that Figure 6 excludes the very top group (i.e., top 0.1%), for which the average welfare gain is roughly $\$5,000{,}000$, with a relatively high contribution of debt of $\$2,000{,}000$. These high welfare gains due to debt arise because some households own private firms that have a lot of debt on their balance sheet and thus benefit from declining interest rates.

Welfare gains as percent of total wealth. The dispersion in welfare gains across individuals may reflect dispersion in asset sales and purchases relative to initial wealth or simply dispersion in initial wealth (i.e. asset sales and purchases in dollar terms may simply scale with wealth). To disentangle between the two, we divide our measure of welfare gain by total wealth, defined as the sum of financial and human wealth (see Section 3.3). Recall that welfare

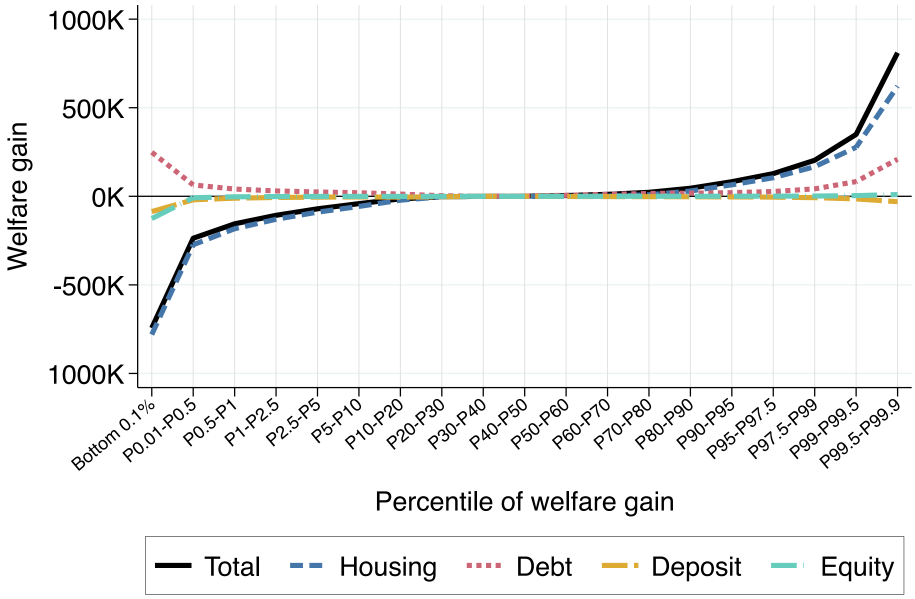


Figure 6: Decomposition of welfare gains by percentile

Notes. Figure 6 plots the decomposition of individual welfare gains, as defined in (19), across percentiles of the welfare gain distribution. The black line labelled "Total" plots the distribution of total welfare gains and losses in dollar terms, i.e. the same quantity as in Figure 5(a). The dashed colored lines for housing, debt, deposits and equity add up to the black line for the total effect. The figure excludes the top 0.1% group because the welfare gains of this group are so large as to make the figure hard to read. The top 0.1% welfare gain equals about \$5, 000,000 with about \$3, 000,000 due to housing and \$2, 000,000 due to debt.

gains can be interpreted as the present value of the change in consumption due to the deviation in asset prices (see Equation 14). As as consequence, this *normalized* version of welfare gains is a proxy for the change in consumption as a fraction of the present value of consumption.

Figure 5 shows significant heterogeneity in welfare gains, even after normalizing by initial wealth. Welfare gain is -6% at the 10th percentile and 15% at the 90th percentile, with a standard deviation of 11%. Similarly, it is -67% at the 1st percentile and 152% at the 99th percentile. The Kelly skewness is positive, at 0.33: while the median individual experiences a near-zero welfare gain, some individuals disproportionately benefit from the rise in asset prices.[29] Finally, the kurtosis of the distribution is approximately equal to 9, which is high compared to a normal distribution, whose kurtosis is 3, reflecting a larger mass in the tails relative to the normal distribution.

Wealth gains. How do *welfare* gains differ from *wealth* gains? Following the discussion in the two-period model of Section 2.1, we define wealth gains as the deviation in the wealth of individuals due to the deviation in asset prices:

$$\text{Wealth Gain} = \sum_{t=0}^{T} R_{0\to t}^{-1} \sum_{k=1}^{K} N_{k,t-1} P_{k,t-1} \Delta R_{k,t} + \sum_{t=1}^{T} R_{0\to t}^{-1} B_{t-1} Q_{t-1} \Delta R_t, \tag{20}$$

where $\Delta R_{k,t}$ denotes the deviation in the realized return at time t of asset k holding dividends $D_{k,t}$ constant, $\Delta R_{k,t} = (\Delta P_{kt} - R_t \Delta P_{k,t-1})/P_{k,t-1}$, and ΔR_t denotes the deviation in the return

[29]Kelly skewness is defined as $(\text{p90} + \text{p10} - 2 \times \text{p50})/(\text{p90} - \text{p10})$ where p10, p50, and p90 are the 10th, 50th and 90th percentiles of the distribution under consideration.

of one period bonds, $\Delta R_t = -\Delta Q_{t-1}$.

While wealth gains are identical to welfare gains for the one period bond, they differ for long-lived assets. As in the two-period model, this is because wealth gains only capture the positive effect of rising valuations on current returns, while welfare gains also take into account their negative effect on returns in the future.[30] In practice, wealth gains tend to overestimate welfare gains in a time of rising asset valuations.

Figure 7a compares the histograms of welfare and wealth gains, both normalized by initial wealth. The main observation is that, while welfare gains are centered close to zero, wealth gains are centered at a positive value (roughly 30% of wealth on average). This reflects the fact that wealth gains accrue to all asset holders while welfare gains only accrue to asset sellers. Also, the distribution of wealth gains tends to be less dispersed than the distribution of welfare gain (standard deviation, skewness, and kurtosis are all lower).

While this exercise shows that welfare and wealth gains have different densities, it is silent on the *ordinal* relationship between the two variables. To focus on this question, Figure 7b plots the average rank of welfare gains in terms of the rank of wealth gains. If welfare gains are monotonically related to wealth gains, the result should be a 45° line from 0 to 1. Conversely, if welfare gains are unrelated to wealth gain, the result should be a horizontal line at 0.5. Reality is somewhere in-between: empirically some individuals with large asset positions buy and hence loose in welfare terms; conversely, others with small positions sell and hence win. This finding also shows up in the wide bands for the 10th and 90th percentile welfare gains: within any given wealth gain rank, some individuals experience a very low welfare gain and others experience a very high one.

4.2 Redistribution across cohorts

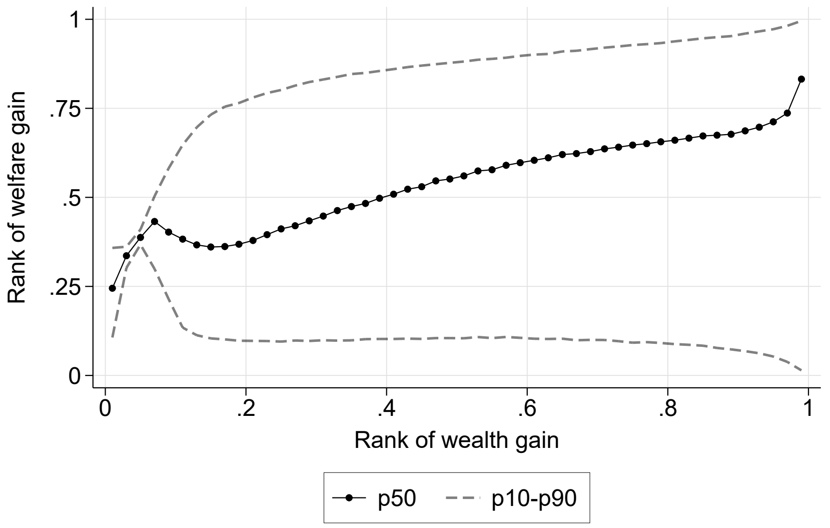
Welfare gains. In the previous section, we documented a large amount of heterogeneity in welfare gains across individuals. We now focus on describing the heterogeneity in welfare gains by observable characteristics. One natural characteristic is age. Indeed, the existing literature on household finance has documented large differences in portfolio holdings over the life cycle (e.g., Flavin and Yamashita, 2011; Cocco et al., 2005). This heterogeneity may naturally generate heterogeneity in trading, and, therefore, in welfare gains.

Figure 8 contains the average welfare gain for different cohorts, indexed by the age of individuals in the cohort in 1994. The main pattern is that welfare gains are negative for the young and positive for the old: rising asset prices redistribute welfare from the young towards the old. This is consistent with standard life cycle models of savings: the young save for retirement by purchasing financial assets while the old sell their financial assets to consume.

Quantitatively, the average welfare gain is approximately −\$15,000 for individuals below 15 years old in 1994 (Millennials), and around \$30,000 for individuals above 50 years old in 1994 (Baby boomers). Decomposing the welfare gains into the contribution of each asset class reveals interesting patterns. On the one hand, consistent with the fact that the young tend to

[30] See Appendix A.5 for a formal expression for the difference between welfare and wealth gains.

(a) Density of normalized welfare gains versus normalized wealth gains



(b) Rank of welfare gain in terms of the rank of wealth gains

Figure 7: Welfare versus wealth gains

Notes. Figure 7a plots the density of welfare gains defined in (19), in black lines, and the density of wealth gains defined in (20), in grey shading, across individuals in Norway. Welfare and wealth gains are normalized by initial wealth, defined as the sum of financial wealth and human capital (e.g. the present value of labor income earned and government benefits received from 1994 to 2015) in 1994. Figure 7b plots the average rank of welfare gains in terms of the rank of wealth gains.

buy houses from the old, higher house prices redistribute from young to old. On the other hand, consistent with the fact that the young tends to borrow from the old, lower mortgage rate redistribute from old to young.[31] Overall, the effect of higher house prices dominates the effect of lower mortgage rates for two reasons. First, as one can see from Figure 3, the yields of house prices decreased more than the interest rate on debt. Second, as young people build equity in their houses, they decrease their mortgage balances over time, which means that they benefit relatively less from the decline in mortgage rates as they age.

[31] As we discuss in Section 4.4, the household sector as a whole is a net debtor. Therefore, the young do not borrow only from the old, but also from another sector of the economy, which turns out to the government sector.

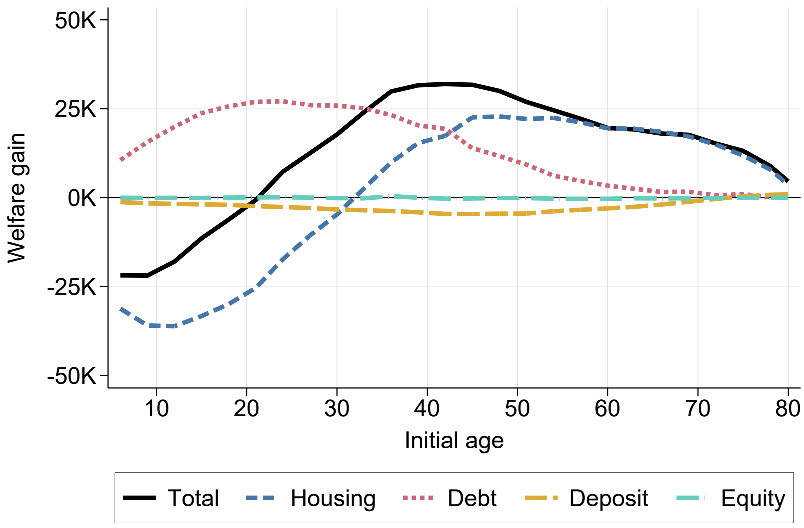


Figure 8: Welfare across cohorts

Notes. Figure 8 plots the average welfare gain (19) for individuals in each cohort. Cohorts are indexed by the age of individuals in 1994. All quantities in 2011 US dollars.

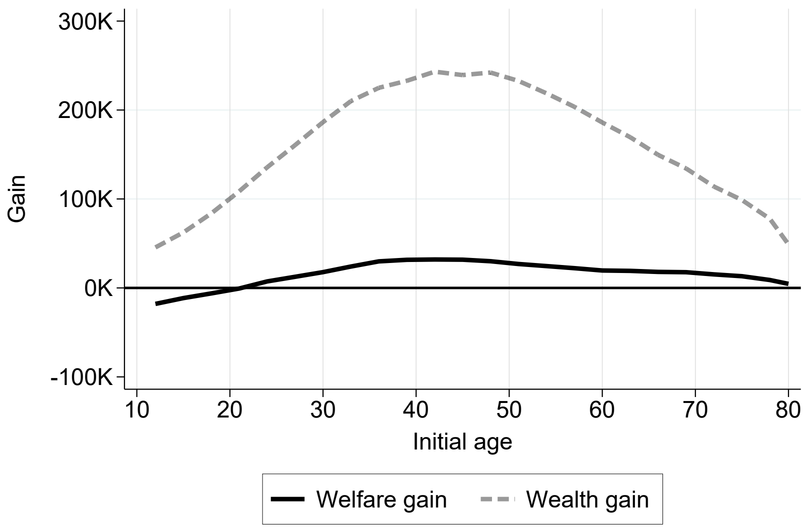


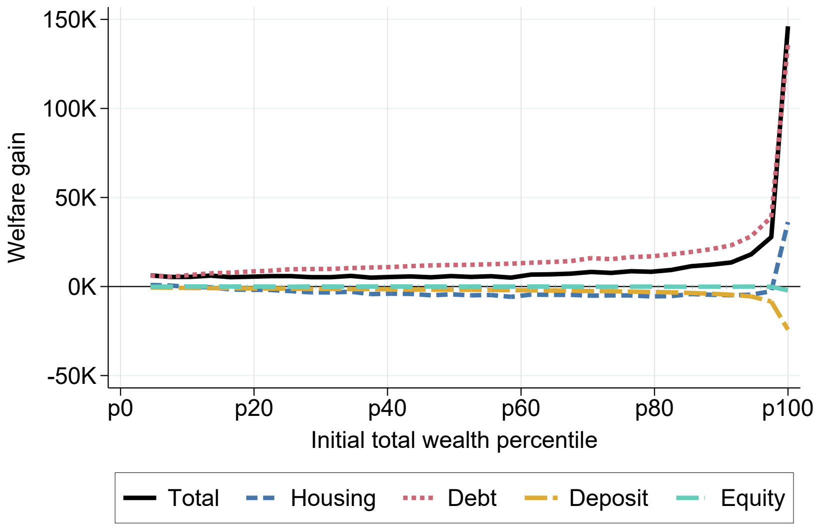
Figure 9: Welfare versus welfare gains across cohorts

Notes. Figure 9 plots the average wealth gain, as defined in (19), and the average wealth gain, as defined in (20), for each cohort in Norway. Cohorts are indexed by the age of individuals in 1994. Units are 2011 US dollars.

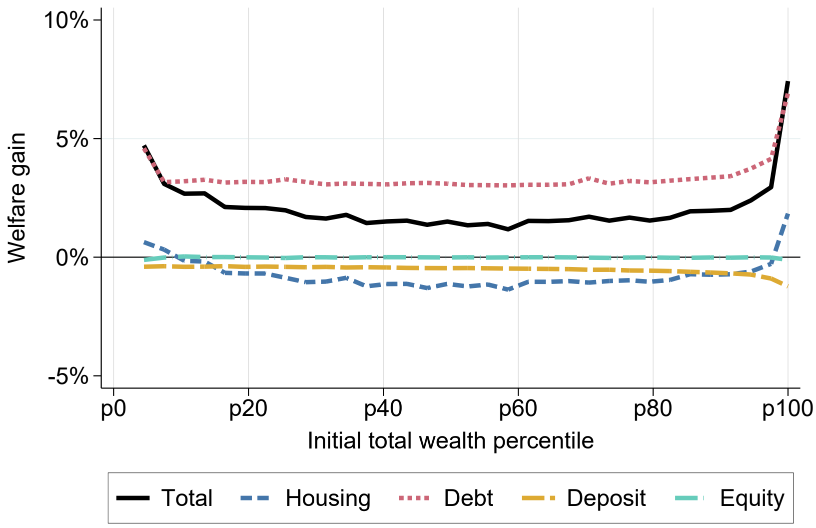
Wealth gains. As in the previous section, we now compare welfare and wealth gains across cohorts. Figure 9 plots the average wealth gain and the average welfare gain for each cohort in our sample. There are two main observations. First, in contrast with welfare gains, wealth gains are positive for everyone, reflecting the overall rise in valuations during the time period. Second, while welfare gains converge to zero as age increases, wealth gains remain large even for 80-year olds. This reflects the fact that, while assets held by these individuals increased in value, these wealth gains did not correspond to any welfare gains as these assets are, for the most part, passed to their children.

4.3 Redistribution across the wealth distribution

A growing literature has emphasized the fact that rising asset valuations affect the distribution of wealth (e.g., Kuhn et al., 2020; Gomez, 2016; Greenwald et al., 2021). A natural question is: are these wealth gains actually welfare gains? To answer this question, we compare wealth and welfare gains along the wealth distribution. To examine the heterogeneity in welfare gains along the wealth distribution, we rank individuals in 1994 according to their total wealth *within their cohort*. By doing so, we isolate the effect of initial wealth from the effect of age.



(a) Level



(b) Normalized by initial wealth

Figure 10: Welfare gains across wealth percentiles

Notes. Figure 10a plots the average welfare gain, as defined in (19), for each wealth percentile in Norway. Wealth percentiles are constructed by ranking individuals within each cohort with respect to total wealth, defined as the sum of financial wealth and human capital (e.g. the present value of labor income earned in our sample) in 1994. Figure 10b plots the same quantities normalized by wealth. Units are in 2011 US dollars.

Figure 10a contains the average welfare gain for each wealth percentile. The main pattern is that wealthier individuals do benefit more from higher asset prices. For instance, individuals in the top 1% of their cohort in terms of total wealth experience a welfare gain of roughly $125,000 on average, which is much higher than the population average of $7,000. Figure 10b shows that, as a fraction of total wealth, welfare gains are u-shaped. Focusing on the contribution of different asset classes reveals interesting patterns. As a proportion of total wealth, both poor and wealthy individuals tend to be net seller of houses, which explains why they are not hurt from rising house prices. Also, wealthy households benefit much more from declining interest rates on debt due to their higher leverage.

Finally, Figure 11 contrasts welfare gains to wealth gains along the wealth distribution. The main finding is that wealth gains are an order of magnitude larger than the actual welfare gains. For instance, the welfare gain for the top 1% is around $125,000, while the wealth gains are nearly $750,000. Overall, our results indicate that welfare gains in the right tail of the wealth distribution have been much smaller than the wealth gains.

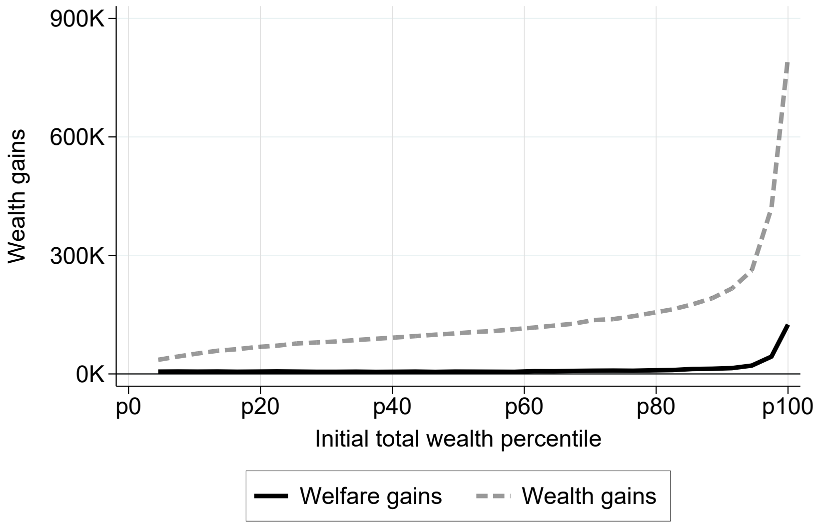


Figure 11: Welfare versus wealth gains across wealth percentiles

Notes. Figure 11 plots the average individual welfare gain, defined in (19), and the average wealth gain, defined in (20), for each wealth percentile in Norway. Wealth is defined as the sum of financial wealth and human capital (e.g. the present value of labor income earned in our sample) in 1994. Units are 2011 US dollars.

4.4 Redistribution across sectors

When all trades are between households, the welfare gain of the household sector as a whole must be zero (see Corollary 2). The logic is that for every household selling an asset that appreciated in value, there is an offsetting household purchasing it. In practice, however, households routinely trade assets with other non-household entities. We now conduct a systematic sectoral investigation of the redistributive effect of asset price changes. To do so, we group all entities in the economy into three sectors: households (H), the government (G), and foreigners (F).

The key accounting identity that we use is that every financial asset is a liability for one sector and an asset for another sector. With this in mind, it is immediate that in a multisector economy, Corollary 2 becomes

$$\text{Welfare Gain}_{\text{H}} + \text{Welfare Gain}_{\text{G}} + \text{Welfare Gain}_{\text{F}} = 0, \tag{21}$$

where the sector-level welfare gain is defined analogously to Equation (19). In words, a positive welfare gain for the household sector must be exactly offset by a welfare loss in another sector.

Sectoral data. We use publicly available data from the *Financial Accounts*, which covers all holdings and transactions of financial assets in the Norwegian economy. For our analysis, we combine the government sector with the central bank and the non-profit sector. Hence, we use the term "government" liberally to include all entities that serve the household sector. Importantly, the government has a sovereign wealth fund—the *Norwegian Pension Fund Global*—which invests in foreign assets, financed by income taxes on the energy (oil and gas) sector.[32] Consistent with what we do in the microdata, we also consolidate the business sector with its ultimate owners.

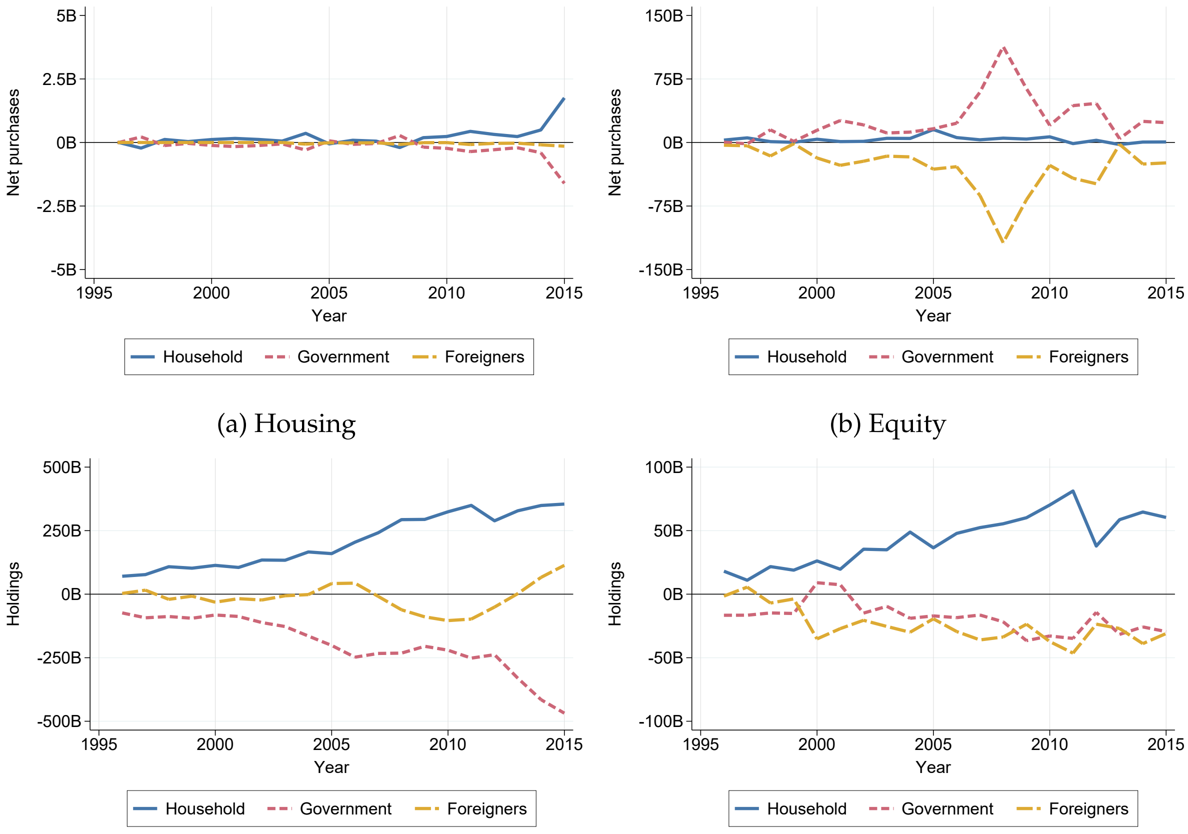
Housing transactions are not recorded in the Financial Accounts. We augment the Financial Accounts with between-sector housing transactions, which we construct by aggregating the housing transaction registry data described in Section 3.3. (See Appendix B.3 for more details on the data construction.)

The resulting dataset covers the total amount of asset holdings and transactions for three sectors (i.e., household, government, and foreigners) and four asset classes (i.e., housing, deposits, debt, equity) over the 1996–2015 period. As we show in Appendix B.3 Figure 20, our aggregated microdata aligns closely with the Financial Accounts data.

Transactions and holdings by sector. Before we quantify the welfare gain by sector, we briefly discuss the main pattern of housing and equity transactions as well as debt and deposit holdings across sectors. Notice that the levels of net housing purchases across sectors are very low (less than 2B per year in absolute value, see Figure 12a). The reason is that most housing transactions are within the household sector, with minimal transactions between the government and the household sector.

Regarding equity purchases, households have a positive but small level of net equity purchase on average (see 12b). In contrast, the government is a net buyer of foreign equities through the sovereign wealth fund described above. Note that the government's large positive purchases at the height of the great financial crisis reflect the fund's mandated portfolio rebalancing from fixed income assets to equities from mid 2007 to early 2009 (see Footnote 32).

[32] Over our sample period, the fund's value grew from approximately zero in 1997 to approximately one 1B$ in 2015. Its portfolio mandate first prescribed 40 percent equities and 60 percent fixed income assets. In 2007 this was changed to 60 percent equities. The rebalancing transition was implemented over 20 months. In 2010, the fund's portfolio was extended to real estate with a 5 percent weight, and the fixed income share was cut to 35. A fiscal policy rule states that the expected real rate of return, first 4% and since 2017 3%, of the current fund value can be spent over the national budget each year. As the fund grew over our sample period, so did government spending. Details regarding the fund's mandate and investment strategy are provided at https://www.nbim.no/en/the-fund/how-we-invest.



(a) Housing (b) Equity

(c) Debt (d) Deposits

Figure 12: Financial transactions by sector (1996–2015)

Notes. Figure 12 plots net transactions per capita data that enters the sufficient statistic (19) by sector (thousands of 2011 US dollars).

Figure 12c reveals that the household sector as a whole has a large and growing level of debt. Moreover, this debt level is approximately equal to the government's net holding of debt securities, with the foreign sector being a marginal holder towards the end of our sample. While households do not typically borrow directly from the government, in terms of welfare redistribution, it has the same effect: a decline in interest rates redistribute from the government towards households.[33]

A similar pattern holds for deposits, although the magnitudes are much smaller. The household sector is a net holder of bank deposits, while the government and foreign sector indirectly hold these bank deposits as liabilities. The reason is that bank deposits are a liability for the banking sector, and since the government and foreigners are important holders of financial business equity, they are ultimately liable for interest payments on these deposits.

[33]Most of household debt is mortgages, who are then securitized into mortgage bonds by private banks. Then, these bonds are for the most part sold to domestic pension funds as well as foreign investors. These foreign bond holdings approximately cancel out against the sovereign wealth fund's bond holdings, which explains why the net foreign debt position is close to zero in Figure 12c. The sovereign wealth fund's holding of foreign bonds then account for most of the government's net holding of debt securities, while a small fraction are held by other public pension funds that invest domestically. The main domestic public pension funds are *Folketrygdfondet* and *Kommunenes Landspensjonskasse* (see Bank (2021) for an overview of Norway's financial system).

Results. Figure 13 presents the welfare gains across sectors, where all numbers are per capita (i.e., scaled by the number of individuals in Norway in 1994). We use the same welfare gain formula and valuation ratios as before. One caveat with this approach is that we implicitely assign the same price deviation for foreign and Norwegian assets.

First, the household sector as a whole has a positive welfare gain of roughly \$7,000 per household. Breaking down the welfare gain by asset class, we find that there is a large positive contribution of debt (\$8,000) and a small contribution of deposits (−\$1,000). Housing and equity purchases have negligible contributions (<\$1,000 in absolute value). The positive welfare gain is therefore entirely due to declining interest rates, which has been beneficial to households since they are net debtors (i.e., their debt exceeds their bank deposits).

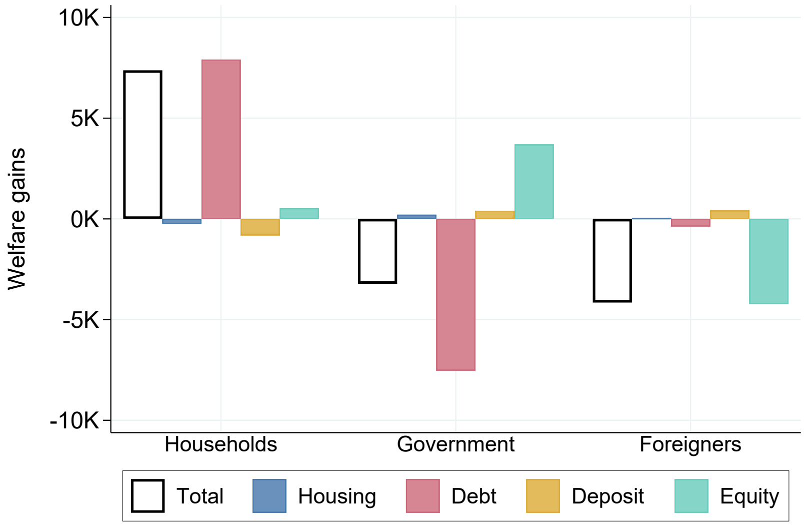


Figure 13: Welfare gains across sectors

Notes. Figure 13 contains the welfare gain for each sector of the economy, as well as the contribution of each asset class. To make it comparable to the other figures in our paper, the aggregate welfare gains of each sector is divided by the number of individuals in Norway. Units are 2011 US dollars.

If households as a whole have experienced a positive welfare gain, who is the counterparty that was hurt? For the most part, it was the government. As discussed earlier, the government is a net saver and is thus hurt by declining interest rates. Overall, the welfare loss of the government is −\$4,000, with a large contribution of debt (−\$8,000) and small positive contribution of equity (\$4,000). The contributions of deposits and housing are negligible (<\$1,000 in absolute value). On the other side, the government has benefited from purchasing foreign equity in 2008, which partially offsets the welfare loss from holding debt securities.

As discussed in Section 2.4, the welfare loss of the government represents a loss of real resources available for net transfers to the household sector. Substracting the government welfare loss (\$4,000) from the household welfare gain (\$8,000), we find that domestic sectors experienced a welfare gain of \$4,000 at the expense of foreigners. While it is beyond the scope of this paper to quantify how the Norwegian government has (and will) adjust net transfers in response to persistently lower interest rates, it is entirely possible that the very households who experienced negative welfare gains (i.e., the young) will also be the one that will bear the brunt of future reductions in government net transfers such as pension benefits.

5 Robustness

Our sufficient statistic relies on a first-order approximation (i.e., we apply the envelope theorem) as well as a truncation (i.e., our sample ends in 2015). In this section, we conduct a number of analyses to probe the robustness of our main results to relaxing these strong assumptions.

5.1 Second-order approximation of welfare gains

Our measure of welfare gains defined in Proposition 1 is exact only for an infinitesimal deviation in asset prices. For non-infinitesimal price deviations, it only corresponds to a first-order approximation of the true welfare gain. The price deviations we observe in the data are quite large. Are these higher-order effects important?

To answer this question, we now formally model a non-infinitesimal deviation in asset prices. The deviation is now indexed by $\theta \in [0,1]$, where $\{P_t(0)\}_{t=0}^{\infty}$ corresponds to the initial price path and $\{P_t(1)\}_{t=0}^{\infty}$ corresponds to the perturbed price path.

Using Proposition 1, we can write the welfare gains between $\theta = 0$ and $\theta = 1$ as the integral of the welfare effect of infinitesimal deviations in prices:

$$\text{Welfare Gain} = \int_0^1 \sum_{t=0}^{\infty} R_{0\to t}(\theta)^{-1} \left(\sum_{k=1}^{K} (N_{k,t-1}(\theta) - N_{k,t}(\theta))\, \mathrm{d}P_{k,t}(\theta) - B_t(\theta)\, \mathrm{d}Q_t(\theta) \right) \mathrm{d}\theta.$$

For a variable x, denote by $\Delta x = x(1) - x(0)$ the difference between the value of the variable under the perturbed price path and in the initial price path. A trapezoidal approximation gives us the following second-order approximation for the welfare gain:

$$\begin{aligned}\text{Welfare Gain} \approx & \sum_{t=0}^{\infty} \left(R_{0\to t} + \frac{\Delta R_{0\to t}}{2} \right)^{-1} \\ & \times \left(\sum_{k=1}^{K} \left(N_{k,t-1} - N_{k,t} + \frac{\Delta(N_{k,t-1} - N_{k,t})}{2} \right) \Delta P_{k,t} - \left(B_t + \frac{\Delta B_t}{2} \right) \Delta Q_t \right)\end{aligned} \tag{22}$$

In contrast to the first-order approximation in (16), this second-order approximation takes into account the welfare gains and losses due to asset transactions responding to asset price changes (e.g., welfare gains from "timing the market" and portfolio reshuffling).[34] For instance, households who sell in response to increasing asset prices, $\Delta(N_{k,t-1} - N_{k,t}) > 0$ when $\Delta P_{k,t} > 0$, or those who buy in response to declining prices, $\Delta(N_{k,t-1} - N_{k,t}) < 0$ when $\Delta P_{k,t} < 0$, benefit in welfare terms. Note that our first-order approximation is valid at the second-order only if financial transactions do not react to price deviations, $\Delta(N_{k,t} - N_{k,t-1}) = \Delta B_t = 0$.

In terms of its empirical implementation, the second-order approximation requires additional assumptions. In contrast to the implementation of the first-order approximation (16),

[34] Martínez-Toledano (2022) empirically studies the implications of "timing the market" and portfolio reshuffling for the evolution of wealth inequality.

we now need to know how financial transactions would have changed if valuations had remained at their 1994 level.

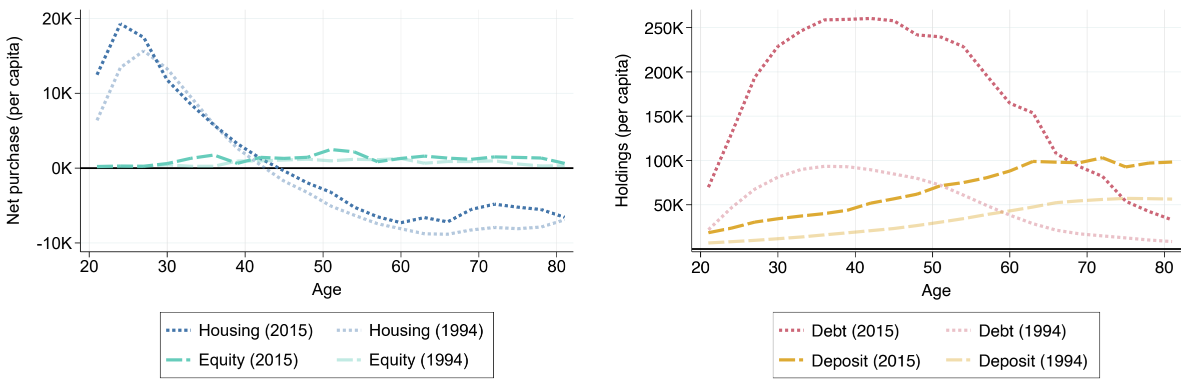
One assumption is that, had valuations remained at their 1994 level, the behavior of a 30-year old in 2015 would be the same as the behavior of 30-year olds in 1994, after accounting for economic growth.[35] Formally, we assume

$$\begin{aligned} N_{k,t-1} - N_{k,t} + \Delta(N_{k,t-1} - N_{k,t}) &= G^t(N_{k,-1} - N_{k,0}), \\ B_t + \Delta B_t &= G^t B_0, \end{aligned} \tag{23}$$

where $G = 1.01$ denotes the real per-capita growth rate of the economy in our sample period.

To examine the effect of this assumption, Figure 14a compares $(N_{k,t} - N_{k,t-1})\, P_{k,t}$ with $G^t(N_{k,0} - N_{k,-1})P_{k,0}$. One result is that real net housing purchases per cohort remains roughly constant over time, despite the rise in house prices during the time period. In contrast, real equity purchases appear to have somewhat increased, albeit from a low initial level. Figure 14b compares B_t with $G^t B_0$. Net debt (debt minus deposits) has increased much more rapidly than one could expect from the growth of the economy. Intuitively, this reflects the fact that the young must now borrow more in order to finance the purchase of houses whose values have increased much faster than the economy.

Figure 15 uses these numbers to compute the welfare effects at the second order, under assumption (23). The main effect of the second-order adjustment is to decrease the welfare gain associated with declining interest rates on debt. Intuitively, this reflects the fact that, if house prices were closer to their 1994 valuations, the buyers would have lower mortgage balances, and, therefore, would benefit less from low mortgage rates. However, this adjustment is small and the results are quantitatively similar to our first-order approximation.



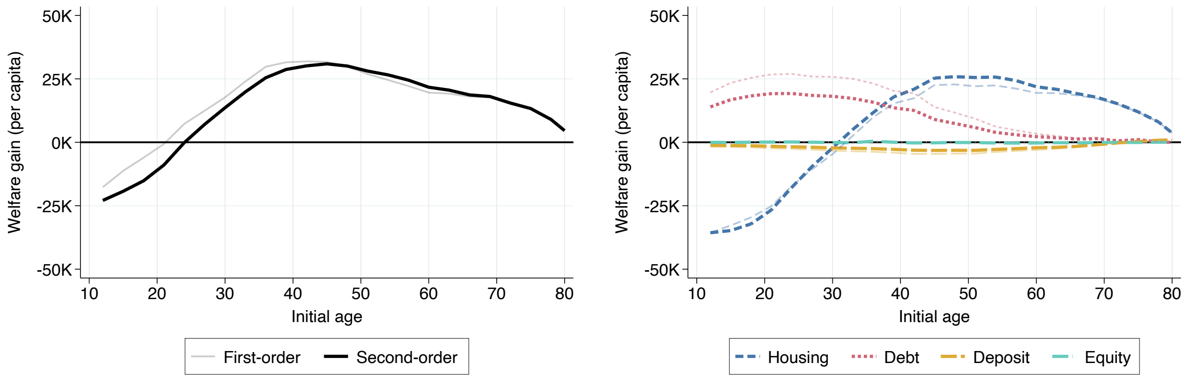
(a) Housing and equity

(b) Debt and deposits

Figure 14: Transactions in 1994 versus 2015 (adjusted for economic growth)

Notes. Figure 14 plots transactions in 1994 versus 2015, after adjusting them for economy growth. Formally, Figure 14a plots $(N_{k,T} - N_{k,T-1})P_{k,T}$ and $G^T(N_{k,0} - N_{k,-1})P_{k,T}$ while Figure 14b plots $B_T Q_T$ and $G^T B_0 Q_T$. Units are 2011 US dollars.

[35]The assumption is that the asset demand curve of Norwegian households remains the same over time. It is equivalent to assuming that the rise in asset prices is purely driven by a shift in asset demand from the government or foreigners, rather than a shift in asset demand from domestic households.



(a) Total (b) Detail by asset class

Figure 15: Welfare gains at the second-order

Notes. Figure 15 plots the average welfare gain at the first-order and at the second-order (22) for individuals in each cohort. The second-order approximation is constructed using Assumption (23), which says that, if valuations were back to their level of 1994, households would trade the same quantity of assets as in 1994. Units are 2011 US dollars.

5.2 Welfare gains with extrapolated valuation changes

Our measure of welfare gains in Proposition 1 expresses the welfare gains as the present value of all future transactions, interacted with the deviation in prices. However, as discussed in Section 3, we only apply our formula on a finite sample that ends in year $T = 2015$. Therefore, our formula should be interpreted as the welfare gain associated with a path of price deviations that ends in 2015 (i.e., valuations revert to the baseline afterwards). In equation (17) and the example in Figure 2, asset prices $P_{k,t}$ revert back to their baseline $\overline{PD}_k \times D_{k,t}$ in 2015.

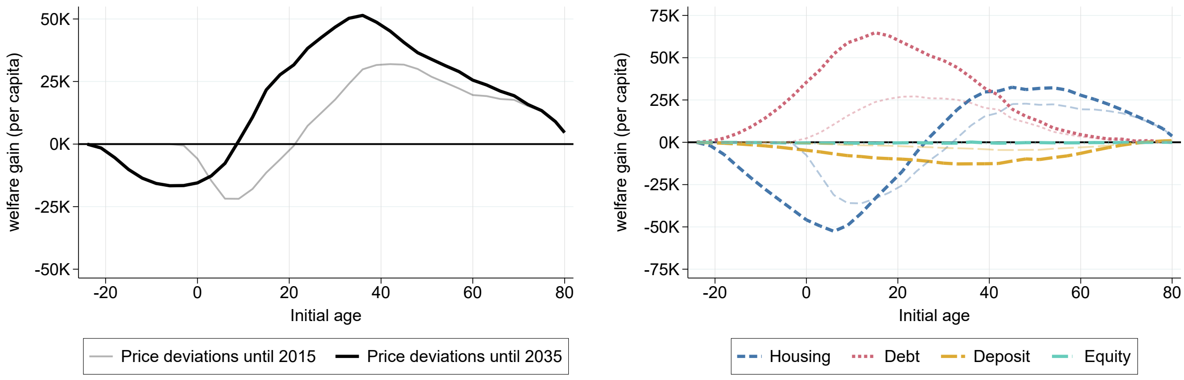
An alternative exercise is to compute the welfare gains of valuations remaining permanently at their 2015 level. To do this exercise, we need to impute the transactions in future years. To do so, we simply assume that future transactions in each cohort will equal the transaction of the cohort with the same age in 2015, after adjusting for economic growth. This assumption is motivated by the fact that, as discussed above, transactions by age groups have remained remarkably stable over our sample period (Figure 14).

Formally, we assume that, for $t \geq T$, we have:

$$\begin{aligned} (N_{k,t} - N_{k,t-1})P_{k,t} &= G^{t-T}(N_{k,T} - N_{k,T-1})P_{k,T}, \\ B_t Q_t &= G^{t-T} B_T Q_T, \end{aligned} \tag{24}$$

where $(N_{k,T} - N_{k,T-1})P_{k,T}$ and $B_T Q_T$ denote asset transactions and bond holdings in year $T = 2015$ and, as before, $G = 1.01$ denotes the real per-capita growth rate of the economy in our sample period.

Figure 16 plots the average welfare gain in each cohort after doing these imputations. The main effect of a permanent rise in valuations is to shift the graph of welfare gains to the left; that is, to redistribute welfare to existing generations at the expense of unborn generations.



(a) Total

(b) Detail by asset classes

Figure 16: Welfare gains with permanent price deviations

Notes. Figure 16 plots the average welfare gain in each cohort for a transitory versus a permanent change in asset prices. Units are 2015 US dollars. Data from Statistics Norway.

6 Conclusion

The main contribution of our paper is to provide a simple framework to quantify the welfare effects of historical asset price fluctuations. Two economic ideas lie at the core of our sufficient statistic approach. First, rising asset prices benefit prospective sellers and harm prospective buyers. Second, because there is a seller for every buyer, they are also purely redistributive. We implement our sufficient statistic formula using administrative data on financial transactions to quantify welfare gains and losses in Norway for the time period 1994 to 2015.

Our empirical implementation generates four main findings. First, rising asset prices had large redistributive effects, i.e., they resulted in some significant welfare gains and losses. At the same time, welfare gains differed substantially from naïvely calculated wealth gains and so did the identity of winners and losers implied by the two approaches. Second, rising asset prices redistributed across cohorts, with the old benefiting at the expense of the young. Third, they redistributed across the wealth distribution, from the poor toward the wealthy. Fourth, they also redistributed across sectors: declining interest rates benefited households at the expense of the government.

We hope that our sufficient statistic approach will also prove useful in other contexts. For example, it could be used to study the welfare consequences of higher-frequency asset price booms and busts rather than the longer-run trends considered here. Work by Kuhn et al. (2020), Martínez-Toledano (2022), Gomez (2016), and Cioffi (2021) has emphasized the importance of asset price fluctuations for wealth inequality dynamics — quantifying the resulting welfare effects would be a valuable exercise.

Finally, the result that rising asset prices benefit asset sellers rather than asset holders raises a number of questions for optimal tax theory. It suggests that taxing wealth or unrealized capital gains (as under the Wyden "Billionaires Income Tax" proposal) may be undesirable from a normative perspective. When asset prices rise, such taxes can redistribute "in the wrong direction": they hit not only households who benefit in welfare terms (those who sell their assets) but also those whose welfare is unaffected or declines (those who do not sell or perhaps even buy). Are there other forms of taxes that are closer to optimal? Perhaps even the existing practice of taxing capital gains on realization (i.e., when a sale occurs) rather than accrual? Answering such questions requires studying environments with changing asset prices using the tools from public finance. Ongoing work by Aguiar et al. (2022) takes some steps in this direction.

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Appendix

A Theory Appendix

A.1 Proofs

Proof of Proposition 1. The proof is an application of the Envelope theorem. The Lagrangian associated with the household problem is

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty} \beta^t U(C_t) \\ &+ \sum_{t=0}^{\infty} \lambda_t \left(\sum_{k=1}^{K} N_{k,t-1} D_t + B_{t-1} + Y_t - C_t - \sum_{k=1}^{K} (N_{k,t} - N_{k,t-1}) P_{k,t} - \sum_{k=1}^{K} \chi_k (N_{k,t} - N_{k,t-1}) - B_t Q_t \right).\end{aligned}$$

The first order conditions are

$$\begin{aligned} \beta^t U'(C_t) &= \lambda_t && (\partial \mathcal{L} / \partial C_t = 0) \\ \lambda_t \Big(\chi_k'(N_t - N_{t-1}) + P_t \Big) &= \lambda_{t+1} \Big(D_{t+1} + \chi_k'(N_{t+1} - N_t) + P_{t+1} \Big) && (\partial \mathcal{L} / \partial N_{kt} = 0) \\ \lambda_t Q_t &= \lambda_{t+1} && (\partial \mathcal{L} / \partial B_t = 0) \end{aligned}$$

Using the Envelope theorem, the infinitesimal change in the value function is given by the infinitesimal change in the Lagrangian:

$$\begin{aligned} \mathrm{d}V &= \sum_{t=0}^{\infty} \left(\sum_{k=1}^{K} \frac{\partial \mathcal{L}}{\partial P_{k,t}} \,\mathrm{d}P_{k,t} + \frac{\partial \mathcal{L}}{\partial Q_t} \,\mathrm{d}Q_t \right) \\ &= \sum_{t=0}^{\infty} \lambda_t \left(\sum_{k=1}^{K} (N_{k,t-1} - N_{k,t}) \,\mathrm{d}P_{k,t} - B_t \,\mathrm{d}Q_t \right) \\ &= \lambda_0 \sum_{t=0}^{\infty} (Q_0 \dots Q_{t-1}) \left(\sum_{k=1}^{K} (N_{k,t-1} - N_{k,t}) \,\mathrm{d}P_{k,t} - B_t \,\mathrm{d}Q_t \right) \\ &= U'(C_0) \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(\sum_{k=1}^{K} (N_{k,t-1} - N_{k,t}) \,\mathrm{d}P_{k,t} - B_t \,\mathrm{d}Q_t \right). \end{aligned}$$

The third equality uses the first-order conditions for B_t, $\lambda_{t+1} = \lambda_t Q_t$, which implies $\lambda_t = \lambda_0 \times Q_0 \dots Q_{t-1}$. The fourth equality uses the first-order conditions for C_t as well as the definition of the cumulative return $R_{0\to t}^{-1} = R_0^{-1} \dots R_{t-1}^{-1} = Q_0 \dots Q_{t-1}$. □

A.2 Interpretation of Price Deviations as Price Changes due to Discount Rates

As explained in Section 2.3 when implementing Proposition 1, we construct the empirical price deviations $\Delta P_{k,t}$ as deviations of asset prices from those that would arise under a constant price-dividend ratio (see equation (17) and Figure 2). We remarked that, under some assumptions, these price deviations isolate variations in asset prices due to variations in discount rates (i.e., pure valuation effects without changes in cashflows).

We now flesh out the underlying logic. We first consider a deterministic case with constant dividend growth and then a stochastic environment in which dividends follow a random walk.

A.2.1 Deterministic Case

Consider the baseline model, which has multiple long-lived assets $k = 1, ..., K$. However, we now drop the k subscripts for notational simplicity. In the main text, we considered an environment with exogenously given time paths for cashflows (dividends) and prices $\{D_t, P_t\}_{t=0}^{\infty}$. We now instead adopt the perspective common in the asset pricing literature to treat prices as determined by cashflows and discount rates $\{D_t, \mathcal{R}_t\}_{t=0}^{\infty}$ where the discount rate $\mathcal{R}_t$ will be defined more precisely momentarily.

More precisely, we assume that the sequence of asset prices $\{P_t\}_{t=0}^{\infty}$ satisfies the recursion $P_t = (D_{t+1} + P_{t+1})/\mathcal{R}_t$ where $\mathcal{R}_t$ is the discount rate between time t and $t+1$ and where the sequences $\{D_t, \mathcal{R}_t\}_{t=0}^{\infty}$ are exogenously given. Assuming a no-bubble condition, the asset price P_t therefore equals the present-discounted value of dividends

$$P_t = \frac{D_{t+1}}{\mathcal{R}_t} + \frac{D_{t+2}}{\mathcal{R}_t \mathcal{R}_{t+1}} + \frac{D_{t+3}}{\mathcal{R}_t \mathcal{R}_{t+1} \mathcal{R}_{t+2}} + \cdots = \sum_{s=1}^{\infty} \mathcal{R}_{t \to t+s}^{-1} D_{t+s},$$

where $\mathcal{R}_{t \to t+s} = \mathcal{R}_t \cdots \mathcal{R}_{t+s-1}$ is the cumulative discount rate between dates t and $t+s$.

We now make one strong maintained assumption, namely that dividends grow at a constant rate:

$$D_{t+s} = D_t G^s \tag{25}$$

Under this constant-growth assumption, the asset price is

$$P_t = D_t \sum_{s=1}^{\infty} \mathcal{R}_{t \to t+s}^{-1} G^s. \tag{26}$$

When discount rates are constant, $\mathcal{R}_t = \mathcal{R}$ for all t with $\mathcal{R} > G$, the asset price is given by

$$\overline{P}_t = D_t \sum_{s=1}^{\infty} \mathcal{R}^{-s} G^s \qquad \text{or} \qquad \overline{P}_t = D_t \times \overline{PD} \quad \text{with} \quad \overline{PD} = \frac{G}{\mathcal{R} - G}, \tag{27}$$

i.e., the price-dividend ratio is constant and the price grows at the same rate as dividends.[36]

[36] The derivation of the second expression for $\overline{P}_t$ in (27) is as follows:

$$\overline{P}_t = D_t \sum_{s=1}^{\infty} \mathcal{R}^{-s} G^s = D_t \frac{G}{\mathcal{R}} \sum_{s=0}^{\infty} \left(\frac{G}{\mathcal{R}}\right)^s = D_t \frac{G}{\mathcal{R}} \frac{1}{1 - G/\mathcal{R}} = D_t \frac{G}{\mathcal{R} - G}.$$

This is the orginal "Gordon growth model", studied in Gordon and Shapiro (1956).

In Proposition 1, we considered the effect of asset price changes dP_t but holding constant dividends (i.e., $dD_t = 0$). In the setup considered here, these can be interpreted as changes in discount rates $\mathcal{R}_{t\to t+s}$ which induce variation in P_t even with constant cash flows D_t.

In this environment with growing cash flows, how can we empirically isolate the effect of asset price changes but holding cashflows constant? The answer is to look at price-dividend ratios. In particular, from (26) the price-dividend ratio $PD_t = P_t/D_t$ satisfies

$$PD_t = \sum_{s=1}^{\infty} \mathcal{R}_{t\to t+s}^{-1} G^s. \tag{28}$$

Hence, changes in the price-dividend ratio pick up price variation due to changes in discount rates (i.e., pure valuation effects).

In Section 2.3, in particular equation (17), we constructed price deviations as deviations of asset prices from a baseline with a constant price-dividend ratio: $\Delta P_t = P_t - \overline{PD} \times D_t$ or equivalently

$$\frac{\Delta P_t}{P_t} = \frac{PD_t - \overline{PD}}{PD_t}.$$

Using (28) and (26), we have

$$\frac{\Delta P_t}{P_t} = \frac{\sum_{s=1}^{\infty} \left(\mathcal{R}_{t\to t+s}^{-1} - \mathcal{R}^{-s}\right) G^s}{\sum_{s=1}^{\infty} \mathcal{R}_{t\to t+s}^{-1} G^s}. \tag{29}$$

Hence these deviations pick up asset price variation due to variation in discount rates $\mathcal{R}_{t\to t+s}$.

A.2.2 Stochastic Case

While our main results, in particular Proposition 1, assume a deterministic environment, we consider a stochastic environment in Section 2.4 and Appendix A.3.1. We therefore turn to the stochastic case, which also allows for a connection with the Campbell-Shiller decomposition.

As in the deterministic case in the previous section, assume that asset prices are determined by cashflows and discount rates $\{D_t, \mathcal{R}_t\}_{t=0}^{\infty}$ but now assume that these are random variables. Analogously to the previous section, the asset price equals the expected present discounted value of cashflows:

$$P_t = \frac{\mathbb{E}_t[D_{t+1} + P_{t+1}]}{\mathcal{R}_t} \quad \text{or} \quad P_t = \mathbb{E}_t\left[\sum_{s=1}^{\infty} \mathcal{R}_{t\to t+s}^{-1} D_{t+s}\right]. \tag{30}$$

In analogy to (25), we now assume constant *expected* dividend growth:

$$\mathbb{E}_t[D_{t+s}] = G^s D_t. \tag{31}$$

When the discount rate is constant, $\mathcal{R}_t = \mathcal{R}$ for all t with $\mathcal{R} > G$, substituting (31) into (30) yields (27), i.e. the same formula for the price and price-dividend ratio as in the deterministic case. This is the Gordon growth model in which price-dividend ratios are constant.

When the discount rate is stochastic, fluctuations in the discount rate $\mathcal{R}_t$ result in deviations from the Gordon growth model (i.e., the price-dividend ratio is no longer constant). Our specification of the price deviations (17) picks up the asset price variation due to this variation in discount rates (i.e., "discount rate shocks" in the language of Campbell and Shiller (1988)).

This point can be made more precise by means of a Campbell-Shiller approximation. To this end, denote by

$$R_{t+1} = \frac{D_{t+1} + P_{t+1}}{P_t}$$

the realized asset return. Note that $\mathcal{R}_t = \mathbb{E}_t[R_{t+1}]$ from (30). Denote by $p_t = \log D_t$ the logarithm of the asset price, by $d_t = \log D_t$ the logarithm of dividends, and by $r_t = \log R_t$ the logarithm of the realized return. Finally, replace the random growth assumption (31) by the (equivalent) assumption that the logarithm of dividends follows a random walk

$$d_t = d_{t-1} + g + u_t, \qquad \mathbb{E}_{t-1}[u_t] = 0. \tag{32}$$

The Campbell-Shiller approximation for the log asset price p_t is

$$p_t = \frac{\kappa}{1-\rho} + \underbrace{\sum_{s=0}^{\infty} \rho^s (1-\rho) d_{t+1+s}}_{\text{part due to cashflows}} - \underbrace{\sum_{s=0}^{\infty} \rho^s r_{t+1+s}}_{\text{part due to discount rates}}, \tag{33}$$

For a derivation, see for example Campbell (2018, Section 5.3.1). This is a log-linear approximation of the asset price in terms of realized future returns and dividends.

This approximation can also be written in terms of expected returns (or discount rates) rather than realized returns. To this end denote by $\mathtt{r}_t = \mathbb{E}_t[r_{t+1}]$ the expected log return. Taking expectations in (33), we have

$$p_t = \frac{\kappa}{1-\rho} + \underbrace{\mathbb{E}_t \left[\sum_{s=0}^{\infty} \rho^s (1-\rho) d_{t+1+s} \right]}_{\text{part due to cashflows}} - \underbrace{\mathbb{E}_t \left[\sum_{s=0}^{\infty} \rho^s \mathtt{r}_{t+s} \right]}_{\text{part due to discount rates}} \tag{34}$$

Finally, using the assumption that dividends follow (32), one can show that

$$p_t = \frac{\kappa + g}{1-\rho} + \underbrace{d_t}_{\text{part due to cashflows}} - \underbrace{\mathbb{E}_t \left[\sum_{s=0}^{\infty} \rho^s \mathtt{r}_{t+s} \right]}_{\text{part due to discount rates}}.$$

This is the stochastic analogue to expression (26) in the deterministic case. Just like there, changes in the price-dividend ratio pick up variation in the asset price due to variation in discount rates. Therefore, under some assumptions, price deviations relative to a world with constant price-dividend ratios (as in equation (17)) pick up changes in discount rates (i.e., pure valuation effects without changing cashflows).

A.3 Model Extensions

A.3.1 Stochastic environment

We now consider an extension of the baseline model in which asset prices and dividends are stochastic. We show that, in this case, our expression for welfare gains can be interpreted as a small noise expansion around a deterministic economy. The result is an application of the envelope theorem for ex-post welfare in general dynamic stochastic optimization problems in Appendix A.4 and we closely follow the general strategy developed there.

Stochastic processes. For simplicity, we consider a two-asset version of the baseline model. We assume that prices and dividends are given by

$$p_t = \overline{p}_t + u_t, \quad q_t = \overline{q}_t + v_t, \quad d_t = \overline{d}_t + w_t. \tag{35}$$

where p_t is the log of the price of the long-lived asset, q_t is the log of the price of the liquid asset, and d_t is the log of dividends. The sequences $\{\overline{p}_t, \overline{q}_t, \overline{d}_t\}_{t=0}^{\infty}$ are deterministic. The sequences $\{u_t, v_t, w_t\}_{t=0}^{\infty}$ are stochastic. Note that the joint process for $\{p_t, q_t, d_t\}_{t=0}^{\infty}$ is a special case of the general stochastic process (43) in Appendix A.4.

Household problem. The household problem is

$$V = \max_{\{C_t, N_t, B_t\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t) \right], \tag{36}$$

subject to budget constraints at each period $t \geq 0$

$$C_t + (N_t - N_{t-1})P_t + B_t Q_t + \chi(N_t - N_{t-1}) = Y_t + N_{t-1}D_t + B_{t-1}, \tag{37}$$

and the stochastic processes for $\{p_t, q_t, d_t\}_{t=0}^{\infty}$. Note that this problem is a special case of the general stochastic problem (41) in Appendix A.4. As there, denote variables in the deterministic case with bars, e.g. $\overline{V}$.

Ex-post welfare Define ex-post welfare (a random variable) as

$$W = \sum_{t=0}^{\infty} \beta^t U(C_t).$$

Note that this is a special case of ex-post welfare in the general model (42) in Appendix A.4.

Welfare gain. We define welfare gains analogously as in the baseline model:

$$\text{Welfare Gain} \equiv \frac{W - \overline{W}}{U'(C_0)}. \tag{38}$$

where $\overline{W}$ denotes ex-post welfare in the deterministic case. To compute this object we simply apply Proposition 8 in Appendix A.4, in particular the second version (47).

Proposition 3. *In the stochastic environment (36) and (37), the welfare gain is*

$$\textit{Welfare Gain} \simeq \sum_{t=0}^{\infty} R_{0\to t}^{-1} \Big((N_{t-1} - N_t) \, P_t u_t - B_t Q_t v_t \Big)$$

where $\simeq$ denotes a first-order approximation that holds for small noise.[37]

Note that this is exactly the same formula as in the baseline model, except that the sequence of perturbations $\{\mathrm{d}P_t, \mathrm{d}Q_t\}_{t=0}^{\infty}$ is replaced by a sequence of stochastic shocks $\{u_t, v_t\}_{t=0}^{\infty}$.

Proof of Proposition 3. The Proposition is a direct corollary of Proposition 8 in Appendix A.4. Proposition 8 provides two different approximations. Here, we use the second one. Applied to the current problem, that Proposition states the following. To obtain the effect of $\{u_t, v_t\}_{t=0}^{\infty}$ on ex-post welfare W, we can use the following two-step procedure:

1. Compute the derivative of welfare V in a deterministic version of the problem with respect to the deterministic sequences $\{\overline{p}_t, \overline{q}_t\}_{t=0}^{\infty}$.
2. Replace the deterministic deviations $\{\mathrm{d}\overline{p}_t, \mathrm{d}\overline{q}_t\}_{t=0}^{\infty}$ with the realizations of the stochastic variables $\{u_t, v_t\}_{t=0}^{\infty}$.

Step 1. This step is essentially the same as in Proposition 1. In particular, in the case of $K = 1$ long-lived assets, equation (15) implies

$$\mathrm{d}V = U'(C_0) \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{t-1} - N_t) \, \mathrm{d}P_t - B_t \, \mathrm{d}Q_t \right).$$

The one missing piece to complete step 1 is that (35) specified a process for the logarithm of asset prices. In the deterministic case $\sigma = 0$, (35) becomes $P_t = \exp(\overline{p}_t)$ and $Q_t = \exp(\overline{q}_t)$. Therefore $\mathrm{d}P_t = P_t \, \mathrm{d}\overline{p}_t$ and $\mathrm{d}Q_t = Q_t \, \mathrm{d}\overline{q}_t$. Plugging in, we obtain

$$\mathrm{d}V = U'(C_0) \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{t-1} - N_t) P_t \, \mathrm{d}\overline{p}_t - B_t Q_t \, \mathrm{d}\overline{q}_t \right).$$

Step 2. Replacing $\{\mathrm{d}\overline{p}_t, \mathrm{d}\overline{q}_t\}_{t=0}^{\infty}$ with $\{u_t, v_t\}_{t=0}^{\infty}$ to obtain the analogue of (47), we obtain

$$W - \overline{W} \simeq U'(C_0) \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{t-1} - N_t) P_t u_t - B_t Q_t v_t \right).$$

Using the definition of welfare gain in (38), this concludes the proof. □

[37] See Appendix A.4 for a formal definition.

A.3.2 Borrowing and collateral constraints

We now examine the welfare effect of price deviations in the presence of borrowing constraints. For simplicity, we consider a two-asset version of the baseline model:

$$V = \max_{\{C_t, N_t, B_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t),$$

subject to budget constraints at each period $t \geq 0$

$$C_t + (N_t - N_{t-1})P_t + B_t Q_t + \chi(N_t - N_{t-1}) = Y_t + B_{t-1} + N_{it-1}D_t,$$

and a borrowing constraint

$$B_t \geq -F(N_t, P_t).$$

The function F governs the nature of the borrowing constraint. For instance, if $F(N_t, P_t) = \underline{B}$, then the borrowing limit is exogenous and fixed over time. If instead $F(N_t, P_t) = \phi N_t P_t$, then the borrowing limit is a fraction ϕ of the value of assets owned (as in collateral constraints models).

Proposition 4. *In the presence of borrowing constraints, the welfare gain is*

$$\text{Welfare Gain} = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_t)}{U'(C_0)} \left((N_{t-1} - N_t)\, \mathrm{d}P_t - B_t\, \mathrm{d}Q_t \right) + \sum_{t=0}^{\infty} \frac{\mu_t}{U'(C_0)} F_P(N_t, P_t)\, \mathrm{d}P_t,$$

where $\mu_t \geq 0$ is the shadow value of relaxing the borrowing constraint.

Proof of Proposition 4. The Lagrangian associated with the household problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t U(C_t) + \sum_{t=0}^{\infty} \lambda_t \left(Y_t + N_{t-1}D_t + B_{t-1} - C_t - (N_t - N_{t-1})\, P_t - B_t Q_t - \chi(N_t - N_{t-1}) \right)$$
$$+ \sum_{t=0}^{\infty} \mu_t \left(F(N_t, P_t) + B_t \right)$$

The first-order conditions are

$$\beta^t U'(C_t) = \lambda_t \qquad (\partial \mathcal{L}/\partial C_t = 0)$$
$$\lambda_t \Big(\chi'(N_t - N_{t-1}) + P_t \Big) - \mu_t F_N(N_t, P_t) = \lambda_{t+1} \Big(D_{t+1} + \chi'(N_{t+1} - N_t) + P_{t+1} \Big) \qquad (\partial \mathcal{L}/\partial N_t = 0)$$
$$\lambda_t Q_t - \mu_t = \lambda_{t+1} \qquad (\partial \mathcal{L}/\partial B_t = 0)$$

Moreover, complementary slackness implies that $\mu_t > 0$ and $B_t = -F(N_t, P_t)$ or $\mu_t = 0$ and $B_t > -F(N_t, P_t)$ for any $t \geq 0$. Totally differentiating the welfare function using the Envelope theorem, we obtain

$$
\begin{aligned}
\mathrm{d}V &= \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}}{\partial P_t}\,\mathrm{d}P_t + \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}}{\partial Q_t}\,\mathrm{d}Q_t, \\
&= \sum_{t=0}^{\infty} \lambda_t \left((N_{t-1} - N_t)\,\mathrm{d}P_t - B_t\,\mathrm{d}Q_t \right) + \sum_{t=0}^{\infty} \mu_t F_P(N_t, P_t)\,\mathrm{d}P_t, \\
&= U'(C_0) \sum_{t=0}^{\infty} \frac{\beta^t U'(C_t)}{U'(C_0)} \left((N_{t-1} - N_t)\,\mathrm{d}P_t - B_t\,\mathrm{d}Q_t \right) + \sum_{t=0}^{\infty} \mu_t F_P(N_t, P_t)\,\mathrm{d}P_t.
\end{aligned}
$$

Using the definition of the welfare gain in (13) concludes the proof. □

Relative to the welfare gain formula in the baseline model (i.e., Equation 15), the formula differs along two dimensions. First, for constrained households, the Euler equation does not hold (i.e., $U'(C_t) > R_t \beta U'(C_{t+1})$). Therefore, the correct discount rate at time t in the welfare gain formula weakly exceeds $R_{0\to t}$.

Second, when the borrowing limit depends directly on asset prices (i.e., $F_P \neq 0$), there is an additional term that accounts for the effect of asset prices on the tightness of the borrowing constraint. For instance, if higher prices allow for more borrowing (as in collateral constraint models), then the second term is strictly positive for constrained households. For these households, higher asset prices relaxes their borrowing constraint and therefore improves their welfare.

A.3.3 Bequests

We now examine the welfare effect of price deviations in the presence of bequest. For simplicity, we consider a two-asset version of the baseline model:

$$
V^* = \max_{\{C_t, N_t, B_t, I_t^-\}} \sum_{t=0}^{\infty} \beta^t U(C_t) + \sum_{t=0}^{\infty} F(I_t^-, \{\{P_{k,t}\}_{k=1}^K, Q_t\}_{t=0}^{\infty}),
$$

subject to budget constraints at each period $t \geq 0$

$$
C_t + (N_t - N_{t-1})P_t + B_t Q_t + \chi(N_t - N_{t-1}) = Y_t + N_{t-1} D_t + B_{t-1} + (I_t^+ - I_t^-) P_t.
$$

Relative to the baseline model, there is an additional choice variable I_t^-, which is the quantity of bequest that the individual decides to give at period t, in units of the long-lived asset N_t. The variable I_t^+ denotes the quantity of bequest received by the household at period t, and is therefore not a choice variable.

The bequest function $F(\cdot)$ governs the "warm glow" utility that households receive from bequest. Note that it is allowed to depend on all prices, and therefore nests the altruistic model, where $F(\cdot)$ would correspond to the value function of the heirs. From now on, denote by $I_t \equiv I_t^+ - I_t^-$ the net inheritance received at time t.

To compute our welfare gain formula (i.e., Equation 15 in the baseline model), we want to exclude the "warm glow" utility associated with bequest and focus only on the utility change due to consumption. Otherwise, we would be double-counting the welfare effect of a bequest event (i.e., positive welfare effect for both the parents and children). The consumption value function is defined as in the baseline model (i.e., $V \equiv \sum_{t=0}^{\infty} \beta^t U(C_t)$)

Proposition 5. *In the presence of bequest, the welfare gain is*

$$\text{Welfare Gain} = U'(C_0) \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{t-1} - N_t + I_t) \, \mathrm{d}P_t - B_t \, \mathrm{d}Q_t \right) + U'(C_0) \sum_{t=0}^{\infty} R_{0\to t}^{-1} P_t \, \mathrm{d}I_t.$$

Proof of Proposition 5. Using the Lagrangian approach as in the baseline model, the first-order conditions are

$$\beta^t U'(C_t) = \lambda_t \qquad (\partial\mathcal{L}/\partial C_t = 0)$$

$$\lambda_t \Big(\chi'(N_t - N_{t-1}) + P_t \Big) = \lambda_{t+1} \Big(D_{t+1} + \chi'(N_{t+1} - N_t) + P_{t+1} \Big) \qquad (\partial\mathcal{L}/\partial N_t = 0)$$

$$\lambda_t Q_t = \lambda_{t+1} \qquad (\partial\mathcal{L}/\partial B_t = 0)$$

$$\lambda_t P_t = F_{I^-}(I_t^-, \{\{P_{k,t}\}_{k=1}^K, Q_t\}_{t=0}^{\infty}) \qquad (\partial\mathcal{L}/\partial I_t^- = 0)$$

Because I_t is not optimally chosen by the individual receiving the bequest, we cannot use the Envelope theorem as in the proof of Proposition 1. Instead, we totally differentiate the expression for welfare:

$$\begin{aligned}
\mathrm{d}V &= \sum_{t=0}^{\infty} \beta^t U'(C_t) \, \mathrm{d}C_t \\
&= \sum_{t=0}^{\infty} \lambda_t \left((N_{t-1} - N_t + I_t) \, \mathrm{d}P_t - B_t \, \mathrm{d}Q_t \right) + \sum_{t=0}^{\infty} \lambda_t P_t \, \mathrm{d}I_t. \\
&- \sum_{t=0}^{\infty} \lambda_t \left(\chi'(N_t - N_{t-1}) + P_t \right) \mathrm{d}N_t + \sum_{t=0}^{\infty} \lambda_t \Big(D_t + \chi'(N_t - N_{t-1}) + P_t \Big) \, \mathrm{d}N_{t-1} - \sum_{t=0}^{\infty} \lambda_t Q_t \, \mathrm{d}B_t + \sum_{t=0}^{\infty} \lambda_t \, \mathrm{d}B_{t-1} \\
&= U'(C_0) \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{t-1} - N_t + I_t) \, \mathrm{d}P_t - B_t \, \mathrm{d}Q_t \right) + U'(C_0) \sum_{t=0}^{\infty} R_{0\to t}^{-1} P_t \, \mathrm{d}I_t \\
&\quad - \sum_{t=0}^{\infty} \lambda_t \left(\chi'(N_t - N_{t-1}) + P_t \right) \mathrm{d}N_t + \sum_{t=0}^{\infty} \lambda_t \Big(D_t + \chi'(N_t - N_{t-1}) + P_t \Big) \, \mathrm{d}N_{t-1} \\
&\quad - \sum_{t=0}^{\infty} \lambda_t Q_t \, \mathrm{d}B_t + \sum_{t=0}^{\infty} \lambda_t \, \mathrm{d}B_{t-1} \\
&= U'(C_0) \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{t-1} - N_t + I_t) \, \mathrm{d}P_t + B_t \, \mathrm{d}Q_t \right) + U'(C_0) \sum_{t=0}^{\infty} R_{0\to t}^{-1} P_t \, \mathrm{d}I_t \\
&\quad + \sum_{t=0}^{\infty} \lambda_t \left(\chi'(N_t - N_{t-1}) + P_t \right) \mathrm{d}N_t + \sum_{t'=-1}^{\infty} \lambda_{t'+1} \Big(D_{t'+1} + \chi'(N_{t'+1} - N_{t'}) + P_{t'+1} \Big) \, \mathrm{d}N_{t'} \\
&\quad - \sum_{t=0}^{\infty} \lambda_t Q_t \, \mathrm{d}B_t + \sum_{t'=-1}^{\infty} \lambda_{t'+1} \, \mathrm{d}B_{t'} \\
&= U'(C_0) \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{t-1} - N_{t-1} + I_t) \, \mathrm{d}P_t - B_t \, \mathrm{d}Q_t \right) + U'(C_0) \sum_{t=0}^{\infty} R_{0\to t}^{-1} P_t \, \mathrm{d}I_t.
\end{aligned}$$

The first equality comes from the definition of consumption welfare. The second equality uses the first-order condition for C_t combined with the budget constraint. The third equality uses a change of variables $t' \equiv t-1$. The fourth equality uses the first order conditions B_t and N_t, combined with $\mathrm{d}N_{-1} = \mathrm{d}B_{-1} = 0$. □

Note that the resulting welfare gain formula differs from the one in the baseline model along two dimensions. First, for the long-lived asset, the decrease in holdings $N_{t-1} - N_t$ is replaced by the net asset sales $N_{t-1} - N_t + I_t$. Second, there is an additional term that accounts for the response of bequest to changes in asset prices. Our baseline assumption is that $\mathrm{d}I_t = 0$, ie that the quantity of bequeathed assets doe not respond to asset prices.

A.3.4 General equilibrium

We now present an overlapping generation model in the spirit of Samuelson (1958b) with a single long-lived asset (i.e., a tree). The goal is to clarify the meaning of our welfare gain formula in a general equilibrium. We use the model to simulate a rise in asset prices due to either demand or supply shocks.

Environment. Consider an economy where at each year $t \geq -1$, a new cohort of measure one is born. Households in cohort t have a subjective discount factor $\beta_t \in (0,1)$ and are endowed with positive income Y_t when young and zero when old. The cohort born at $t=-1$ is endowed with a long-lived asset that pays a stream of positive dividends $\{D_t\}_{t\geq 0}$. Denote the (ex-dividend) price of the asset at time t as P_t and the one-period holding return on the asset by $R_{t+1} = \frac{D_{t+1}+P_{t+1}}{P_t}$. Denote N_t the share of the asset owned by cohort t at the end of period t.

Household problem. The problem of the young in period $t \geq 0$ is

$$\begin{aligned} V_t = \max_{C_t, C'_t, N_t} \; & (1-\beta_t)\log(C_t) + \beta_t \log(C'_t), \\ \text{s.t.} \quad & C_t + N_t P_t = Y_t \\ & C'_t = N_t(D_{t+1} + P_{t+1}) \end{aligned}$$

where C_t and C'_t denote respectively consumption when young and old. The Lagrangian is

$$\mathcal{L}_t = (1-\beta_t)\log(C_t) + \beta_t \log(C'_t) + \lambda_t\big(Y_t - C_t - N_t P_t\big) + \lambda'_t\big(N_t(D_{t+1}+P_{t+1}) - C'_t\big).$$

The optimal consumption level is a fixed fraction of labor income

$$C_t = (1-\beta_t)Y_t.$$

For the initial old, the solution is simply given by $C_{-1} = Y_{-1}$ and $C'_{-1} = N_{-1}(D_0 + P_0)$ and we define $\lambda'_{-1} = \beta_{-1}\frac{1}{C'_{-1}}$

Equilibrium. At every period $t \geq 0$, the asset market clearing condition is

$$N_t = N_{t-1} = 1,$$

which says that the purchases of the young N_t must equal the sales of the old N_{t-1}, which is equal to 1. Using the optimal consumption of the young combined with their budget constraint and the market clearing condition, we have that the equilibrium price is

$$P_t = \beta_t Y_t.$$

Therefore, P_t is affected by preferences β_t and endowment Y_t.

Demand shock. We now consider the effect of a MIT shock to the subjective discount factor of the young $\mathrm{d}\beta_0$. A rise in β_0 implies a rise in the desire to save, which increases the equilibrium price P_0. Hence, we interpret this exercise as simulating a "demand shock". The welfare gain (i.e., $\lambda_t^{-1}\,\mathrm{d}V_t$) for each cohort is

$$\begin{aligned} \text{Welfare Gain}_{-1} &= (N_{-1} - 0)\,\mathrm{d}P_0, \\ \text{Welfare Gain}_0 &= (0 - N_0)\,\mathrm{d}P_0 + \lambda_0^{-1}\log(C_0'/C_0)\,\mathrm{d}\beta_0. \end{aligned}$$

Since market clearing implies that $N_{-1} = N_0 = 1$, the aggregate welfare gain is

$$\sum_{i=-1,0} \text{Welfare Gain}_i = \underbrace{\lambda_0^{-1}\log(C_0'/C_0)\,\mathrm{d}\beta_0}_{\text{Direct effect of demand shock}} + \underbrace{(\mathrm{d}P_0 - \mathrm{d}P_0)}_{\text{Redistributive effect via asset prices}}$$

Note that the first term in the welfare gain formula is exactly as in the formula in the two-period model (6), which is equal to asset sales times price deviation. However, for the initial young, there is also a second term, which accounts for the direct effect of the preference shock on welfare. Summing the welfare gain across both young and old, we have that the aggregate welfare gain is nonzero in general. As in the baseline model, the effect of asset price changes is purely redistributive. The old (who are sellers) benefit from the rise in P_0 at the expense of the young (who are buyers). However, the direct effect of the change in β_0 is not zero. Its sign depends on the initial growth rate of within-cohort consumption.

Supply shock. We now consider the effect of a MIT shock to the endowment of the young $\mathrm{d}Y_0$. A rise in Y_0 implies a rise in the total amount of desired saving, which increases the equilibrium price P_0. Hence, we interpret this exercise as simulating a "supply shock". The welfare gain for each cohort is

$$\begin{aligned} \text{Welfare Gain}_{-1} &= (N_{-1} - 0)\,\mathrm{d}P_0, \\ \text{Welfare Gain}_0 &= (0 - N_0)\,\mathrm{d}P_0 + \mathrm{d}Y_0, \end{aligned}$$

The aggregate welfare gain is

$$\sum_{i=-1,0} \text{Welfare Gain}_i = \underbrace{\mathrm{d}Y_0}_{\text{Direct effect of supply shock}} + \underbrace{(\mathrm{d}P_0 - \mathrm{d}P_0)}_{\text{Redistributive effect via asset prices}}$$

The redistributive effect is exactly as for the demand shock. The direct effect for the currently young is unambiguously positive.

A.3.5 Government

We now examine the welfare effect of price deviations in the presence of government transfers. For simplicity, we consider a two-asset version of the baseline model. Suppose that the government makes targeted transfers to households $i \in \{1, \ldots, I\}$, where T_{it} denotes the net amount of resources transfered from the government to household i at time t. The household problem is now given by

$$V_i = \max_{\{C_{it}, N_{it}, B_{it}\}} \sum_{t=0}^{\infty} \beta^t U(C_{it}),$$

subject to budget constraints at each period $t \geq 0$

$$C_{it} + (N_{it} - N_{it-1})P_t + B_{it}Q_t + \chi(N_{it} - N_{it-1}) = Y_{it} + T_{it} + N_{it-1}D_t + B_{it-1},$$

We assume that the government can trade both assets and thus faces, at each period $t \geq 0$, the following budget constraint:

$$(N_{Gt} - N_{Gt-1})P_t + B_{Gt}Q_t = N_{Gt-1}D_t + B_{Gt-1} - \sum_{i=1}^{I} T_{it} - \chi(N_{Gt} - N_{Gt-1}). \tag{39}$$

We do not fully specify the government problem, but we assume that the government's portfolio choice satisfies the following cost-minimization condition

$$Q_t^{-1} = \frac{D_{t+1} + P_{t+1} - \chi'(N_{t+1} - N_t)}{P_t + \chi'(N_t - N_{t-1})}, \tag{40}$$

at every $t \geq 0$. The idea is that the government minimizes the cost of borrowing (or alternatively maximizes the return on saving) by adjusting portfolio shares until the marginal return on the long-lived asset (net of adjustment costs) is equalized with the bond return.

The following proposition characterizes the welfare gain in the presence of government transfers.

Proposition 6. *In the presence of government transfers, the individual welfare gain of household i is*

$$\text{Welfare Gain}_i = \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{it-1} - N_{it})\, \mathrm{d}P_t - B_{it}\, \mathrm{d}Q_t\right) + \sum_{t=0}^{\infty} R_{0\to t}^{-1}\, \mathrm{d}T_{it}.$$

Moreover, the aggregate contribution of deviations in government transfers $\mathrm{d}T_{it}$ *to household welfare is*

$$\sum_{t=0}^{\infty} R_{0\to t}^{-1} \sum_{i=1}^{I} \mathrm{d}T_{it} = \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(\left(N_{Gt-1} - N_{Gt} \right) \mathrm{d}P_t - B_{Gt}\, \mathrm{d}Q_t \right).$$

Proof of Proposition 6. The welfare gain formula follows immediately from the Envelope theorem, as in the baseline model. This proof focuses on the second equation. Differentiating the government budget constraint (39), we obtain

$$\sum_{i=1}^{I} \mathrm{d}T_{it} = \left(N_{Gt-1} - N_{Gt} \right) \mathrm{d}P_t - B_{Gt}\, \mathrm{d}Q_t$$
$$- \left(\chi'(N_t - N_{t-1}) + P_t \right) \mathrm{d}N_{Gt} + \Big(D_t + \chi'(N_{Gt} - N_{Gt-1}) + P_t \Big) \mathrm{d}N_{Gt-1} - Q_t\, \mathrm{d}B_{Gt} + \mathrm{d}B_{Gt-1}.$$

The sum of aggregate net transfer deviations discounted using the liquid asset return is

$$\begin{aligned}
\sum_{t=0}^{\infty} R_{0\to t}^{-1} \sum_{i=1}^{I} \mathrm{d}T_{it} &= \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(\left(N_{Gt-1} - N_{Gt} \right) \mathrm{d}P_t - B_{Gt}\, \mathrm{d}Q_t \right) \\
&\quad - \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(\chi'(N_t - N_{t-1}) + P_t \right) \mathrm{d}N_{Gt} + \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(D_t + \chi'(N_{Gt} - N_{Gt-1}) + P_t \right) \mathrm{d}N_{Gt-1} \\
&\quad - \sum_{t=0}^{\infty} R_{0\to t}^{-1} Q_t\, \mathrm{d}B_{Gt} + \sum_{t=0}^{\infty} R_{0\to t}^{-1}\, \mathrm{d}B_{Gt-1} \\
&= \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(\left(N_{Gt-1} - N_{Gt} \right) \mathrm{d}P_t - B_{Gt}\, \mathrm{d}Q_t \right) \\
&\quad - \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(\chi'(N_t - N_{t-1}) + P_t \right) \mathrm{d}N_{Gt} \\
&\quad + \sum_{t'=-1}^{\infty} R_{0\to t'+1}^{-1} \left(D_{t'+1} + \chi'(N_{Gt'+1} - N_{Gt'}) + P_{t'+1} \right) \mathrm{d}N_{Gt'} \\
&\quad - \sum_{t=0}^{\infty} R_{0\to t}^{-1} Q_t\, \mathrm{d}B_{Gt} + \sum_{t'=-1}^{\infty} R_{0\to t'+1}^{-1}\, \mathrm{d}B_{Gt'} \\
&= \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(\left(N_{Gt-1} - N_{Gt} \right) \mathrm{d}P_t - B_{Gt}\, \mathrm{d}Q_t \right) \\
&\quad - \sum_{t=0}^{\infty} R_{0\to t}^{-1} \Big(\chi'(N_t - N_{t-1}) + P_t - Q_t \left(D_{t'+1} + \chi'(N_{Gt'+1} - N_{Gt'}) + P_{t'+1} \right) \Big) \mathrm{d}N_{Gt} \\
&\quad \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(-Q_t\, \mathrm{d}B_{Gt} + Q_t\, \mathrm{d}B_{Gt} \right) \\
&= \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(\left(N_{Gt-1} - N_{Gt} \right) \mathrm{d}P_t - B_{Gt}\, \mathrm{d}Q_t \right)
\end{aligned}$$

The second equality uses a change of variables $t' \equiv t - 1$. The third equality uses the fact that $R_{0\to t+1}^{-1} = R_{0\to t}^{-1} Q_t$ as well as $\mathrm{d}N_{G-1} = \mathrm{d}B_{G-1} = 0$. The fourth equality uses the cost-minimization assumption (40). □

The formula for the welfare gain of household i differs from the one in the baseline model since it includes the present-value of deviations in net government transfers. The reason is that the government might respond to a change in asset prices by adjusting net transfers. Moreover, the second part of Proposition 6 states that the discounted sum of aggregate net transfers to the household sector is equal to the "welfare gain of the government". Note that we obtain this result without making assumptions on the objective of the government. It is merely a consequence of the budget constraint of the government.

A.3.6 Housing and wealth in the utility function

We now examine the welfare effect of price deviations in the presence of "assets in the utility function" (i.e., joy of asset ownership). For simplicity, we consider a two-asset version of the baseline model:

$$V = \max_{\{C_t, N_t, B_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t, F(N_t, P_t)),$$

subject to budget constraints at each period $t \geq 0$

$$C_t + (N_t - N_{t-1})P_t + B_t Q_t + \chi(N_t - N_{t-1}) = Y_t + B_{t-1},$$

We assume that $U(\cdot, \cdot)$ is strictly increasing and concave in both arguments. The function F governs the sensitivity of flow utility to asset ownership. For instance, if $F(N_t, P_t) = \overline{F}$, then the model coincides with the baseline (i.e., assets ownership does not affect flow utility directly). If $F(N_t, P_t) = N_t$, then households value the quantity of assets that they own directly, but not their market value. This is the natural assumption in the case of housing. If $F(N_t, P_t) = P_t N_t$, then household value the market value of their wealth directly.

Proposition 7. *In the presence of assets in the utility function, the welfare gain is*

$$\text{Welfare Gain} = \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{t-1} - N_t) \, \mathrm{d}P_t - B_t \, \mathrm{d}Q_t \right) + \sum_{t=0}^{\infty} \beta^t \frac{U_F(C_t, F(N_t, P_t))}{U_C(C_0, F(N_0, P_0))} F_P(N_t, P_t) \, \mathrm{d}P_t$$

Proof of Proposition 7. The Lagrangian associated with the household problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t U(C_t, F(N_t, P_t)) + \sum_{t=0}^{\infty} \lambda_t \left(Y_t + N_{t-1} D_t + B_{t-1} - C_t - (N_t - N_{t-1}) P_t - B_t Q_t - \chi(N_t - N_{t-1}) \right)$$

The first-order conditions are

$$\beta^t U_C(C_t, F_t) = \lambda_t \qquad (\partial \mathcal{L}/\partial C_t = 0)$$

$$\lambda_t \Big(\chi'(N_t - N_{t-1}) + P_t \Big) = \beta^t U_F(C_t, F_t) F_N(N_t, P_t) + \lambda_{t+1} \Big(D_{t+1} + \chi'(N_{t+1} - N_t) + P_{t+1} \Big) \qquad (\partial \mathcal{L}/\partial N_t = 0)$$

$$\lambda_t Q_t - \mu_t = \lambda_{t+1} \qquad (\partial \mathcal{L}/\partial B_t = 0)$$

Totally differentiating the welfare function using the Envelope theorem, we obtain

$$\begin{aligned}
\mathrm{d}V &= \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}}{\partial P_t} \mathrm{d}P_t + \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}}{\partial Q_t} \mathrm{d}Q_t, \\
&= \sum_{t=0}^{\infty} \lambda_t \left(-\left(N_t - N_{t-1}\right) \mathrm{d}P_t - B_t \,\mathrm{d}Q_t \right) + \sum_{t=0}^{\infty} \beta^t U_F(C_t, F(N_t, P_t)) F_P(N_t, P_t) \,\mathrm{d}P_t \\
&= U(C_0, F(N_0, P_0)) \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{t-1} - N_t) \,\mathrm{d}P_t - B_t \,\mathrm{d}Q_t \right) + \sum_{t=0}^{\infty} \beta^t U_F(C_t, F(N_t, P_t)) F_P(N_t, P_t) \,\mathrm{d}P_t
\end{aligned}$$

□

Relative to the welfare gain formula in the baseline model (i.e., Equation 15), the formula has an additional term, which accounts for the direct effect of price deviations on utility. Note that, when flow utility only depends on the quantity of assets, not their market value (i.e., $F_P = 0$), the welfare gain formula coincides with the formula in the baseline model.

A.3.7 Duration mismatch

Auclert (2019) examines the effect of a one-time perturbation in the path of interest rates on consumption and welfare. We now discuss how this result relates to our Proposition 1. Consider an economy where, at time $t = 0$, households can trade bonds of all maturities. Denote Q_h the price of the bond with maturity $h \geq 1$. That is, the long-term interest rate between 0 and h is $R_{0\to h} = 1/Q_h$.

As in the baseline model, the household receives labor income Y_t at time t and they initially own N_{-1} shares of a long lived asset that pays a sequence of dividends $\{D_t\}_{t=0}^{\infty}$. The household chooses consumption and holdings to maximize utility

$$V = \max_{\{C_t, N_t, B_t\}_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t U(C_t),$$

with the following sequence of budget constraints

$$\begin{aligned}
C_0 + \sum_{h=1}^{\infty} B_h Q_{h0} &= N_{-1} D_0 + Y_0 && \text{for } t = 0, \\
C_t &= N_{-1} D_t + B_h + Y_t && \text{for } t \geq 1,
\end{aligned}$$

where B_h denotes the number of bonds with maturity h bought at time $t = 0$. Proposition 1 states that the welfare effect of a perturbation in the price of bonds with different maturities depends on transactions:

$$\begin{aligned}\text{Welfare Gain} &= -\sum_{h=1}^{\infty} B_h \, \mathrm{d}Q_{h0} \\ &= \sum_{h=1}^{\infty} (N_{-1}D_h + Y_h - C_h) \, \mathrm{d}Q_{h0} \\ &= \sum_{h=1}^{\infty} R_{0\to h}^{-1} (C_h - Y_h - N_{-1}D_h) \, \mathrm{d}\log R_{0\to h}.\end{aligned}$$

This corresponds to Appendix formula (A.37) in Auclert (2019).

In the special case in which the perturbation is a level shift in the yield curve (i.e., $\mathrm{d}\log R_{0\to h} = h\,\mathrm{d}\log R$ for $h > 1$), the formula simplifies to

$$\text{Welfare Gain} = \left(\sum_{h=1}^{\infty} R_{0\to h}^{-1} (C_h - Y_h - N_{-1}D_h) h \right) \mathrm{d}\log R.$$

This formula expresses the welfare gain of a permanent rise in interest rate on welfare, as a share of total wealth, as the difference between the duration of consumption and the duration of income, where "duration" is defined as the value-weighted time to maturity of a sequence of cash flows (see Greenwald, Leombroni, Lustig and Van Nieuwerburgh, 2021).[38]

A.4 An envelope theorem for ex-post welfare in stochastic environments

Appendix A.3.1 considers an extension of the baseline model in which asset prices and dividends are stochastic. Its main result is that our expression for welfare gains can be interpreted as a small noise expansion around a deterministic environment. This is, in fact, a more general result that holds in general stochastic dynamic optimization problems. This appendix spells out this more general result.

General environment. Consider a general stochastic dynamic optimization problem with state vector $x_{t-1} \in \mathbb{R}^n$ and period-return function $F(x_{t-1}, x_t, z_t)$ where $z_t \in \mathbb{R}^m$ is a vector of random variables. The timing is as follows: z_t is realized before choosing x_t and z_{t+1} is realized afterwards. For future reference, we denote the partial derivatives of F with respect to its first, second and third arguments by F_1, F_2 and F_3.

Ex-ante welfare is

$$V = \max_{\{x_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t F(x_t, x_{t-1}, z_t) \right]. \tag{41}$$

Denote the policy function by $x_t = g(x_{t-1}, z_t)$. Ex-post welfare is given by

$$W = \sum_{t=0}^{\infty} \beta^t F(x_t, x_{t-1}, z_t), \tag{42}$$

[38] More specifically, duration of consumption is $\sum_{h=1}^{\infty} R_{0\to h}^{-1} \frac{C_h}{\sum_{h=1}^{\infty} R_{0\to h}^{-1} C_h} h$ while the duration of income is $\sum_{h=1}^{\infty} R_{0\to h}^{-1} \frac{Y_h + N_{-1}D_h}{\sum_{h=1}^{\infty} R_{0\to h}^{-1} (Y_h + N_{-1}D_h)} h$.

where $\{z_t\}_{t=0}^{\infty}$ is a particular realized sequence of the random variable and where $x_t = g(x_{t-1}, z_t)$ and therefore the x_t's depend on the entire history of realized z_t's. Therefore, ex-post welfare depends on the particular realization of z_t's, i.e. $W = W(\{z_t\}_{t=0}^{\infty})$ is itself a random variable.

In what follows, we will approximate ex-post welfare around a deterministic setup. It will therefore be useful to split z_t into a deterministic component and a stochastic one:

$$z_t = \bar{z}_t + \varepsilon_t, \tag{43}$$

where the sequence $\{\bar{z}_t\}_{t=0}^{\infty}$ is deterministic, and where the sequence $\{\varepsilon_t\}_{t=0}^{\infty}$ is stochastic. In the setup in Appendix A.3.1, the deterministic component $\bar{z}_t$ is the vector of deterministic prices $\{\bar{z}_t\}_{t=0}^{\infty} = \{\bar{p}_t, \bar{q}_t\}_{t=0}^{\infty}$ and the stochastic component ε_t is the vector of shocks to prices $\{\varepsilon_t\}_{t=0}^{\infty} = \{u_t, v_t\}_{t=0}^{\infty}$.

Goal of the exercise. The goal is to derive an envelope theorem for ex-post welfare W. If the environment was fully deterministic (i.e., $\varepsilon_t \equiv 0$ for all t), things would be straightforward. In particular, ex-ante welfare would equal ex-post welfare and a change in the time path of $\bar{z}_t$ would generate a welfare change

$$\mathrm{d}V = \mathrm{d}W = \sum_{t=0}^{\infty} \beta^t F_3(\bar{x}_t, \bar{x}_{t-1}, \bar{z}_t)\, \mathrm{d}\bar{z}_t, \tag{44}$$

where we denote the sequence of endogenous state variables in the deterministic case by $\{\bar{x}_t\}_{t=0}^{\infty}$. We show below that, even in the stochastic environment, a very similar expression can be interpreted as a small noise expansion around a deterministic environment.

Perturbed model. To define the concept of welfare gain in the stochastic environment, we need to define the concept of "perturbed environment". Consider the same model as before, but where the stochastic processes are:

$$z_t(\sigma) = \bar{z}_t + \sigma\varepsilon_t, \tag{45}$$

where σ is a perturbation parameter. Note that $\sigma = 0$ corresponds to the deterministic environment while $\sigma = 1$ corresponds to the stochastic environment. From now on, we denote objects in the perturbed model by $y_t(\sigma)$, with the convention that $y_t = y_t(1)$ and $\bar{y}_t = y_t(0)$.

Envelope theorem for ex-post welfare. The goal is to derive expressions for the welfare change $W - \overline{W} = W(1) - W(0)$ (i.e., the change in ex-post welfare relative to the deterministic baseline resulting from a particular sequence of shock realizations). The strategy is to approximate ex-post welfare $W(\sigma)$ in the stochastic case $\sigma = 1$ around the deterministic case $\sigma = 0$. The approximations therefore have the interpretation of small-noise approximations.[39]

[39] Denoting the stochastic case by $\sigma = 1$ and stating that "$\sigma = 1$ is small" is simply a normalization. The key is that the "noise" $\sigma\varepsilon_t$ in (45) is "small." An alternative notation fixes the variability of ε_t—e.g., normalize the variance of ε_t to one—and denotes the stochastic case by $W(\sigma)$. With this notation, our results hold for $\sigma \approx 0$.

Proposition 8. *In the general stochastic setup (41), consider the effect on ex-post welfare W defined in (42) of a particular realization of the random variables $\{\varepsilon_t\}_{t=0}^{\infty}$. The following two first-order approximations of W around the deterministic case $\sigma = 0$ (which we denote by $\simeq$) hold:*

1. *Denoting by $\{\overline{x}_t, \overline{z}_t\}_{t=0}^{\infty}$ the state sequences in the deterministic case $\sigma = 0$, we have*

$$W - \overline{W} \simeq \sum_{t=0}^{\infty} \beta^t F_3(\overline{x}_t, \overline{x}_{t-1}, \overline{z}_t)\varepsilon_t. \tag{46}$$

2. *Denoting by $\{x_t, z_t\}_{t=0}^{\infty}$ the realized state sequences in the stochastic case $\sigma = 1$, we have*

$$W - \overline{W} \simeq \sum_{t=0}^{\infty} \beta^t F_3(x_t, x_{t-1}, z_t)\varepsilon_t. \tag{47}$$

In both versions, the effect of $\{\varepsilon_t\}_{t=0}^{\infty}$ on ex-post welfare is simply given by formula (44) from the deterministic case, except that the sequence of perturbations $\{\mathrm{d}\overline{z}_t\}_{t=0}^{\infty}$ is replaced by a sequence of stochastic shocks $\{\varepsilon_t\}_{t=0}^{\infty}$.

Proposition 8 states the following. Consider any general stochastic dynamic optimization problem of the form (41). Then we can use the following two-step procedure to approximate the effect of $\{\varepsilon_t\}_{t=0}^{\infty}$ on ex-post welfare W:

1. Compute the derivative of welfare V in a deterministic version of the problem with respect to the deterministic sequence $\{\overline{z}_t\}_{t=0}^{\infty}$ as in equation 44.
2. Replace the deterministic deviations $\{\mathrm{d}\overline{z}_t\}_{t=0}^{\infty}$ with the realizations of the stochastic variables $\{\varepsilon_t\}_{t=0}^{\infty}$.

roposition 8 features two such approximations, namely (46) and (47). In empirical applications, an advantage of the second approximation (47) is that it is evaluated at the *realized* state sequences in the stochastic case $\{x_t, z_t\}_{t=0}^{\infty}$. In particular, in our main empirical application, this allows for the use of realized data on $\{x_t, z_t\}_{t=0}^{\infty}$ (in our case data on net asset purchases and prices) rather than hypothetical data $\{\overline{x}_t, \overline{z}_t\}_{t=0}^{\infty}$ in a counterfactual deterministic world.

Proof of Proposition 8. Both (46) and (47) approximate the random variable $W(1)$ with a first-order approximation of $W(\sigma)$ around $\sigma = 0$ evaluated at $\sigma = 1$. We use the symbol $\simeq$ to denote these approximations.

Proof of Part 1 of Proposition 8. We begin by deriving the first approximation (46). For general σ, a standard first-order approximation is

$$W(\sigma) \approx W(0) + W'(0)\sigma. \tag{48}$$

The key step is to compute $W'(0)$. Once this is done, we simply set $\sigma = 1$ in (48).

Before computing $W'(0)$, note that the first-order condition (Euler equation) in the perturbed problem is

$$\mathbb{E}_t \left[F_2(x_{t-1}(\sigma), x_t(\sigma), z_t(\sigma)) + \beta F_1(x_t(\sigma), x_{t+1}(\sigma), z_{t+1}(\sigma))\right] = 0, \tag{49}$$

where the expectation $\mathbb{E}_t$ is over realizations of ε_{t+1} (recall the timing convention: ε_t is known when choosing x_t, but ε_{t+1} is not). Importantly x_{t+1} depends on the realization of ε_{t+1}. Furthermore, note that the first-order condition (49) holds only in expectation rather than for each ε_{t+1} realization (this is what creates the difficulty).

In the deterministic limit $\sigma = 0$ the first-order condition becomes a standard deterministic one

$$F_2(\overline{x}_{t-1}, \overline{x}_t, \overline{z}_t) + \beta F_1(\overline{x}_t, \overline{x}_{t+1}, \overline{z}_{t+1}) = 0. \tag{50}$$

We now turn to ex-post welfare in the perturbed model $W(\sigma)$ and to computing its derivative $W'(\sigma)$. It is useful to write out ex-post welfare as:

$$W(\sigma) = F(x_{-1}, x_0, \overline{z}_0 + \sigma\varepsilon_0) + \beta F(x_0, x_1, \overline{z}_1 + \sigma\varepsilon_1) + \ldots + \beta^t F(x_{t-1}, x_t, \overline{z}_t + \sigma\varepsilon_t) + \ldots$$

and where we suppress the dependence of $\{x_t\}_{t=0}^{\infty}$ on σ for notational simplicity.

Differentiate this expression with respect to σ:

$$\begin{aligned}
W'(\sigma) &= F_3(x_{-1}, x_0, \overline{z}_0 + \sigma\varepsilon_0)\varepsilon_0 + F_2(x_{-1}, x_0, \overline{z}_0 + \sigma\varepsilon_0)x_0'(\sigma) + \\
&\qquad \beta\left[F_3(x_0, x_1, \overline{z}_1 + \sigma\varepsilon_1)\varepsilon_1 + F_2(x_0, x_1, \overline{z}_1 + \sigma\varepsilon_1)x_1'(\sigma) + F_1(x_0, x_1, \sigma\varepsilon_1)x_0'(\sigma)\right] + \ldots \\
&= F_3(x_{-1}, x_0, z_0)\varepsilon_0 + \beta F_3(x_0, x_1, z_1)\varepsilon_1 + \left[F_2(x_{-1}, x_0, z_0) + \beta F_1(x_0, x_1, z_1)\right] x_0'(\sigma) + \ldots \\
&= \sum_{t=0}^{\infty} \beta^t F_3(x_{t-1}, x_t, z_t)\varepsilon_t + \sum_{t=0}^{\infty} \beta^t \left[F_2(x_{t-1}, x_t, z_t) + \beta F_1(x_t, x_{t+1}, z_{t+1})\right] x_t'(\sigma).
\end{aligned}$$

The difficulty is that, for given realizations of uncertainty $\{\varepsilon_t\}_{t=0}^{\infty}$, it is generally *not* true that $F_2(x_{t-1}(\sigma), x_t(\sigma), z_t(\sigma)) + \beta F_1(x_t(\sigma), x_{t+1}(\sigma), z_{t+1}(\sigma)) = 0$. Instead, as already noted, the first-order condition (49) holds only in expectation. This is where perturbing around the deterministic case $\sigma = 0$ comes in. Setting $\sigma = 0$ and recalling the convention that $x_t(0) = \overline{x}_t$ and $z_t(0) = \overline{z}_t$ we have

$$\begin{aligned}
W'(0) &= \sum_{t=0}^{\infty} \beta^t F_3(\overline{x}_{t-1}, \overline{x}_t, \overline{z}_t)\varepsilon_t + \sum_{t=0}^{\infty} \beta^t \left[F_2(\overline{x}_{t-1}, \overline{x}_t, \overline{z}_t) + \beta F_1(\overline{x}_t, \overline{x}_{t+1}, \overline{z}_t)\right] x_t'(0) \\
&= \sum_{t=0}^{\infty} \beta^t F_3(\overline{x}_{t-1}, \overline{x}_t, \overline{z}_t)\varepsilon_t,
\end{aligned}$$

where the second equality uses the deterministic first-order condition (50). Plugging into (48) we have

$$W(\sigma) \approx W(0) + \sigma \sum_{t=0}^{\infty} \beta^t F_3(\overline{x}_{t-1}, \overline{x}_t, \overline{z}_t)\varepsilon_t. \tag{51}$$

Finally, we set $\sigma = 1$ and obtain (46) where, as already noted, the symbol $\simeq$ denotes the small-noise expansion to approximate $W(1)$ around $\sigma = 0$.

Proof of Part 2 of Proposition 8. We next derive the second approximation (47). This approximation makes use of Lemma 9 (see below). We start by rewriting (51) as

$$W(\sigma) \approx W(0) + \sigma \sum_{t=0}^{\infty} \beta^t F_3(x_{t-1}(0), x_t(0), z_t(0))\varepsilon_t. \tag{52}$$

Next, define the function ϕ as

$$\phi(\sigma) := \sum_{t=0}^{\infty} \beta^t F_3(x_{t-1}(\sigma), x_t(\sigma), z_t(\sigma)).$$

By (56) in Lemma 9, we have

$$\sum_{t=0}^{\infty} \beta^t F_3(x_{t-1}(\sigma), x_t(\sigma), z_t(\sigma))\sigma = \phi(\sigma)\sigma \approx \phi(0)\sigma = \sum_{t=0}^{\infty} \beta^t F_3(x_{t-1}(0), x_t(0), z_t(0))\sigma.$$

Substituting into (52) we have

$$W(\sigma) \approx W(0) + \sigma \sum_{t=0}^{\infty} \beta^t F_3(x_{t-1}(\sigma), x_t(\sigma), z_t(\sigma))\varepsilon_t. \tag{53}$$

Intuitively, (52) is not the only valid first-order approximation around the case $\sigma = 0$ and (53) is another one (in the same way as (54) and (55) in Lemma 9 are two valid approximations). Evaluating (53) at $\sigma = 1$ yields (47). This completes the proof. □

Lemma 9. *[Three useful approximations] For any function $\Phi(\sigma)$ that is continuously differentiable in a neighborhood of $\sigma = 0$, we have not only the standard first-order approximation around $\sigma = 0$*

$$\Phi(\sigma) = \Phi(0) + \Phi'(0)\sigma + \mathcal{O}(\sigma^2) \tag{54}$$

but also the alternative first-order approximation around $\sigma = 0$

$$\Phi(\sigma) = \Phi(0) + \Phi'(\sigma)\sigma + \mathcal{O}(\sigma^2). \tag{55}$$

Furthermore, defining $\phi(\sigma) := \Phi'(\sigma)$ and assuming that $\phi(0) = \Phi'(0)$ exists, from (54) and (55) we also have

$$\phi(\sigma)\sigma = \phi(0)\sigma + \mathcal{O}(\sigma^2). \tag{56}$$

Proof of Lemma 9. Equation (54) is simply the standard first-order Taylor-series approximation of Φ around $\sigma = 0$. Equation (54) is another valid first-order approximation that simply evaluates the tangent Φ' at an alternative point, namely at σ rather than at 0.[40] Finally, (56) follows directly from combining (54) and (55). □

[40] Assuming that Φ is twice continuously differentiable around $\sigma = 0$, the first Taylor-series approximation of $\Phi'(\sigma)$ around $\sigma = 0$ is $\Phi'(\sigma) = \Phi'(0) + \Phi''(0)\sigma + \mathcal{O}(\sigma^2)$. Combining with (54) yields

$$\Phi(\sigma) = \Phi(0) + \left(\Phi'(\sigma) - \Phi''(0)\sigma - \mathcal{O}(\sigma^2)\right)\sigma + \mathcal{O}(\sigma^2),$$

which simplifies to (55).

A.5 Welfare and wealth gains in multiperiods model

We now discus the relationship between welfare gains, as defined in (15), and wealth gains, as defined in (20). Consider a variation in prices $(\mathrm{d}P_{k,t})_{0\le t\le T}$. As in the two period model discussed in Section 2.1, we can decompose the associated welfare gain into two terms: one that depends on returns from 0 and T, and one that depends on the terminal return at $T+1$.

Proposition 10. *Consider a variation in prices $(\mathrm{d}P_{k,t})_{0\le t\le T}$. For an asset $1\le k\le K$, welfare gain equals wealth gains up to T adjusted for the effect of valuations on the terminal return at $T+1$:*[41]

$$\underbrace{\sum_{t=0}^{T} R_{0\to t}^{-1}\left(N_{k,t-1}-N_{k,t}\right)\mathrm{d}P_{k,t}}_{\text{Welfare gain}} = \underbrace{\sum_{t=0}^{T} R_{0\to t}^{-1} N_{k,t-1} P_{k,t-1}\,\mathrm{d}R_{k,t}}_{\text{Wealth gain}} + \underbrace{R_{0\to T+1}^{-1} N_{k,T} P_{k,T}\,\mathrm{d}R_{k,T+1}}_{\text{Effect of } T+1 \text{ return}}. \tag{57}$$

where $\{\mathrm{d}R_{k,t}\}_{t\ge 0}$ denotes the perturbation in asset returns corresponding to the perturbation in prices $\{\mathrm{d}P_t\}_{t\ge 0}$; that is,

$$\mathrm{d}R_{k,t} = \left(\mathrm{d}P_{k,t} - R_t\,\mathrm{d}P_{k,t-1}\right)/P_{k,t-1}. \tag{58}$$

Equation (57) generalizes Equation (8) from a two-period model to an infinite-horizon model. The key message is that, in a period of rising asset prices, wealth gains overestimate welfare gains because these rising valuations imply lower future returns on wealth. Finally, the proposition highlights a duality between sales interacted with price deviations (the left-hand-side) versus asset holdings interacted with return deviations (the right-hand side). As $T\to\infty$, welfare wealth gains $\sum_{t=0}^{\infty} R_{0\to t}^{-1}\left(N_{k,t-1}-N_{k,t}\right)\mathrm{d}P_{k,t} = \sum_{t=0}^{\infty} R_{0\to t}^{-1} N_{k,t-1} P_{k,t-1}\,\mathrm{d}R_{k,t}$. But when $T<\infty$, welfare gains differ from wealth gains.

Finally, as one can see in the proof, the proposition also holds true with non-infinitesimal differences in prices and returns (i.e., with $\Delta P_{k,t}$ and $\Delta R_{k,t}$ instead of $\mathrm{d}P_{k,t}$ and $\mathrm{d}R_{k,t}$).

Proof of Proposition 10. Using summation by part, welfare gains for asset k can be rewritten as:

$$\begin{aligned}\sum_{t=0}^{T} R_{0\to t}^{-1}\left(N_{k,t-1}-N_{k,t}\right)\mathrm{d}P_{k,t} &= \sum_{t=0}^{T} R_{0\to t}^{-1} N_{k,t-1}\left(\mathrm{d}P_{k,t} - R_t\,\mathrm{d}P_{k,t-1}\right) - R_{0\to T}^{-1} N_{k,T}\,\mathrm{d}P_{k,T} \\ &= \sum_{t=0}^{T} R_{0\to t}^{-1} N_{k,t-1} P_{k,t-1}\,\mathrm{d}R_{k,t} + R_{0\to T+1}^{-1} N_{k,T} P_{k,T}\,\mathrm{d}R_{k,T+1}.\end{aligned}$$

□

The relationship between welfare gains and wealth gains in equation (57) is represented graphically in Figure 17 for the case $T=3$. On the x-axis, the Figure plots a particular sequence of asset holdings $N_{k,t}$ for $t=0,1,2,3$ starting from $N_{k,-1}$. In this particular example, the sequence of holdings decreases over time, i.e. the household is a net seller. On the y-axis, the Figure plots price-deviation $R_{0\to t}^{-1}\,\mathrm{d}P_{k,t}$ for $t=0,1,2,3$. In this particular example, the sequence of price deviations increases over time, i.e. the in-sample deviation in returns are positive.

[41] Note that this equation is, in spirit, similar to the Campbell-Shiller formula. See also Knox and Vissing-Jorgensen (2021).

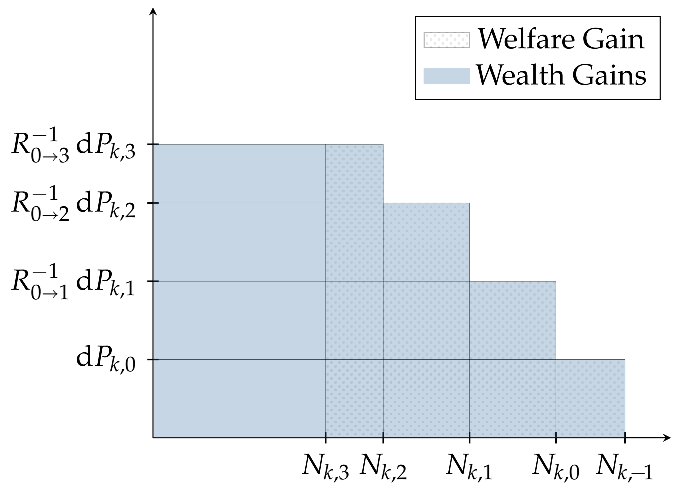


Figure 17: Geometric representation of welfare gains and wealth gains in Equation (57)

Notes. The figure graphically represents the relation between welfare and wealth gains in Equation (57) in an environment with increasing price deviations for the case $T = 3$. As explained in more detail in the text, welfare gains, $\sum_{t=0}^{T} R^{-1}_{0\to t}\,(N_{k,t-1} - N_{k,t})\,\mathrm{d}P_{k,t}$, equal the size of dotted area. In contrast, wealth gains, $\sum_{t=0}^{T} R^{-1}_{0\to t} N_{k,t-1} P_{k,t-1}\,\mathrm{d}R_{k,t}$, equal the size of the blue area. The figure shows that, in an environment with increasing price deviations, wealth gains (the blue area) exaggerate welfare gains (the dotted area).

This graph allows us to represent the relationship between welfare and wealth gains in (57) graphically. Welfare gains, $\sum_{t=0}^{T} R^{-1}_{0\to t}\,(N_{k,t-1} - N_{k,t})\,\mathrm{d}P_{k,t}$, equal the size of dotted area: the integral of the curve $(N_t, R^{-1}_{0\to t}\,\mathrm{d}P_{k,t})$ with respect to the x-axis. In contrast, wealth gains, $\sum_{t=0}^{T} R^{-1}_{0\to t} N_{k,t-1} P_{k,t-1}\,\mathrm{d}R_{k,t}$, equal the size of the blue area: the integral of the curve $(N_t, R^{-1}_{0\to t}\,\mathrm{d}P_{k,t})$ with respect to the y-axis. Finally, the difference between welfare and wealth gains, i.e. the last term in equation (57) $R^{-1}_{0\to 4} N_{k,3} P_{k,3}\,\mathrm{d}R_{k,4} = -R^{-1}_{0\to 3} N_{k,3}\,\mathrm{d}P_{k,3}$, is given by the difference between the two areas, i.e. the area of the rectangle starting at $(0,0)$ and with opposite corner $(N_{k,3}, R^{-1}_{0\to 3}\,\mathrm{d}P_{k,3})$.

Figure 17 shows clearly what we have already discussed above: in an environment with increasing price deviations, wealth gains (the blue area) exaggerate welfare gains (the dotted area).

Empirical implementation. We now detail how we construct wealth gains in the data. As for welfare gains, we are interested in pertubations around a constant price-dividend ratio path. In this case, Equation (20) gives:

$$\text{Wealth Gain} = \sum_{t=0}^{T} R^{-1}_{0\to t} \sum_{k=1}^{K} N_{k,t-1} P_{k,t-1} \times \left(\frac{P_{k,t}}{P_{k,t-1}} \frac{PD_{k,t} - \overline{PD}_k}{PD_{k,t}} - R_t \frac{PD_{k,t-1} - \overline{PD}_k}{PD_{k,t-1}} \right) - \sum_{t=0}^{T} R^{-1}_{0\to t} B_t Q_t \times \frac{\Delta Q_t}{Q_t}.$$

To implement it empirically, we estimate $P_{k,t}/P_{k,t-1}$ as $PD_{k,t}/PD_{k,t-1} \times G$, where $G = 1\%$, the growth rate of the Norwegian economy per capita, approximates the growth rate of dividends.

B Data Appendix

B.1 Additional figures

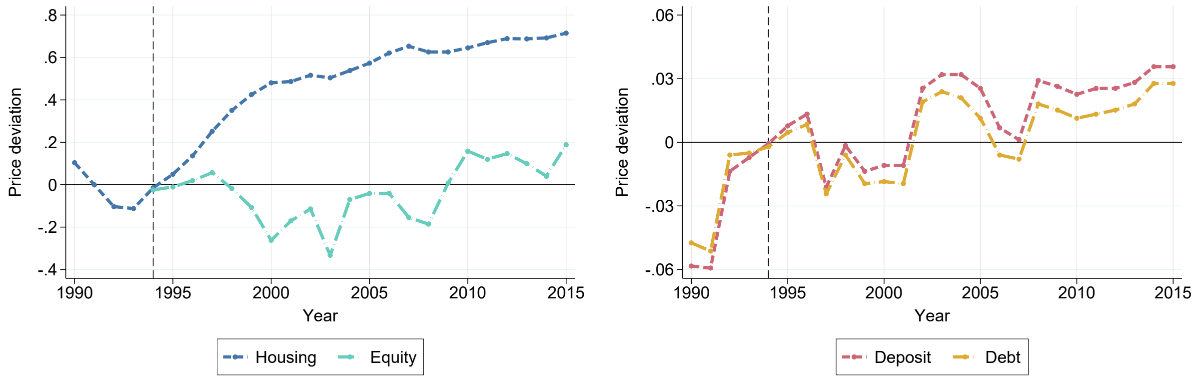


Figure 18: Price deviations

B.2 Imputing indirect holdings and transactions

Households who own firms are indirectly exposed to asset price changes through the asset holdings and transactions of the firms they own. We now describe how we impute indirect holdings and transactions.

Private businesses. Starting in 2005, we observe the balance sheet of private firms owned by households. We assign the holdings and transactions of financial assets by private firms to their ultimate owners, proportionally to their ownership share. To compute ownership shares, we divide the number of shares that households own (directly or indirectly through another private business firm) by the total number of shares outstanding in each specific firm.

Table 2 reports the average value of indirect holdings and transactions as a fraction of the tax assessed value of the equity in the firm over the 2005-2015 period. Private firms have, on average, positive net leverage (i.e., debt exceeds deposits). Moreover, private firms hold a significant amount of housing and stocks on their balance sheet. In terms of transactions, they are on average net buyers of stocks and net sellers of housing.

Table 2: Indirect holdings (private businesses, share of business value, 2005–2015 average)

Asset class	Holdings	Transactions
Deposits	0.453	–
Debt	1.231	–
Housing	0.722	-0.027
Stocks	0.172	0.008

From 1994 to 2004, we observe the tax assessed value of the equity that households have in private firms, but we do not observe the balance sheets of private firms. We therefore impute indirect holdings and transactions over this period by using the values in Table 2 multiplied by the tax assessed value of equity.

Public businesses. To impute indirect holdings and transactions due to ownership of publicly-traded stocks, we use data from the Financial Accounts (see Section B.3 for more details). Table 3 reports the holdings and transactions of financial assets by the corporate sector as a whole, expressed as a share of total equity outstanding, averaged over the 1996–2015 period. We impute indirect holdings and transactions over this period by using these values multiplied by equity holdings in public firms. One thing to note is that the corporate sector as a whole has a negative position in deposits. This is because deposits are a liability for private banks. Moreover, public firms do not transact houses (more on this shortly).

Table 3: Indirect holdings (corporations, share of equity outstanding, 1996–2015 average)

Asset class	Holdings	Transactions
Deposits	−0.266	–
Debt	0.209	–
Stocks	–	0.009

B.3 Financial accounts

Definitions. The *Financial Accounts* are produced by Statistics Norway and provide consistent measures of stocks and flows in financial markets. We use Table 10788, which provides annual data on (i) financial assets and liabilities by sector and (ii) financial transactions between sectors. We consider the following asset categories:

1. Deposits (22);
2. Loans and debt securities (30, 40);
3. Public equity shares (511);
4. Private equity shares (512);
5. Fund equity shares (520);
6. Other (10, 21, 519, 610–800).

The numbers in parentheses denote the line items from the Financial Accounts that we sum. The category "other" contains assets that are either quantitatively unimportant or illiquid). We consider the following sectors of the economy:

1. Government (121, 13, 15);
2. Households (14);

3. Non-financial corporations (11);
4. Financial corporations (122–129);
5. Foreigners (2).

The numbers in parentheses denote the sector codes from the Financial Accounts that we consolidate. Note that our definition of "Government" includes the Central Bank as well as the non-profit sector (i.e., institutions that serve the domestic household sector).

Incorporating housing transactions. Housing is a real asset rather than a financial asset, which means that it is not included in the Financial Accounts. For our analysis, we augment the Financial Accounts by aggregating the housing transaction registry data described in Section 3. Figure 19 plots the value of net housing purchases across sectors. Note that, unlike in the Financial Accounts, we can distinguish private firms from public firms. The key takeaway is that essentially all housing transactions are between households (or between households and private firms ultimately owned by households).

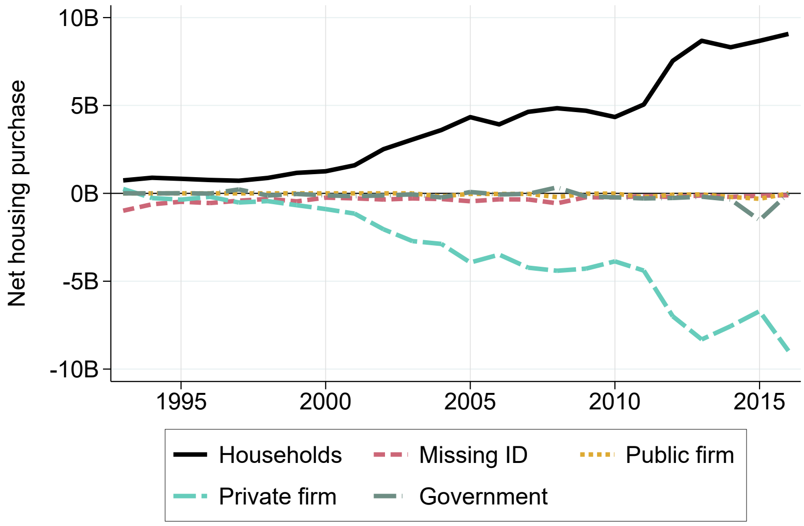


Figure 19: Aggregated housing registry data

To harmonize the sector definitions with the Financial Accounts, we combine add "Missing ID" and "Private firms" to the household sector. The underlying assumption is that private firms are entirely owned by the household sector (i.e., not owned by foreigners and the government).

Sectoral accounting identity. The subscript $j \in \{1, \ldots, J\}$ denotes a "sector", $k \in \{1, \ldots, J\}$ denotes a "counterparty sector", and $s \in \{1, \ldots, S\}$ denotes an asset class. Let A_{jks} denote the value of securities in asset class s held by sector j and issued by sector k. Similarly, let L_{jks} denote the value of securities in asset class s issued by sector k and held by sector j. The following identity holds:

$$A_{jks} = L_{kjs}. \tag{59}$$

In words, it means that every security is an asset for one sector and a liability for another sector. For instance, when a household holds an equity share issued by a business, it represents an asset for the household and a liability for the business.

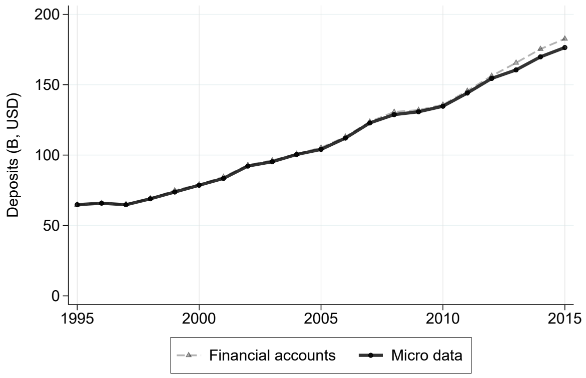
Consolidating the business sector. We now describe how we consolidate the business sector with its ultimate owner. Denote by $j \in \{G, H, Bnf, Bf, F\}$ the government, household, non-financial corporation, financial corporation, and foreign sectors. The consolidation process consists of adjusting measures of stocks and flows held by the sectors $\{G, H, F\}$ to account for their indirect holdings/transactions through their ownership of the sectors $\{Bnf, Bf\}$. Denote by $x_{js} \in \{A_{jks}, L_{jks}\}$ the adjusted value of assets/liabilities securities in asset class s held by sector j. We adjust the data according to

$$\widetilde{x}_{js} = \underbrace{x_{js}}_{\text{Directly held}} + \underbrace{\sum_{k \in \{Bnf, Bf\}} \omega_{jk} x_{ks}}_{\text{Indirectly held}}.$$

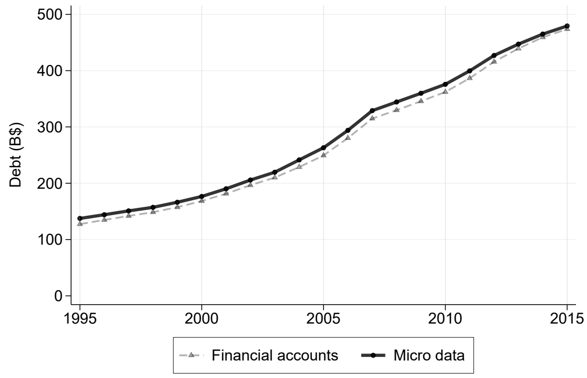
where $\omega_{jk} \in [0, 1]$ denotes the share of the equity issued by sector k that is held by sector j.

B.4 Microdata validation

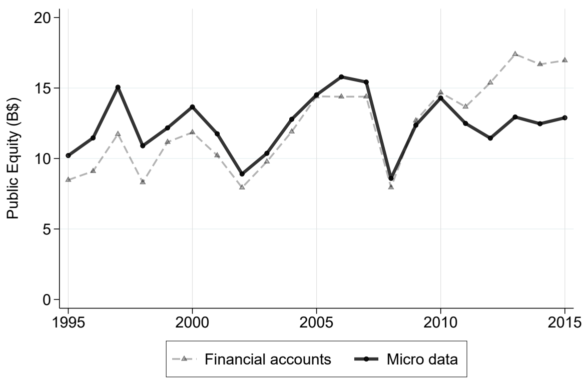
Figure 20 plots the aggregate value of household net assets for each asset category implied by both the Financial Accounts and the aggregate microdata. Overall, the microdata aligns closely with the Financial Account data. The only notable discrepancies are public equity which is higher in the micro data than in the National accounts after 2010, and mutual fund equity which is higher in the Financial Accounts than in our micro data throughout our sample period.



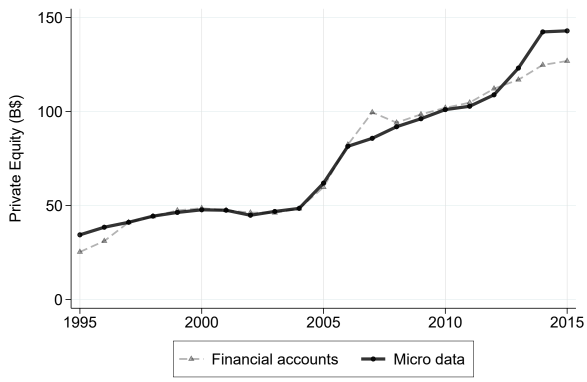
(a) Deposits



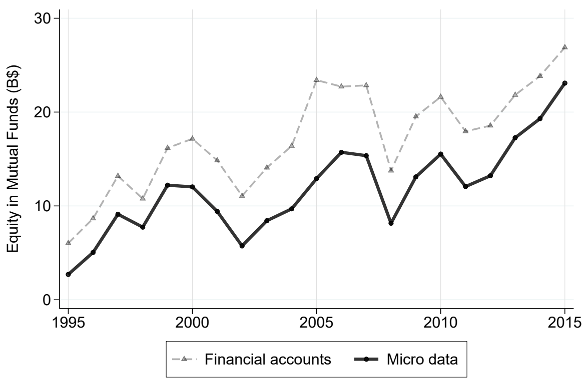
(b) Debt



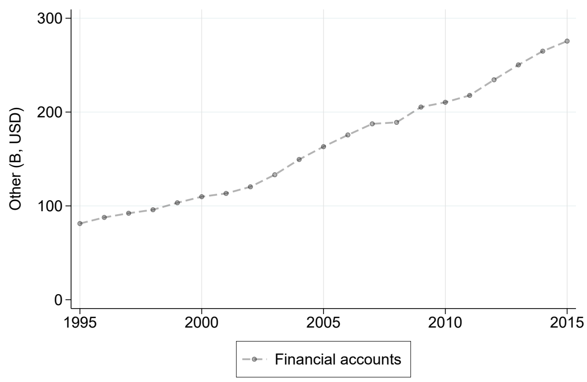
(c) Public equity



(d) Private equity



(e) Equity in mutual funds



(f) Other

Figure 20: Aggregated administrative microdata versus the Financial Accounts